Mehrdad MOHAMMADI, PhD Candidate
E-mail: mehrdadmohamadi@ut.ac.ir
Professor Jafar RAZMI, PhD
E-mail: jrazmi@ut.ac.ir
Professor Reza TAVAKKOLI-MOGHADDAM, PhD
School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran
E-mail: tavakoli@ut.ac.ir

MULTI-OBJECTIVE INVASIVE WEED OPTIMIZATION FOR STOCHASTIC GREEN HUB LOCATION ROUTING PROBLEM WITH SIMULTANEOUS PICK-UPS AND DELIVERIES

Abstract. *In this paper, we consider a variant of the Location-Routing Problem (LRP), namely the stochastic green hub LRP (SGHLRP). The SGHLRP seeks to minimize both total cost and total environmental effect by simultaneously locating the hubs, allocating non-hub nodes to located hubs and designing the vehicle routes between non-hub nodes at each hub considering stochastic travel times while capacity and time windows constraints are secured. We propose a multi-objective mixed integer linear programming formulations for the under study problem. Due to the complexity of the proposed problem, a hybrid multi-objective meta-heuristic algorithm, namely multi-objective invasive weed optimization (MOIWO), is applied for obtaining Pareto optimal solutions. Computational results show that the proposed algorithm outperforms the basic multi-objective algorithms in the literature such as NSGA-II, PAES and SPEA.*

Keywords*: Multi-objective optimization, Hub location problem, vehicle routing problem, Green transportation, Parallel simulated annealing.*

JEL classification: C44, C61, L91

1. Introduction

In supply chain and logistics management, the design of transportation networks is one of the most important problems because it offers a great potential to reduce costs and to improve service quality. For example, one of the most important aspects of designing a transportation network in postal delivery systems is the determination of the location of consolidation centers called hubs. The consolidation is carried using many transportation “lines” and this structure requires plenty of vehicles. Thus, in need of additional vehicles, the postal organizations work with subcontractor companies for delivery and pickup services. In order to reduce the number of vehicles operating in the system, it is decided to implement transportation routes instead of lines. On the other hand, in many applications arising in practice, the demand of each customer may be less than a truckload, in which, the transportation cost raised. So, in this paper, locating hubs and generating routes of allocated cities for each hub are considered. In the related literature, few of such an integrated approach for hub location and vehicle routing are studied. Hence, the aim of this paper is to design an alternative, much more effective postal delivery system in terms of transportation cost and environmental effects.

A location-routing problem (LRP) may be defined as a special case of Vehicle Routing Problem (VRP) in which the optimal number and location of depots are determined simultaneously with finding distribution routes. LRP is an NP-hard problem, as it contains two NP-hard problems (facility location and vehicle routing) (Nagy & Salhi, 2007). LRP applied in many industries such as: applications in food and drink distribution (Watson-Gandy & Dohrn, 1973), blood bank location (Or & Pierskalla, 1979), newspaper distribution (Jacobsen & Madsen, 1980), waste collection (Caballero et al., 2007), and medical evacuation (Chan et al., 2001).

There exists some review papers on LRP literature. These surveys include Balakrishnan et al. (1987), Laporte (1989), Min et al., (1998), and Nagy and Salhi (2007). Among these studies, (Nagy & Salhi, 2007) is stated as the most recent and comprehensive review. Interested readers are referred to these references in order to get more detailed knowledge of LRP. According to literature, it can be seen that fewer studies are focused on the LRPs in comparison with various vehicle routing or location problems. Some of more recent studies in LRPs are as follows:

- Alumur and Kara (2007) proposed a new model for the hazardous waste location-routing problem in which a multi-objective LRP for collection, transportation, treatment and disposal of hazardous material was studied. They solved a real-world sample with 92 generation nodes of their presented mixed integer programming model.
- Using a combination of Particle Swarm Optimization (PSO), Greedy Randomized Adaptive Search Procedure (GRASP), Expanding Neighborhood Search (ENS) and Path Relinking (PR), Marinakis and Marainaki (2008) solve LRP.
- Cappanera et al. (2004) presented an Obnoxious Facility Location-Routing (OFLR) problem in which Lagrangean Relaxation (LR) was presented to decompose the problem into two sub-problems of location and routing.
- Duhamel et al. (2010) solved a capacitated LRP using a combination of GRASP and Evolutionary Local Search (ELS).
- Schwardt and Fischer (2009) studied a single depot LRP and used Self-Organizing Map (SOM) approach to solve the problem.
- Also, several mathematical models and exact solution procedures have been developed for small-and medium-size LRPs in the literature (Labbe et al., 2004; Ambrosino and Scutella, 2005; Belenguer et al., 2006; Berger et al., 2007; Karaoglan et al., 2009; Yu et al., 2010; Karaoglan et al., 2012).
- Hassan-Pour et al. (2009) proposed a simulated annealing algorithm combined with genetic algorithm to solve the multi-objective multi-depot vehicle routing

problem. They presented a novel mathematical model for a stochastic location-routing problem that minimizes the facilities establishing cost and transportation cost, and maximizes the probability of delivery to customers.

- Tavakkoli-Moghaddam et al. (2010) presented a new integrated mathematical model for a bi-objective multi-depot location-routing problem where the total demand served is to be maximized and the total cost, consisting of start-up of the facility, fixed and variable depots and variable delivery cost, is to be minimized. The authors then proposed a multi-objective scatter search algorithm for solving their model.

Some of the contributions of this paper are as follows:

- All the above mentioned papers are considered classical objective functions such as minimizing the transportation cost, minimizing the total travel time in the network. To the best of our knowledge, no paper has considered the environmental effects of transportation network in their studies. In this paper, the vehicles' environmental effects are taken into account in the problem as a new objective function in which, the total travelling distance is minimized. Erdogan and Miller-Hooks (2012) studied the green vehicle routing problem in which, the location of fuel stations and the travelling route of vehicles are determined.
- In all the above-mentioned problems, the deterministic LRP cannot cover real-life applications when some features of the LRP are random or stochastic. In this paper, the travelling time of vehicles are considered as normal distribution. Also, the waiting time of vehicles at each customer's site is stochastic.
- To the best of author's knowledge, no paper has considered the LRP as a hub location structure considering stochastic travel time and environmental effects. Therefore, a new structure of LRP based hub location problem is presented in this paper.
- For solving the under hand SGHLRP, a hybrid multi-objective invasive weed optimization (MOIWO) are introduced for obtaining Pareto solutions.

The rest of the paper is organized as follows. Section 2 gives the mathematical formulation of the SGHLRP. Some assumptions of multi-objective problems and domination procedures are defined in Section 3. Section 4 contains the details of proposed MOIWO. The detailed implementation of MOIWO for solving the SGHLRP is described in Section 5. The computational results are reported in Section 6. The paper is concluded in Section 7.

