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A GREY-BASED ROLLING PROCEDURE FOR SHORT-TERM FORECASTING USING LIMITED TIME SERIES DATA

Abstract. *In a globally competitive environment, it is necessary for enterprises to grasp the price trend of material for drawing up an effective overall manufacturing plan. However, material prices are often dramatically unstable making it difficult to obtain accurate, short-term predictions using long historical observations. Facing the challenge, forecasting with new limited data is considered more effective, efficient, and of considerable interest. Grey theory is one useful approach that can effectively handle uncertain problems under small data sets, and the AGM(1,1) is a forecasting model based on this theory that can obtain better forecasting outcomes. However, this technique is initially suggested for use with very short-term forecasting. To improve this drawback for a longer forecasting, this study aims to extend the applicability of the AGM(1,1) by combining the AGM(1,1) with the rolling framework technique into a useful model referred to, here, as AGMRF. The forecasting performance of AGMRF is confirmed in this study using aluminum price data collected from the London Metal Exchange (LME). The results are compared with the other four methods, GM(1,1), AGM(1,1), LR, and BPN, and show that the proposed procedure can effectively deal with the problem*

when the sample size is limited.

***Keywords**: Aluminum price; Forecasting; Time series; Grey theory; Small data set.*

JEL Classification: CO1, C13, C22, C34, C53

1. Introduction

In a globally competitive environment, an effective and efficient overall production plan is required for businesses. In fact, the input of resources to a manufacturing system is the start of the process operation, and efficient purchasing with a low price will assist companies in earning profits. So managers must grasp the updated price curve for obtaining good purchasing performance.

Popular forecasting techniques can be roughly divided into three categories: multivariate analysis, time series model, and data mining (Bermudez et al., 2011; Chui et al., 2009; Costa et al., 2011; Gu et al., 2011). Multivariate analysis explores the relationship between independent and dependent variables by building a causal model for estimating the dependent variables. The forecasting performance of this model depends on the selection of independent variables. One will obtain a model with high variance if these variables can not effectively explain the variation of the dependent variables. In contrast, time series approaches, such as the autoregressive integrated moving average (ARIMA) model, require only the historical data of the variable of interest to forecast its future evolution. However, it is usually essential to have many observations to produce satisfied forecasting results. Data mining techniques, such as artificial neural networks and the support vector regression, are widely used as forecasting approaches and have extremely good forecasting performance. However, their forecasting outcomes depend highly on the number of training data and their representativeness. In all of the above methods, the forecasting performances are obviously affected by the sample size and quality, which indeed restrict their applicability.

The price of materials generally influenced by various factors, and how to clearly clarify the relations among factors is still a difficult issue. Furthermore, even though a considerable amount of historical data is available, there usually exist inconsistencies between the old data pattern and the currently developing trend in material prices. Therefore, the old information will often mislead the model and thus affect the forecasting accuracy. On the other hand, when building the model, employing only the most current small samples will be reflect the actual developing situation. Therefore, to establish a forecasting model using limited samples is valuable for an enterprise's decision making.

Grey system theory, proposed by Deng (1982), is especially used to deal with uncertain and insufficient information. Its main principle is to process the data indirectly through the accumulating generation operator (AGO) to identify their hidden regularity (Deng, 1989). Because of the convenience of its modeling procedure, grey system theory has been successfully applied in various fields, such as engineering, manufacturing, electricity, and so on (Badescu et al., 2010; Chen et al., 2008; Delcea & Scarlat, 2010; Hsu, 2011; Jin et al., 2012; Kayacan et al., 2010; Li et al., 2010; Tsaur & Liao, 2007; Yamaguchi et al., 2007). The GM(1,1) is the primary forecasting approach in grey theory having a good prediction ability, even if there are only four data (Li et al., 2011; Liu & Lin, 2006). It has thus become one of the most important methods to deal with small-data-set forecasting.

As a practical application, this study utilizes the monthly average price of aluminum for cash buyers from the London Metal Exchange (LME) to explore the forecasting performance of the proposed forecasting model. The experiment indicates that the proposed approach has precise forecasting results using small samples, so it is considered a useful and proper forecasting tool.