2. Mathematical formulation of the SGHLRP

This section describes the mathematical formulation of the SGHLRP. In a logistic system assume that the number, location, and demand of customers, the number, and location of all potential hubs, as well as the vehicles type and capacity are given. The consolidation and distribution based on routing plan must be designed so that:

I. the demand (flow received from other nodes) of each customer must be satisfied,
II. each customer is served by exactly one vehicle,
III. the total demand on each route is less than or equal to the capacity of the vehicle assigned to that route, and
IV. each route begins and ends at the same hub.

The problem is to simultaneously determine the number, locations of hubs, assignment of customers to hubs, vehicle types to routes, and the corresponding delivery routes, so that the total costs consisting of hub-establishing cost, transportation cost, dispatching cost for vehicles as a first objective and environmental effects as a second objective are minimized. We first define some frequently used notations for the SGHLRP, before giving its formulation.

Sets:

I Set of all potential hub nodes

J Set of all customers (i.e., non-hub nodes)

K Set of all vehicles

Parameters:

N Number of customers (non-hub nodes)

$dC_{jj'}$ Distance between customer j and customer j'. $j, j' \in J$.

$CC_{jj'}$ Fixed transportation cost between customer j and customer j'. $j, j' \in J$.

$dH_{ii'}$ Distance between hub i and hub i'. $i, i' \in I$.

$CH_{ii'}$ Transportation cost between hub i and hub i'. $i, i' \in I$.

dCH_{ij} Distance between hub i and customer j. $i \in I, j \in J$.

$Flow_{jj'}$ Flow between customer j and customer j'. $j, j' \in J$.

FH_i Fixed costs of establishing hub i.

FV_k Fixed cost of using vehicle k.

$Chub_i$ Capacity of hub i.

Cap_k Capacity of vehicle (or route) k.

t_{ij} Travel time between nodes i and j. In this paper, t_{ij} is assumed to be a continues random variable with normal distribution, i.e., $t_{ij} \sim N(\mu_{ij}^t, {\sigma_{ij}^t}^2)$.

r_i Maximum collection/distribution distance between hub i and customers that are allocated to hub i (radius of hub i).

s_{jk} Service time for vehicle k at customer j; it is also a random variable with normal distribution, namely, $s_{jk} \sim N(\mu_{jk}^s, {\sigma_{jk}^s}^2)$.

$[e_j, l_j]$ Time window of the vehicle to serve customer $j \in J$. e_j and l_j denote the earliest and the latest time when customer j will permit the start of the service, respectively.

w_{jk} Waiting time for vehicle k at customer j; it is also a random variable with normal distribution, namely, $w_{jk} \sim N(\mu_{jk}^{w}, \sigma_{jk}^{w\,2})$.

$O_j = \sum_{j'} Flow_{jj'}$ Pick-up demand of customer j.

$D_j = \sum_{j'} Flow_{j'j}$ Delivery demand of customer j.

B_k A given upper bound of driver's duration of vehicle route k. the value of B is composed of travel and service times, and waiting times, etc.

Decision variables:

x_{ijk} 1, if point i immediately precedes point j on route k ($i, j \in I \cup J$); 0 otherwise.

y_i 1, if hub i is established; 0 otherwise.

z_{ij} 1, if customer j is allocated to hub i; 0 otherwise.

$q_{ii'}$ 1, if there is a link between hub i and i'.

$Y_{ii'}^{j}$ total amount of flow from customer j (origin) that is routed via hubs i and i'.

a_{jk} the arrival time of each vehicle k at customer j.

L_{ijk} Load of vehicle k after visiting point i, before visiting point j.

U_{ik} Auxiliary variable for sub-tour elimination constraints in route k.

According to above notations, the multi-objective mathematical formulation of the SGHLRP is as follow:

$$Min \sum_{i\in I} FH_i y_i + \sum_{k\in K} FV_k \sum_{i\in I}\sum_{j\in J} x_{ijk} + \sum_{j\in J}\sum_{j'\in J}\sum_{k\in K} CC_{jj'} x_{jj'k} + \sum_{j\in J}\sum_{i\in I}\sum_{i'\in I} CH_{ii'} Y_{ii'}^{j} \quad (1)$$

$$Min \sum_{j\in J}\sum_{j'\in J}\sum_{k\in K} dC_{jj'} x_{jj'k} + \sum_{i\in I}\sum_{i'\in I} dH_{ii'} q_{ii'} \quad (2)$$

s.t.

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1 \qquad \forall j \in J \quad (3)$$

$$\sum_{j\in I\cup J} x_{ijk} - \sum_{j\in I\cup J} x_{jik} = 0 \qquad \forall k \in K, i \in I \cup J \quad (4)$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \qquad \forall k \in K \quad (5)$$

$$\sum_{j\in J} O_j + D_j \;\; z_{ij} \le Chub_i y_i \qquad \forall i \in I \quad (6)$$

$$-z_{ij} + \sum_{u\in I\cup J} x_{iuk} + x_{ujk} \;\; \le 1 \qquad \forall i \in I, j \in J, k \in K \quad (7)$$

$$P\left\{\sum_{i\in I\cup J}\sum_{j\in I\cup J} t_{ij}x_{ijk} + \sum_{j\in J} s_{jk} + w_{jk} \sum_{i\in I\cup J} x_{ijk} \le B_k\right\} \ge \beta \qquad \forall k \in K \tag{8}$$

$$\sum_{k\in K}\sum_{j'\in I\cup J} x_{j'jk} \;\; a_{j'k} + t_{jj'} + s_{jk} + w_{jk} \;\; \le a_{jk} \qquad \forall j \in J/\{j'\} \tag{9}$$

$$\begin{cases} P\{e_j \le a_{jk} + w_{jk} \le l_j\} \ge \alpha \\ P\{a_{ik} + w_{ik} \le l_i\} = 1 \end{cases} \qquad \forall i \in I, j \in J, k \in K \tag{10}$$

$$w_{ik} = s_{ik} = 0 \qquad \forall i \in I, k \in K \tag{11}$$

$$\sum_{j\in J} L_{ijk} = \sum_{j\in I\cup J}\sum_{g\in J} x_{jgk} D_j \qquad \forall i \in I, k \in K \tag{12}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} L_{ijk} - D_j = \sum_{k\in K}\sum_{i\in I\cup J} L_{jik} - O_j \qquad \forall j \in I \cup J \tag{13}$$

$$0 \le L_{ijk} \le Cap_k \qquad \forall i, j \in I \cup J, k \in K \tag{14}$$

$$dCH_{ij} z_{ij} \le r_i \qquad \forall i \in I, j \in J \tag{15}$$

$$\sum_{i'\in I} Y_{ii'}^{j} + \sum_{j'\in J} Flow_{jj'} z_{jj'} = O_j z_{ij} + \sum_{i'\in I} Y_{i'i}^{j} \qquad \forall i \in I, j \in J \tag{16}$$