The remainder of this paper is organized as follows. In Section 2, the modeling procedure of the proposed method is introduced. In Section 3, the experimental results are given. Finally, the conclusions are presented in Section 4.

2. Methodology

Although the GM(1,1) has been successfully applied in various fields, it can still be further improved in its forecasting performance. Li et al. (2009) employed the trend and potency tracking method (TPTM) (Li & Yeh, 2008) to analyze data behavior to develop an improved grey forecasting model, AGM(1,1), where A stands for adaptive. This approach corrected a weakness of the ordinary modeling process to better reflect data growth trends at different stages. It can obtain more accurate forecasts than the original model, except it is convenient to use. However, this model is only suggested using in a very short-term forecasting. This study thus combines the AGM(1,1) with rolling framework (RF) to deal with the price forecasting problems under small sample sets. The proposed building procedure called AGMRF consists of three parts: the computation of the trend and potency value, the construction of the grey model, and the use of the rolling process. All of these will be described in the following subsections.

2.1. Trend and potency tracking method

Li and Yeh (2008) proposed the trend and potency tracking method (TPTM), which systematically estimates the variations in sequential data to obtain hidden information. TPTM is an analysis method based on the data features and behavior, and it is the basis of the AGM(1,1) model.

The detailed procedure of TPTM is as follows:

1. Assume the original data series are $X = \{x_1, x_2, \ldots, x_n\}$. Let $x_{\min}$ is the minimal observation in X, and $x_{\max}$ is the maximal observation in X.

2. Compute the variations σ_i between the paired observations (x_{i-1}, x_i), $i = 2,3,\ldots,n$, to obtain the increasing or decreasing potencies according to the data sequence.

3. Give the incremental weights according the time sequences to represent the intensity of different phases because the latest datum dominates the features of the upcoming datum. The importance w_i of the datum at phase i equals $i-1$, $i = 2,3,\ldots,n$. For example, at phase 6 the importance (or the weight) of the datum $w_6 = 6-1 = 5$.

4. Let $A_i = \sigma_i \times w_i$, $i = 2,3,\ldots,n$ as an accession to strengthen the data trend and potency at different phases by multiplying both weights and variations. $A_i > 0$ is the increasing potency (IP), and $A_i < 0$ is the decreasing potency (DP).

5. Find the central location (CL) of existing data using the following equation:

$$\mathrm{CL} = \frac{x_{\min} + x_{\max}}{2}.$$

The CL is then utilized as the main point to conduct the asymmetric domain range expansion.

6. Calculate the average of increasing potencies (AIP) and the average of the decreasing potencies (ADP), and then use them to asymmetrically expand the domain range. The upper limit of the expanded domain range is $\mathrm{EDR\_UL} = x_{\max} + \mathrm{AIP}$, and the lower limit is $\mathrm{EDR\_LL} = x_{\min} + \mathrm{ADP}$.

7. CL, EDR_UL, and EDR_LL are used to build a triangular TP function. Here we set the TP value of the CL to be 1, and we thus can obtain the TP values of existing data through the ratio rule of a triangle. Figure 1 demonstrates a simple instance for which the TP value of $x_{\min}$ is $t = \frac{p}{p+q}$, where p is the distance between EDR_LL and $x_{\min}$, and q is the distance between $x_{\min}$ and CL. The range of the TP value is between 0 and 1, and the TP value represents the current datum's intensity close to the CL.

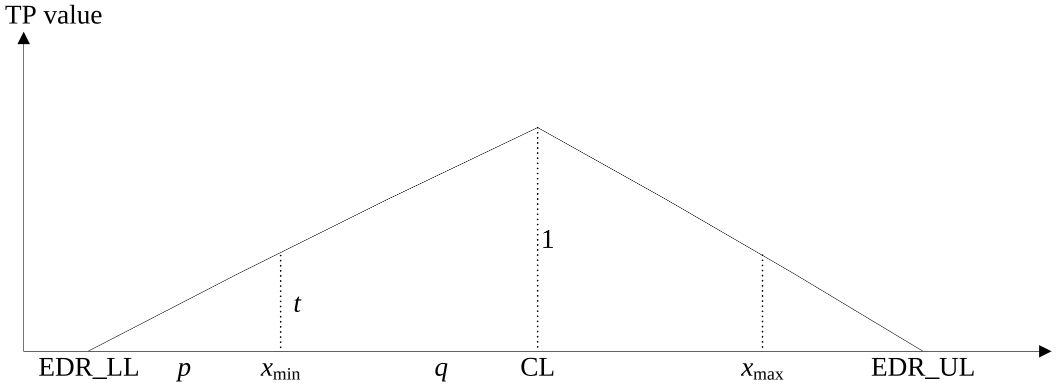

Figure 1: A triangular TP function and the TP value corresponding to the datum $x_{\min}$

2.2. Adaptive grey forecasting model

The GM(1,1) is a popular time-series model due to its simpleness, and it can create good forecasting results with only four raw data. In its modeling procedure, the background value is the key factor to affect the model construction and final forecasting results. Li et al. (2009) studied the impact of different background values on forecasting performance and proposed an improved grey model, AGM(1,1), by adding the concept of TPTM into the formula for determining the background values. The AGM(1,1) has a better forecasting accuracy among all existing grey models, so this study chose it to deal with small-data-set forecasting problems.

The modeling process of the AGM(1,1) model is described in detail as below:

1. Given a data set $\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$.

2. Calculate the TP values by TPTM as $\{TP_i\} = \{TP_1, TP_2, \ldots, TP_n\}$, $i = 1, 2, \ldots, n$.

3. Computed α_k by

$$\alpha_k = \frac{\sum_{i=1}^{k} 2^{i-1} TP_i}{\sum_{i=1}^{k} 2^{i-1}}, \quad k \geq 2 \tag{1}$$

4. Use the accumulating generation operator to obtain a new data series as:

$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$, $x^{(1)}(1) = x^{(0)}(1)$;

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 2, 3, \ldots, n \tag{2}$$

5. Determine the background values $z^{(1)}(k)$ using the following equation:

$$z^{(1)}(k) = x^{(1)}(k-1) + \alpha_k x^{(0)}(k), \quad \alpha \in (0,1), \quad k = 2, 3, \ldots, n \tag{3}$$

6. Establish the grey differential equation and estimate the developing coefficient a and the grey input b from Eq. (4) by the least-squares method and establish the grey differential equation from Eq. (5).

$$x^{(0)}(k) + a z^{(1)}(k) = b \tag{4}$$

$$\frac{dx^{(1)}}{dt} + a x^{(1)} = b \tag{5}$$

To estimate *a* and *b*, we expand Eq. (4) as

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & 1 \\ -z^{(1)}(n) & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} \tag{6}$$

Let

$$Y = [x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)]^T \tag{7}$$

$$\hat{a} = [a, b]^T \tag{8}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & 1 \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{9}$$

$$\hat{a} = (B^T B)^{-1} B^T Y \tag{10}$$

7. Solve Eq. (5) together with the initial condition $x^{(0)}(1) = x^{(1)}(1)$, to acquire the desired forecasting output at step $k+1$ by Eq. (11).

$$\begin{cases} \hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \dfrac{b}{a})e^{-ak} + \dfrac{b}{a} \\ \hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \end{cases} \tag{11}$$

2.3. Rolling framework

Grey theory generally emphasizes the immediateness of information, and the rolling framework is a technique used in grey forecasting model for achieving this purpose. For example, given four data, $\{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), x^{(0)}(4)\}$, to predict the next output $\hat{x}^{(0)}(5)$ by AGM(1,1). After the result is found, we add the newly predicted value to the data set and remove the oldest datum $x^{(0)}(1)$ to ensure that the information is timely. This revised data set, $\{x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), \hat{x}^{(0)}(5)\}$, is applied to obtain the new predicted value, $\hat{x}^{(0)}(6)$. The procedure is repeated until all desired forecasting outputs are found.

3. Data and experimental results

The modeling approach is explained with the details in the following subsections.