$$q_{ii'} \le y_i \qquad \forall i, i' \in I \tag{17}$$

$$q_{ii'} \le y_{i'} \qquad \forall i, i' \in I \tag{18}$$

$$y_i + y_{i'} - q_{ii'} \le 1 \qquad \forall i, i' \in I \tag{19}$$

$$Y_{ii'}^{j} \le M q_{ii'} \qquad \forall i, i' \in I, j \in J \tag{20}$$

$$z_{ij} \le y_i \qquad \forall i \in I, j \in J \tag{21}$$

$$U_{gk} - U_{jk} + N x_{gjk} \le N - 1 \qquad \forall g, j \in J, k \in K \tag{22}$$

$$x_{ijk}, y_i, z_{ij} \in \{0,1\} \qquad \forall i, j \in I \cup J, k \in K \tag{23}$$

$$Y_{ii'}^{j}, a_{jk}, U_{jk} \ge 0 \qquad \forall i, i' \in I, j \in J, k \in K \tag{24}$$

The objective function (1) minimizes the sum of the fixed hub-establishing cost, vehicle-using cost, collecting/distributing cost and transportation cost between hubs, respectively. The objective function (2) minimizes the total travelling distance that is considered as the green object for minimizing pollution emission from vehicles in terms of less travelling in the network by vehicles. Equations (3) require that each customer be assigned to a single route. Flow conservation constraints are expressed in (4). Constraints (5) assure that each route can be served at most once. Capacity constraints for the hubs are given in (6). Constraints (7) specify that a customer can be assigned to a hub only if there is a route from that hub going through that customer. Constraints (8) require that there is at least a probability of β that the route duration exceeding the threshold of the driver duration is at most B_k. Constraints (9)-(11) require that the probability of the arriving a vehicle during the customer's time window is at least α. The constraints in Equation (12) enforce that the load capacity of each vehicle departing from a hub is equal to its total delivery demand. The load-preservation constraints in Equation (13) express the proper movement of load delivery and/or pick-up. The capacity constraints in Equation (14) ensure that the vehicle load at any given time does not exceed the vehicle capacity. Constraints (15) make sure that node *j* can only be allocated to hub *i*, if the distance between *i* and *j* (dCH_{ij}) is at most the radius r_i. Equations (16) are the divergence equations for customer *j* assigned to hub *i* in a complete graph, when the demand and supply at the node is determined by the allocations z_{ij}. Constraints (17) to (19) linearize the product of $y_i \times y_{i'}$. Constraints (20) ensures that the value of $Y_{ii'}^{j}$ can be more than zero if nodes *i* and i' are the valid hubs. Constraints (21) assure that no customer is assigned to a location unless a hub is opened at that site. Equations (22) are the new sub-tour elimination constraint set. Constraints (23) and (24) are domain constraints.

To solve the SGHLRP, the traditional method is to convert the chance constraints into their corresponding deterministic equations. Under some special assumptions, the chance constraint programming of special SGHLRP can equivalently be transformed into a deterministic LRP. To illustrate this, we give the proposition, which holds for the LRP with time windows and stochastic travel, service and waiting times as follows:

Proposition 1. If travel, service and waiting times, t_{ij}, s_{jk} and w_{jk} are independent and normally distributed random variables, then chance constraints (8) has the deterministic equivalent as equation (25) and equation (10) has the deterministic equivalent as equations (26) and (27), simultaneously.

$$\phi^{-1}(\beta)\sqrt{\sum_{i\in I\cup J}\sum_{j\in I\cup J} V(t_{ij})x_{ijk} + \sum_{j\in J}(V(s_{jk})+V(w_{jk}))\sum_{i\in I\cup J} x_{ijk}} + \\ + \sum_{i\in I\cup J}\sum_{j\in I\cup J} E(t_{ij})x_{ijk} + \sum_{j\in J}(E(s_{jk})+E(w_{jk}))\sum_{i\in I\cup J} x_{ijk} \le B_k \quad (25)$$

$$\phi^{-1}(\alpha)\sqrt{V(a_{jk})+V(w_{jk})} + E(a_{jk}) + E(w_{jk}) \le L_j \quad (26)$$

$$\phi^{-1}(1-\alpha)\sqrt{V(a_{jk})+V(w_{jk})} + E(a_{jk}) + E(w_{jk}) \le e_j \quad (27)$$

where ϕ is the standardized normal distribution function and ϕ^{-1} is the inverse function of ϕ. $E(.)$ and $V(.)$ denote the expected and variance values, respectively.

Proof. We first define variable $\sum_i \sum_j t_{ij}x_{ijk} + \sum_j (s_{jk}+w_{jk})\sum_i x_{ijk} - B_k$. Because t_{ij}, s_{jk} and w_{jk} are independent and normally distributed random variables, variable *y* is also normally distributed, namely $y \sim N(E(y),V(y))$. The expected and variance value are calculated as follows:

$$E(y) = \sum_{i\in I\cup J}\sum_{j\in I\cup J} E(t_{ij})x_{ijk} + \sum_{j\in J}(E(s_{jk})+E(w_{jk}))\sum_{i\in I\cup J} x_{ijk} - B_k$$

$$V(y)=\sum_{i\in I\cup J}\sum_{j\in I\cup J}V(t_{ij})x_{ijk}+\sum_{j\in J}(V(s_{jk})+V(w_{jk}))\sum_{i\in I\cup J}x_{ijk}$$

so we have $\sum_i\sum_j t_{ij}x_{ijk}+\sum_j(s_{jk}+w_{jk})\sum_i x_{ijk}-B_k-E(y)\big/\sqrt{V(y)}\sim N(0,1)$.