3.1. The aluminum price data

The experiment aims to examine the effectiveness of the proposed method for price forecasting. This study selects the monthly average price of aluminum for cash buyers from the London Metal Exchange (LME) to explore the forecasting performance (http://www.lme.co.uk). Aluminum is the most heavily consumed non-ferrous metal in the world, and its annual consumption is more than 24 million tons in one year (Totten & Mackenzie, 2003). It is one of the most important production materials in manufacturing industries, so grasping its price curve is a significant issue for many enterprises. The price of aluminum is usually severely influenced by some uncontrollable factors, which cause older historical observations to not be able to reflect the existing data trend and become noise in the learning procedure. Therefore, creating a model using limited new samples to forecast aluminum prices is valuable for enterprise policies.

In this research, we use the data from January 2001 to December 2010 for the experimental analysis, and all 120 observations shown in Table 1. In this table, the unit of measurement for prices is U.S. dollars per tonne (US$/tonne).

3.2. The moving frame

The purpose of this research is to create a price trend model to forecast the aluminum price using limited samples. The moving cross-validation procedure (Hu et al., 1999; Lauret et al., 2008) is employed to evaluate the proposed approach and its schematic diagram is shown in Figure 2. In this scheme, we use four observations as the training set to construct a model to predict the next four incoming data. For example, the data from January 2001 to April 2001 are used to generate a model for forecasting the potential values from May 2001 to August 2001 in the first round. Next, the training set is moved and the predicted values from June 2001 to September 2001 are obtained using the data from February 2001 to May 2001. The forecasting procedure is repeated until all 452 desired outputs are acquired.

Table 1: The monthly average price of aluminum for cash buyer (unit: US$/tone)

Months	Prices	Months	Prices	Months	Prices
2001/01	1615.65	2004/05	1623.22	2007/09	2390.68
2001/02	1604.36	2004/06	1677.72	2007/10	2441.9
2001/03	1509.17	2004/07	1709.27	2007/11	2506.28
2001/04	1496.91	2004/08	1692.19	2007/12	2381.14
2001/05	1538.77	2004/09	1723.6	2008/01	2445.08
2001/06	1466.13	2004/10	1819.57	2008/02	2776.46
2001/07	1416.39	2004/11	1813.9	2008/03	3004.86
2001/08	1377.08	2004/12	1849.18	2008/04	2958.82
2001/09	1344.56	2005/01	1833.94	2008/05	2902.43
2001/10	1282.5	2005/02	1882.39	2008/06	2957.46
2001/11	1327.45	2005/03	1981.88	2008/07	3070.68
2001/12	1344.63	2005/04	1893.9	2008/08	2763.96
2002/01	1368.59	2005/05	1743.36	2008/09	2525.48
2002/02	1369.34	2005/06	1730.99	2008/10	2120.83
2002/03	1405	2005/07	1778.4	2008/11	1852.08
2002/04	1369.99	2005/08	1867.45	2008/12	1490.02
2002/05	1343.3	2005/09	1839.5	2009/01	1412.79
2002/06	1353.97	2005/10	1928.3	2009/02	1329.81
2002/07	1338.09	2005/11	2050.16	2009/03	1335.5
2002/08	1291.6	2005/12	2247.11	2009/04	1420.5
2002/09	1301.25	2006/01	2377.45	2009/05	1460.09

2002/10	1310.58	2006/02	2454.91	2009/06	1573.33
2002/11	1372.2	2006/03	2428.77	2009/07	1667.53
2002/12	1375.07	2006/04	2621.11	2009/08	1933.39
2003/01	1378.28	2006/05	2860.93	2009/09	1833.6
2003/02	1422.16	2006/06	2477.01	2009/10	1878.18
2003/03	1389.27	2006/07	2512.17	2009/11	1948.94
2003/04	1332.01	2006/08	2459.48	2009/12	2179.69
2003/05	1398.49	2006/09	2472.36	2010/01	2234.84
2003/06	1409.85	2006/10	2653.82	2010/02	2048.58
2003/07	1436.09	2006/11	2701.99	2010/03	2205.21
2003/08	1456.31	2006/12	2813.14	2010/04	2316.4
2003/09	1415.57	2007/01	2808.34	2010/05	2040.14
2003/10	1474.25	2007/02	2831.61	2010/06	1931.02
2003/11	1508.34	2007/03	2760.97	2010/07	1987.78
2003/12	1554.9	2007/04	2814.17	2010/08	2117.61
2004/01	1606.49	2007/05	2794.25	2010/09	2161.98
2004/02	1685.63	2007/06	2676.93	2010/10	2346.07
2004/03	1655.99	2007/07	2732.44	2010/11	2332.62
2004/04	1729.74	2007/08	2514.88	2010/12	2350.1

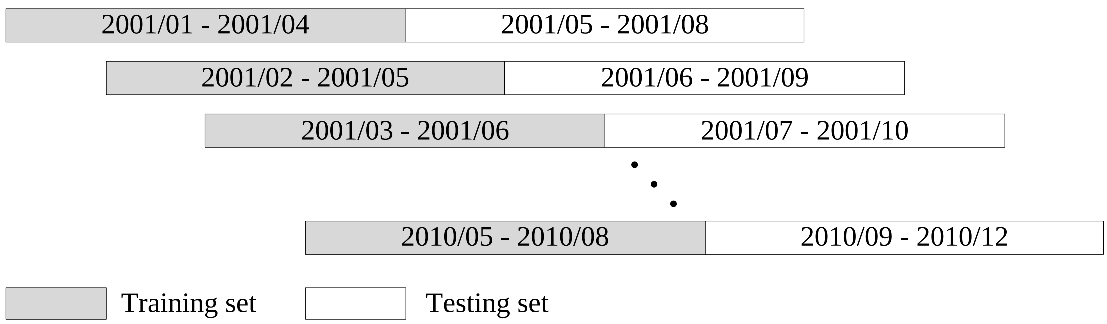

Figure 2: The schematic diagram of the moving cross-validation procedure

3.3. Experiment studies

In this subsection, we use one example to describe the modeling process using various methods and adopt the results in the comparison.

3.3.1 The GM(1,1) model

GM(1,1) is a popular model in grey theory that can provide adequate forecasting results even when there are few observations. This study utilizes four data with $\alpha = 0.5$ to build the GM(1,1) model for predicting $\hat{x}_5$, $\hat{x}_6$, $\hat{x}_7$, and $\hat{x}_8$.

For instance, using $\{1615.65,\ 1604.36,\ 1509.17,\ 1496.91\}$ to create the model to predict the next four observations,
$\{\hat{x}_5,\ \hat{x}_6,\ \hat{x}_7,\ \hat{x}_8\}=\{1440.56,\ 1423.11,\ 1388.47,\ 1371.11\}$, where the built predictor is $\hat{x}^{(1)}(k+1)=(-3137.05)e^{-0.03527k}+47529.69$.

3.3.2 The AGM(1,1) model

We first use the TPTM method to compute the TP value for each observation and use it to generate a forecasting model. When utilizing $\{1615.65,\ 1604.36,\ 1509.17,\ 1496.91\}$ as the example to create the AGM(1,1) model, the next four outputs are predicted as $\{\hat{x}_5,\ \hat{x}_6,\ \hat{x}_7,\ \hat{x}_8\}=\{1440.56,\ 1397.93,\ 1356.55,\ 1316.41\}$, where the built AGM(1,1) model is $\hat{x}^{(1)}(k+1)=(-53265.74)e^{-0.03004k}+54881.39$.

3.3.3 The AGMRF model

The modeling procedure of AGMRF is similar to AGM(1,1). The main difference is that AGM(1,1) uses a model to gain four outputs and AGMRF uses four different models to forecast the desired values. They are in order as $\hat{x}^{(1)}(k+1)=(-53265.74)e^{-0.03004k}+54881.39$, $\hat{x}^{(1)}(k+1)=(-69656.09)e^{-0.02207k}+71260.49$, $\hat{x}^{(1)}(k+1)=(-68483.41)e^{-0.02189k}+69992.58$, and $\hat{x}^{(1)}(k+1)=(-80410.27)e^{-0.01817k}+81907.18$, respectively. Note that the first predicted value is obtained from the first model, and the second one is obtained from the second model, and so on. These four outputs are $\{\hat{x}_5,\ \hat{x}_6,\ \hat{x}_7,\ \hat{x}_8\}=\{1440.56,\ 1423.11,\ 1388.47,\ 1371.11\}$.