Constraints $\sum_i\sum_j t_{ij}x_{ijk}+\sum_j(s_{jk}+w_{jk})\sum_i x_{ijk}\le B_k$ is equivalent to the following equation:

$$\frac{\sum_i\sum_j t_{ij}x_{ijk}+\sum_j(s_{jk}+w_{jk})\sum_i x_{ijk}-B_k-E(y)}{\sqrt{V(y)}}\le-\frac{E(y)}{\sqrt{V(y)}},$$

so the chance constraint is equivalent to $P\{\theta\le -E(y)\big/\sqrt{V(y)}\}\ge\beta$, where $\theta\sim N(0,1)$. So, the chance constraint holds if and only if $\phi^{-1}\ \beta\ \le -E(y)\big/\sqrt{V(y)}$, namely

$$\phi^{-1}(\beta)\sqrt{\sum_{i\in I\cup J}\sum_{j\in I\cup J}V(t_{ij})x_{ijk}+\sum_{j\in J}(V(s_{jk})+V(w_{jk}))\sum_{i\in I\cup J}x_{ijk}}+$$
$$+\sum_{i\in I\cup J}\sum_{j\in I\cup J}E(t_{ij})x_{ijk}+\sum_{j\in J}(E(s_{jk})+E(w_{jk}))\sum_{i\in I\cup J}x_{ijk}\le B_k$$

□

Based on Proposition 1, similar deterministic equivalents of chance constraints for time windows can also be derived as follows:

$$\phi^{-1}(\alpha)\sqrt{V(a_{jk})+V(w_{jk})}+E(a_{jk})+E(w_{jk})\le L_j$$

$$\phi^{-1}(1-\alpha)\sqrt{V(a_{jk})+V(w_{jk})}+E(a_{jk})+E(w_{jk})\le e_j$$

where $E(a_{jk})$ and $V(a_{jk})$ are iteratively computed as equations (28) and (29), respectively.

$$E(a_{jk})=\sum_{\substack{i\in I\cup J\\ i\ne j}}\ E(a_{ik})+E(t_{ij})+E(s_{jk})+E(w_{jk})\ \ x_{ijk} \tag{28}$$

$$V(a_{jk})=\sum_{\substack{i\in I\cup J\\ i\ne j}}\ V(a_{ik})+V(t_{ij})+V(s_{jk})+V(w_{jk})\ \ x_{ijk} \tag{29}$$

Also, in the equation (9), the parameters t_{ij}, s_{jk} and w_{jk} are replaced with their expected value.

3. Multi-Objective Problem-MOP

The main difference between single and multi-objective optimization problems is the number of the obtained optimal solutions. In a single-objective optimization problem, the decision maker (DM) is looking for one and only one optimal solution, while in multi-objective optimization problems, a set of solutions depending on non-dominance criterion are found that is named the Pareto solutions. In the following section, we provide a summary of some basic definitions of Pareto optimality and fuzzy domination to better understand the multi-objective invasive weed optimization. Kundu et al. (2010) has proposed some definitions as follows:

Definition 1 (*Pareto-optimality*). Consider $\vec{s} \in S$ be a sample solution vector.

(1) The solution vector $\vec{s}$ is determined to be non-dominated based on set $S^{'} \subseteq S$ if and only if there is no solution in $S^{'}$ which can dominate $\vec{s}$.

(2) The solution vector $\vec{s}$ is said to be Pareto-optimal if $\vec{s}$ is non-dominated in the whole solution space *S*.

Definition 2 (*Fuzzy k-Dominance by a Solution*). The mapping $\mu_k^{dom} = f_k(S) \to [0,1]$, where $k \in \{1,2,...,n\}$, defines a given uniform non-decreasing membership function. A solution $\vec{s} \in S$ is determined as *k*-dominance solution $\vec{p} \in S$, if $f_k(\vec{s}) < f_k(\vec{p})$. This relationship is represented as $\vec{s} \prec_k^F \vec{p}$. If $\vec{s} \prec_k^F \vec{p}$, the degree of fuzzy *k*-dominance is equal to $\mu_k^{dom}(f_k(\vec{p}) - f_k(\vec{s})) \equiv \mu_k^{dom}(\vec{s} \prec_k^F \vec{p})$.

Definition 3 (Fuzzy Dominance by a solution). Solution $\vec{s} \in \Psi$ is determined as fuzzy dominate solution $\vec{p} \in \Psi$ if and only if $\forall k \in \{1,2,...,n\}$, $\vec{s} \prec_k^F \vec{p}$. If $\vec{s} \prec_k^F \vec{p}$, the degree of fuzzy dominance $\mu_k^{dom}(\vec{s} \prec_k^F \vec{p})$ is calculated by computing the intersection of the fuzzy relationships $\vec{s} \prec_k^F \vec{p}$ for each *k*. Also, the fuzzy intersection operation, denoted with "$\bigcap$", is performed using a family of functions called *t*-norms and is calculated as:

$$\mu^{dom}(\vec{s} \prec_k^F \vec{p}) = \bigcap_{k=1}^{n} \mu_k^{dom}(\vec{s} \prec_k^F \vec{p})$$

Definition 4 (Fuzzy Dominance in a Population). Let *S* be a population of solutions. A solution $\vec{p} \in P$ is defined to be fuzzy dominated is *S* if it is fuzzy dominated by any other solution $\vec{s} \in P$. In this case, the degree of fuzzy dominance can be calculated by performing a union operation $\bigcup$ over every possible $\mu^{dom}(\vec{s} \prec_k^F \vec{p})$ implemented with *t*-conforms. Hence, the degree of fuzzy dominance of a solution $\vec{p} \in P$ in the set *S* is given by:

$$\mu^{dom}(S \prec^F \vec{p}) = \bigcup_{\vec{s} \in S} \mu^{dom}(\vec{s} \prec^F \vec{p})$$

4. Multi-objective IWO for SGHLRP

The invasive weed optimization (IWO) was first introduced by Mehrabian and Lucas (2006) in which, a numerical stochastic population-based meta-heuristic algorithm imitating the colonizing behavior of weeds is applied. In this section, we adapt the IWO for multi-objective SGHLRP by associated fuzzy dominance based sorting technique (Kundu et al., 2010) for obtaining Pareto optimal solutions. The steps of proposed MOIWO are described below in detail.

4.1. Solution representation and initialize a population

One important decision in designing a meta-heuristic algorithm is to decide how to represent and relate solutions in an efficient way to the search space. In this paper, a solution uses a string of number for its representation. In at hand SGHLRP, solution representation must determine the assigned customers to each vehicle, the hubs to be established and the sequence of customers to be served by a specific vehicle starting and ending at a hub. Considering c customers, v vehicles and h potential locations for being hubs, a solution representation is a $1 \times (c + 2v)$ matrix which includes three parts. The first two parts of the solution (first $c + v$ elements) are used together to decode the customers sequence and assigning to vehicles, on other words, the first part represents the customers sequencing and the second part shows the indices of customers to be served by a vehicle. Finally, the third v elements are considered as the hubs locations and are coded separately.

On the other hand, for applying the IWO for the SGHLRP, the value of solution representation must be real and randomly distributed of 0 to 1. For converting the real solution to understandable integer representation, we act according to the below three steps and each for one part of the solution:

1. *First part*: the first part is filled with real random value of 0 to 1. After that, the random values are sorted in ascending order and the returned array of indices shows the integer representation of customer sequencing so that the place of the biggest value includes number 1, 2 for the next biggest value, and so on for other random bits.
2. *Second part*: the second part is also filled with real random value of 0 to 1. After that, the random values are sorted in ascending order. Next, whole values are multiplied by c and then decimal values are round to bigger value. This procedure results in an understandable integer representation for second part.
3. *Third part*: similar to second part, all the actions are performed, but the values are multiplied by h.

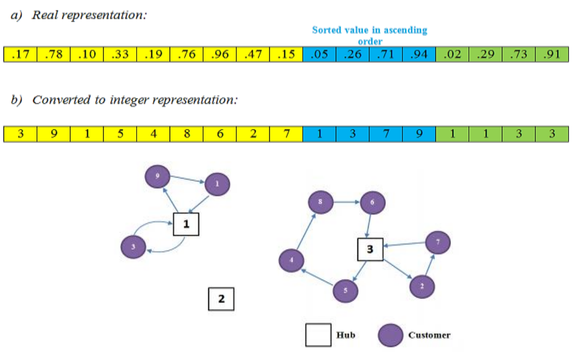

Figure 1. A sample solution

To clarify the encoding, a simple example of nine customers, four vehicles and three potential locations for a hub is illustrated in figure 1. According to figure 1, the first part of the solution shows that the order of customer to be served is [3 9

1 5 4 8 6 2 7]. The second part determines which customers are served by each vehicle, in which, the number of each bit represents the number of vehicle and the value of each bit represents the indices of customer to be served. On the other words, the customers with indices lying between the values of *i*th and (*i*+1)th bits in the second part are served by a vehicle *i*+1. It is easy to guess that the last bit of the second part must be equal to *c*, considering *c* customers. In addition, the values in the second part must be sorted in ascending from. Finally, for initializing the population, a limited number of weeds, called *PopSize*, are randomly created.

4.2. Reproduction

The weeds are allowed to produce other seeds depending on their fitness. Then, every seed grows to become a new plant (Weed). In this paper, the weed with minimum fitness function will produce maximum possible seed (S_{max}). Likewise the weed with maximum fitness function will produce minimum possible seed (S_{min}). The number of seeds associated with each weed is produced depending on the value of the fitness function and is generated using a linear function varying in a range between S_{min} and S_{max}. Figure 2 illustrates the relationship between the number of seed and value of fitness function.

The number of weeds in the each iteration is equal to *PopSize*. Each weed produces some seeds according to its rank in population based on the eligibility of fitness function. Therefore, the number of allowed seeds to be produced by each weed is calculated as follows:

$$seeds_i = floor\left(S_{\min} + \left(S_{\max} - S_{\min}\right) \times \left(PopSize - rank_i\right) \big/ \left(PopSize - 1\right)\right) \quad (28)$$

where, S_{max} and S_{min} are the maximum allowed and minimum allowed seeds to be produced by each weed, respectively. Also, $rank_i$ shows the rank of *i*th weed in the population at the each iteration.

The generated seeds are distributed over the solution space by normally distributed random numbers with mean equal to zero but varying variance. The standard deviation is reduced at the each iteration as equation (29):

$$\sigma_{iter} = \left(iter_{\max}\right)^{pow} \times \left(\sigma_{initial} - \sigma_{final}\right) \big/ \left(iter_{\max} - 1\right)^{pow} + \sigma_{final} \quad (29)$$

where, $iter_{max}$ is the maximum number of iteration and *iter* is the current iteration. Also, $\sigma_{initial}$ and σ_{final} are the initial and final sigma, respectively. Besides, *pow* is the non-linear modulation index.

4.3. Evolution of the weeds

Humans have recently created an entirely new category of very nasty weeds: herbicide resistant weeds. These resistant weeds are produced by evolving in them. In this paper, a Neighborhood Search Structure (NSS) is added to IWO for the first time. NSS is a mechanism to get new solutions by slightly changing the current solution. The NSS is done on the weeds in the each iteration using variable neighborhood search (VNS) algorithm. For searching the neighbors in the VNS algorithm, four difference operators including mutation (used for whole three parts), reversion, insertion and swap (used for first and third parts) are applied according to figure 3. The evolution is done on the percent of population which is shown by P_E. The number of VNS outer and inner iteration is equal to $ItVNS_{max}$ and *nRepeat*, respectively.

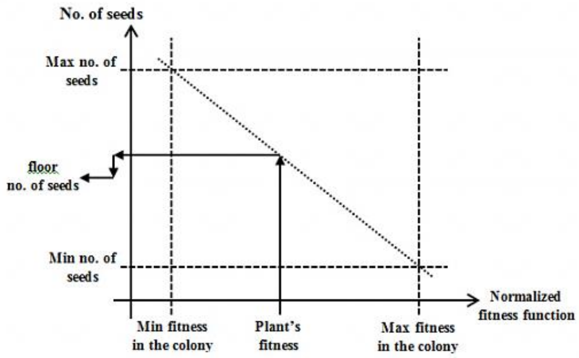

Figure 2. Relationship between the number of seed and value of fitness function (Mehrabian and Lucas, 2006)

Sample solution	.17	.76	.59	.33	.81	.03
Mutation	.17	.76	.59	.33	.81	.41
Reversion	.17	.76	.81	.33	.59	.03
Insertion	.17	.76	.03	.59	.33	.81
Swap	.17	.81	.33	.59	.76	.03

Figure 3. The four NSS used in this paper

4.4. Competitive exclusion

In the each iteration, three population including initial weeds, produced seeds and evolved weeds are merged together. Afterward, the population is ranked using fuzzy dominance sorting, which is introduced in the next subsection, the best *PopSize* weeds are chosen, and the rest discarded. Finally, selected population is the final population which is used in next iteration.

4.5. Fuzzy dominance based sorting

Initially, the fuzzy dominances of the solutions in the population must be computed as a first step of fuzzy dominance sorting. Next, the solutions are sorted by fuzzy dominance into ascending order. Afterward, diversity is calculated by sorting the solutions based on crowding distance. The fuzzy dominance of each solution is computed as following pseudo-code:

for $k=1:n$ // Compute Fuzzy dominance of each solution in the population
$\mu(k)=0;$
for $i=1:n$
$\mu(i)=1;$
for $j=1:m$ // Compute Fuzzy *j*-dominance of each solution
if $f_j \ \vec{x}_i \ - f_j \ \vec{x}_k \ < 0$
$\mu_i^{dom}(\vec{x}_k \prec_i^F \vec{x}_i) = 0$
else if $f_j \ \vec{x}_i \ - f_j \ \vec{x}_k \ < p_j$
$\mu_i^{dom}(\vec{x}_k \prec_i^F \vec{x}_i) = \ f_j \ \vec{x}_i \ - f_j \ \vec{x}_k \ \big/ p_j$
else
$\mu_i^{dom}(\vec{x}_k \prec_i^F \vec{x}_i) = 1$
$\mu^{dom}(\vec{x}_k \prec_i^F \vec{x}_i) = \mu^{dom}(\vec{x}_k \prec_i^F \vec{x}_i) \times \mu_i^{dom}(\vec{x}_k \prec_i^F \vec{x}_i)$
end
$\mu(k) = \mu(k) + \mu^{dom}(\vec{x}_k \prec_i^F \vec{x}_i) - \mu(k) \times \mu^{dom}(\vec{x}_k \prec_i^F \vec{x}_i)$
end
end

4.6. Stopping criteria

In the proposed MOIWO, the maximum number of iteration is the stopping criteria. The Pseudo code of proposed MOIWO is presented in figure 4.

5. Experimental results

This section evaluates the performances of the proposed MOIWO. The performance of the proposed MOIWO is compared with well-known multi-objective evolutionary algorithms (MOEA), namely NSGA-II, PAES and SPEA. Before running the algorithms, the tuned parameters of the proposed MOIWO, employing well known response surface methodology (RSM), for small (S) and large (L) are: *PopSize*=15(S),40(L); pop_{max}=70(S),110(L); *itermax*=125(S),250(L); S_{max}=7(S),9(L); S_{min}=1(S),2(L); $\sigma_{initial}$ =0.55(S),0.8(L); σ_{final} =0.01(S),0.02(L); *pow*=1(S),1(L); P_E=0.3(S),0.65(L); *ItVNSmax*=17(S),20(L); *nRepeat*=6(S),8(L);

5.1. Data generation

The value and distribution of input parameters of the model are as table 2. Some special number of hubs has been considered for each number of nodes. Also, each problem instance is shown as "*Number of customer # number of hubs*", for example, 50#8 means 50 customers and 8 hubs.

Table 2. Value and distribution of input parameters.

	Parameter							
	N		*dC*	*CC*	*dH*	*CH*	*dCH*	*Flow*
Value & Distribution	10	40	U ~ (10,20)	U ~ (100,200)	U ~ (20,40)	U ~ (400,800)	U ~ (200,400)	U ~ (40,80)
	15	50						
	20	70	*FH*	*FV*	*Chub*	*Cap*	*t*	
	25	100	U ~ (1000,4000)	U ~ (200,600)	U ~ (200,600)	U ~ (120,360)	N ~ (30,10)	
	30		*r*	*s*	*w*	*e*	*l*	*B*
			U ~ (200,400)	N ~ (20,5)	N ~ (10,4)	U ~ (30,120)	U ~ (80,150)	U ~ (50,150)

```
set the parameters of MOIWO (PopSize, pop_max, iter_max, S_max, S_min, σ_initial , σ_final , pow,  P_E, ItVNSmax, nRepeat) ;

Generate initial Population (Pop_1) ← PopSize;
Evaluate fitness of each weed;
Rank the population using fuzzy sort;
Archive the solutions;
Calculate the Number of allowed seeds for each member of PopSize (PopSize, S_max, S_min);
terminate ← false
while (terminate = false) do for each weed
        Update the σ_iter (iter_max, σ_initial , σ_final , pow);
        Produce allowed seeds for each weed based on their rank in the population (Pop_2);
        Evolve weeds as follow: (Pop_3);
        for i=1:(PopSize* P_E)
        choose a weed randomly among the population;
            for j=1: ItVNSmax
                for k=1:4
                    for r=1:nRepeat
                        if k=1 then
                            use Mutation operator;
                        else if k=2 then
                            use Reversion operator;
                        else if k=3 then
                            use Insertion operator;
                        else
                            use Swap operator;
                        end if
                    end
                end
            end
        end
        Merge all created population (Pop_1, Pop_2, Pop_3);
        Evaluate fitness of each member of merged population;
        Rank the population using fuzzy sort;
        Update Archive ← (Archive = Archive + merged population);
        if numel(merged population) > pop_max then
            Archive = Archive (1: pop_max);
        end if
        Archive = Archive (1: pop_max);
        it=it+1;
        if it> iter_max then
            terminate = true;
        end if
end while
```

Figure 4. The Pseudo code of the proposed MOIWO

5.2. Computational results

To validate the reliability of the proposed MOIWO, the four well known comparison metrics (Tavakkoli-Moghaddam et al., 2011; Mohammadi et al, 2013), namely quality metrics, mean ideal distance (MID), diversification metric (DM) and spacing metric (SM), are applied. The proposed MOIWO is applied for a number of test problems and its performance is compared with NSGA-II, PAES and SPEA. Table 3 shows the quality metric and spacing metric for small-sized problems. Table 4 lists the diversification metric and mean ideal distance for small-sized problems. Tables 5 and 6 show the comparison metrics for the problem size 50 and 100, respectively. All the tables 3 to 6 show that the proposed MOIWO is superior to NSGA-II, PAES and SPEA in the each test problem.

From tables 3 to 6 these following benefits of proposed MOIWO can be shown:

- Proposed MOIWO can achieve a greater number of Pareto optimal solutions with higher qualities than NSGA-II, PAES and SPEA.
- Proposed MOIWO provides non-dominated solutions that have less average values of the spacing metric.
- These data reveal that non-dominated solutions obtained by the proposed MOIWO are more uniformly distributed in comparison with NSGA-II, PAES and SPEA.
- The average values of the diversification metric in our proposed MOIWO are considerably greater than NSGA-II, PAES and SPEA (i.e., MOIWO finds non-dominated solutions that diversity more).
- In most of the test problems, the values of MID in the proposed MOIWO are smaller than those of NSGA-II, PAES and SPEA.

Table 3. Quality and spacing metrics for small-sized problems

Problem No.	Quality Metric (QM)				Spacing Metric (SM)			
	SPEA	PAES	NSGA-II	MOIWO	SPEA	PAES	NSGA-II	MOIWO
10#3	0	0	0.24	0.76	0.69	1.06	0.74	0.36
10#4	0	0	0.17	0.83	0.43	0.53	0.35	0.41
15#3	0	0	0.07	0.93	0.52	0.67	0.40	0.50
15#4	0	0.1	0.15	0.65	0.56	1.05	0.79	0.26
20#3	0	0	0	1	1.17	0.88	0.64	0.26
20#4	0	0	0	1	0.80	1.09	0.84	0.59
20#5	0	0	0.12	0.88	0.37	0.32	0.62	0.53
25#3	0	0	0.21	0.79	0.37	0.86	0.85	0.23
25#4	0	0	0.14	0.86	0.75	0.69	0.82	0.24
25#5	0	0	0	1	0.92	1.07	0.51	0.45
30#3	0	0	0	1	0.83	0.80	0.52	0.23
30#4	0	0	0	1	0.51	0.78	0.47	0.68
30#5	0	0	0.22	0.78	0.54	0.58	0.84	0.42
30#6	0	0	0.34	0.66	1.08	0.83	0.31	0.26

Table 4. Diversification and MID metrics for small-sized problems

Problem No.	Diversification Metric (DM)				Mean Ideal Distance (MID)			
	SPEA	PAES	NSGA-II	MOIWO	SPEA	PAES	NSGA-II	MOIWO
10#3	0.50	0.78	1.11	1.39	0.69	0.93	0.51	0.34
10#4	1.00	0.58	1.11	1.06	0.72	0.83	0.51	0.25
15#3	0.50	0.76	1.06	0.80	0.62	0.93	0.61	0.25
15#4	1.06	1.08	0.87	1.30	0.60	0.78	0.82	0.57
20#3	0.55	0.70	1.02	1.03	0.92	0.66	0.78	0.46
20#4	0.62	0.92	0.87	1.44	0.89	0.74	0.79	0.56
20#5	1.02	0.92	0.82	1.00	0.63	0.81	0.68	0.34
25#3	0.75	0.93	0.76	1.56	0.77	0.88	0.63	0.48
25#4	0.82	0.85	0.85	1.27	0.88	0.71	0.61	0.27
25#5	0.73	0.52	1.02	1.43	0.89	0.61	0.92	0.42
30#3	0.63	0.73	0.76	1.03	0.99	0.77	0.64	0.21
30#4	0.52	0.73	0.85	1.16	0.97	0.76	0.81	0.35
30#5	0.64	0.64	1.01	1.56	0.61	0.78	0.51	0.40
30#6	0.43	0.53	0.89	1.46	0.94	0.65	0.65	0.52

Table 5. Comparison metrics for 50-sized problems

Problem No.	Quality Metric (QM)				Spacing Metric (SM)			
	SPEA	PAES	NSGA-II	MOIWO	SPEA	PAES	NSGA-II	MOIWO
50#4	0	0	0	1	0.75	0.76	0.59	0.44
50#5	0	0	0	1	0.84	0.74	0.61	0.50
50#6	0	0	0	1	0.98	0.84	0.40	0.57
50#7	0	0	0.10	0.90	0.79	0.91	0.59	0.60
50#8	0	0	0	1	0.67	0.69	0.63	0.44
Problem No.	Diversity Metric (DM)				Mean Ideal Distance (MID)			
	SPEA	PAES	NSGA-II	MOIWO	SPEA	PAES	NSGA-II	MOIWO
50#4	0.73	0.77	1.06	1.02	1.02	0.81	0.74	0.48
50#5	0.73	0.60	0.73	1.14	1.06	1.06	0.81	0.45
50#6	0.78	0.78	1.18	1.22	1.06	0.83	0.88	0.38
50#7	0.84	0.60	0.85	0.83	1.04	0.98	0.89	0.30
50#8	0.70	0.82	0.70	1.05	0.91	1.01	0.77	0.59

Table 6. Comparison metrics for 100-sized problems

Problem No.	Quality Metric (QM)				Spacing Metric (SM)			
	SPEA	PAES	NSGA-II	MOIWO	SPEA	PAES	NSGA-II	MOIWO
100#5	0	0	0	1	0.67	0.68	0.55	0.64
100#6	0	0	0	1	0.68	0.75	0.53	0.36
100#7	0	0	0	1	0.74	0.97	0.63	0.55
100#8	0	0	0.20	0.80	0.69	0.74	0.78	0.45
100#9	0	0	0	1	0.84	0.71	0.47	0.28
100#10	0	0	0	1	0.75	0.92	0.46	0.49
Problem No.	Diversity Metric (DM)				Mean Ideal Distance (MID)			
	SPEA	PAES	NSGA-II	MOIWO	SPEA	PAES	NSGA-II	MOIWO
100#5	0.88	0.88	1.08	1.08	0.73	0.90	0.42	0.62
100#6	0.85	0.87	0.82	1.32	0.88	1.01	0.40	0.28
100#7	0.97	0.91	1.03	1.02	1.07	1.06	0.64	0.57
100#8	0.85	0.60	1.14	0.90	1.01	0.90	0.88	0.32
100#9	0.67	0.95	0.87	0.97	0.98	1.04	0.83	0.22
100#10	0.78	0.99	0.97	1.40	0.71	0.83	0.80	0.69

5.3. Paired *t* test

A paired *t* test was conducted to see whether the significant difference exists between the obtained solutions by proposed MOIWO and those of the

NSGA-II, PAES and SPEA or not considering all four comparison metrics. We conducted the paired *t* test by 67 test problems in the SPSS software. By referencing to *t* table, for 66° of freedom the significances (2-tailed) are closed to 0.000. The detailed statistics are shown in Table 7. These tests show that there are statistical significant difference between solutions obtained by MOIWO and those of the NSGA-II, PAES and SPEA.

Table 7. Detailed statistics of paired *t* test

Metric	Pair	Paired Differences					*t*	*df*	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	Upper			
Quality	MOIWO NSGA-II	.90582	.17078	.02086	.86416	.94748	43.414	66	**.000**
SM		-.16254	.18876	.02306	-.20858	-.11649	-7.048	66	**.000**
DM		.22140	.22468	.02745	.16660	.27621	8.066	66	**.000**
MID		-.27746	.18909	.02310	-.32359	-.23134	-12.011	66	**.000**
Quality	MOIWO PAES	.94955	.09564	.01168	.92622	.97288	81.269	66	**.000**
SM		-.41716	.20142	.02461	-.46629	-.36803	-16.953	66	**.000**
DM		.40081	.23964	.02928	.34235	.45926	13.690	66	**.000**
MID		-.45418	.23003	.02810	-.51029	-.39807	-16.161	66	**.000**
Quality	MOIWO SPEA	.95030	.09055	.01106	.92821	.97239	85.900	66	**.000**
SM		-.38194	.21005	.02566	-.43318	-.33070	-14.884	66	**.000**
DM		.40170	.29630	.03620	.32943	.47397	11.097	66	**.000**
MID		-.47045	.22207	.02713	-.52461	-.41628	-17.341	66	**.000**

In order to illustrate the high performance of the MOIWO, two examples with 1 quality are shown in figure 5.

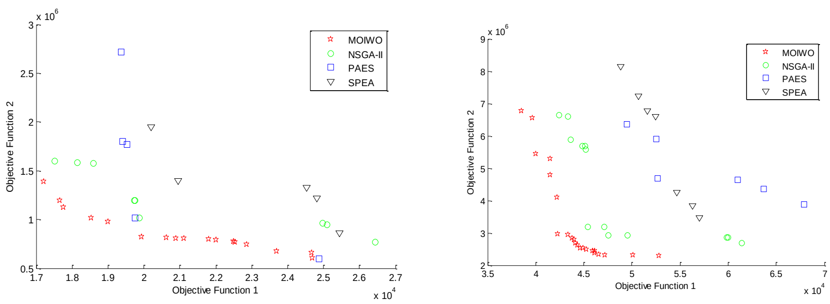

Figure 5. Pareto optimal solution of two sample problems

6. Conclusion

This paper proposed a new model for stochastic green hub location routing problem with simultaneous pick-ups and deliveries while for solving the underhand NP-Hard problem, a multi-objective invasive weed optimization (MOIWO) combined with variable neighborhood search (VNS) was designed. A non-linear stochastic mathematical model was first introduced minimizing both the transportation cost and the environmental effect of the problem. Some preposition then added to the model based on normal standard distribution for making the model linear.

To validate our proposed MOIWO, various test problems were designed to evaluate its performance and reliability in comparison with conventional multi-objective evolutionary algorithms, known as NSGA-II, PAES and SPEA. Furthermore, some useful comparison metrics, such as quality metric, spacing metric, mean ideal distance and diversification metric, were applied to validate the efficiency of the proposed MOIWO. The experimental results indicate that the proposed MOIWO outperformed the NSGA-II, PAES and SPEA and was able to improve the quality of the obtained solutions.

REFERENCES

[1]. **Alumur, S., & Kara, B.Y. (2007),** ***A new model for the hazardous waste location-routing problem.*** *Computers & Operations Research, 34(5): 1406–1423;*

[2]. **Ambrosino, D., Scutella, M.G. (2005),** ***Distribution network design: new problems and related models.*** *European Journal of Operational Research, 165(3): 610–624;*

[3]. **Balakrishnan, A., Ward, J.E., Wong, R.T. (1987),** ***Integrated facility location and vehicle routing models: Recent work and future prospects.*** *American Journal of Mathematical and Management Sciences, 7: 35–61;*

[4]. **Belenguer, J.M., Benavent, E., Prins, C., Prodhon, C., Wolfler-Calvo, R. (2006),** ***A branch and cut method for the capacitated location-routing problem.*** *International Conference on Service Systems and Service Management, 2:1541–1556;*

[5]. **Berger, RT., Coullard, CR., Daskin, M. (2007),** ***Location-Routing Problems with Distance Constraints.*** *Transportation Science, 41(1):29–43;*

[6]. **Caballero, R., Gonzalez, M., Guerrero, F.M., Molina, J., Paralera, C., (2007),** ***Solving a multi-objective location routing problem with a meta-heuristic based on tabu search: Application to a real case in Andalusia.*** *European Journal of Operational Research, 177: 1751–1763.*

[7]. **Chan, Y., Carter, W.B., Burnes, M.D., (2001), *A multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands.*** *Computers and Operations Research, 28(8): 803–826.*

[8]. **Cappanera, P., Gallo, G., Maffioli, F. (2004), *Discrete facility location and routing of obnoxious activities.*** *Discrete Applied Mathematics, 133, 3–28;*

[9]. **Duhamel, C., Lacomme, P., Prins, C., Prodhon, C. (2010), *A GRASP-ELS approach for the capacitated location-routing problem.*** *Computers & Operations Research, 37(11): 1912–1923;*

[10]. **Erdogan, S., Miller-Hooks, E. (2012), *A Green Vehicle Routing Problem.*** *Transportation Research Part E, 48: 100–114;*

[11]. **Hassan-Pour HA, Mosadegh-Khah M, Tavakkoli-Moghaddam R. (2009), *Solving a multi-objective multi-depot stochastic location-routing problem by a hybrid simulated annealing algorithm.*** *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 223(B8): 1045–1054;*

[12]. **Jacobsen, S.K., Madsen, O.B.G. (1980), *A comparative study of heuristics for a two-level routing-location problem.*** *European Journal of Operational Research, 5(6): 378–387;*

[13]. **Karaoglan, I., Altiparmak, F., Kara, I., Dengiz, B. (2009), *Formulations for a location-routing problem with simultaneous pickup and delivery.*** *Research paper, /http://w3.gazi.edu.tr/_fulyaal/Papers/ LRPSPD_MIP Formulations.pdfS.*

[14]. **Karaoglan, I., Altiparmak, F., Kara, I., Dengiz, B. (2012), *The location routing problem with simultaneous pickup and delivery:*** *Omega, 40: 465–477;*

[15]. **Kundu, D., Suresh, K., Ghosh, S., Swagatam, D., Panigrahi, B.K., Das, S. (2011), *Multi-objective optimization with artificial weed colonies.*** *Information Sciences, 181: 2441–2454;*

[16]. **Labbe M., Rodriguez-Martin, I., Salazar-Gonzalez, J. (2004), *A branch-and-cut algorithm for the plant cycle location problem.*** *Journal of the Operational Research Society, 55(5): 513–520;*

[17]. **Laporte, G. (1989), *A survey of algorithms for location-routing problems.*** *Investigation Operation, 1: 93–123;*

[18]. **Marinakis, Y., Marainaki, M. (2008), *A particle swarm optimization algorithm with path relinking for the location routing problem.*** *Journal of Mathematical Modelling and Algorithms, 7: 59–78;*

[19]. **Mehrabian A.R. and Lucas, C. (2006), *A novel numerical optimization algorithm inspired from weed colonization,*** *Ecological Informatics, 1: 355–366;*

[20]. **Mohammadi, M., Jolai, F., Tavakkoli-Moghaddam, R. (2013), *Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm.*** *Applied Mathematical Modelling, 10.1016/j.apm.2013.05.063;*

[21]. **Nagy, G., Salhi, S. (2007),** ***Location-routing: Issues, models and methods.*** *European Journal of Operational Research, 177: 649–672;*

[22]. **Or, I., Pierskalla, W.P. (1979),** ***A transportation location-allocation model for regional blood banking.*** *AIIE Transactions, 11: 86–94;*

[23]. **Schwardt, M., Fischer, K. (2009),** ***Combined location-routing problems – A neural network approach.*** *Annals of Operations Research, 167: 253–269;*

[24]. **Tavakkoli-Moghaddam, R., Makui, A., Mazloomi, Z. (2010),** ***A new integrated mathematical model for a bi-objective multi-depot location-routing problem solved by a multi-objective scatter search algorithm.*** *Journal of Manufacturing Systems, 29: 111–119;*

[25]. **Tavakkoli-Moghaddam, R., Azarkish, M., adeghnejad-Barkousaraie, A. (2011),** ***Solving a multi-objective job shop scheduling problem with sequence-dependent setup times by a Pareto archive PSO combined with genetic operators and VNS.*** *International Journal of Advance Manufacturing Technology, 53:733–750;*

[26]. **Watson-Gandy, C.D.T., Dohrn, P.J. (1973),** ***Depot location with van salesmen – A practical approach.*** *Omega, 1(3): 321–329;*

[27]. **Yu, V. F., Lin, S. W., Lee, W., Ting, C.J.A. (2010),** ***Simulated Annealing Heuristic for the Capacitated Location Routing Problem.*** *Computers and Industrial Engineering, 58(2): 288–299.*