3.3.4 The LR model

Linear regression (LR) is a general numerical forecasting method in statistics. It is easy to calculate and is the fundamental prototype for further methods. In our example, we designate month as the independent variable and price as the dependent variable and utilize the least squares principle to determine the best fit of regression equation. In this case, the regression model is formed as $\hat{x}_k=-45.141k+1669.375$, and the next four observations are predicted as $\{\hat{x}_5,\ \hat{x}_6,\ \hat{x}_7,\ \hat{x}_8\}=\{1443.67,\ 1398.53,\ 1353.39,\ 1308.25\}$.

3.3.5 The BPN model

The back propagation neural network (BPN) is a famous type of neural network that is also popular for its convenience in use. This research uses Pythia software as the learning tool to create its forecasting models. Using the four training samples to learn the BPN, each sample includes one input attribute and one output attribute. Under the restriction of limited training set, this study employs the BPN with a 1-2-1 structure as shown in Figure 3 (three layers, one hidden layer with two neurons, training with 1000 repetitions, learning rate: 0.5) to conduct the prediction task. When BPN is trained in 1-2-1 structure, the learning samples are $\{(1,\ 1615.65),\ (1, 1604.36),\ (1, 1509.17),\ (1, 1496.91)\}$. After the training work, we input $k = 5,\ 6,\ 7,\ 8$ to output the predicted values as $\{\hat{x}_5,\ \hat{x}_6,\ \hat{x}_7,\ \hat{x}_8\} = \{1556.30,\ 1556.23,\ 1556.22,\ 1556.22\}$. The process is shown in Table 2.

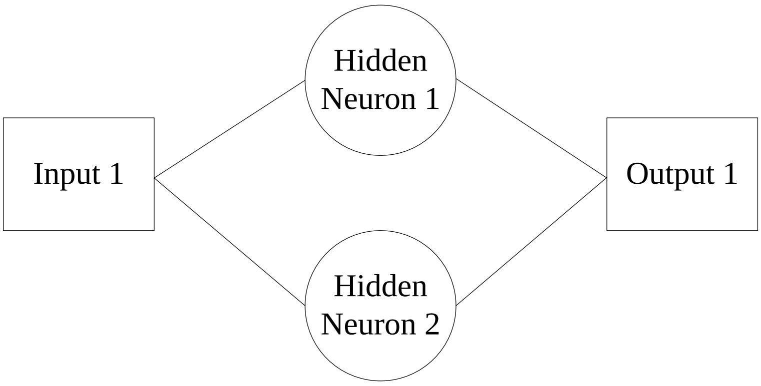

Figure 3: The 1-2-1 BPN structure

Table 2: Experimental study of BPN

	Input	Output
Training set	1 (2001/01)	1615.65 (x_1)
	2 (2001/02)	1604.36 (x_2)
	3 (2001/03)	1509.17 (x_3)
	4 (2001/04)	1496.91 (x_4)
Prediction	5 (2001/05)	1556.30 ($\hat{x}_5$)
	6 (2001/06)	1.556.23 ($\hat{x}_6$)
	7 (2001/07)	1.556.22 ($\hat{x}_7$)
	8 (2001/08)	1.556.22 ($\hat{x}_8$)

3.4. Experimental results: comparison among the forecasting methods

Yokum and Armstrong (1995) noted that the accuracy is an important index for measuring the forecasting ability. This study uses two measurements to evaluate the accuracy and stability of the forecasting methods: namely, the mean absolute percentage error (MAPE), and the standard deviation (SD). Table 3 shows the results of the two measurements for the forecasting methods. In the experiment, the proposed approach outperforms the other four methods, and its standard deviation is also the smallest. This information indicates that AGMRF is superior with regard to both accuracy and stability.

We also find that the MAPE of AGMRF improves by 7.69% compared with that of AGM(1,1) in Table 3; therefore, AGMRF has a better ability for short-term forecasting. To statistically verify the accuracy of the proposed method, we perform the following statistical hypothesis test: $H_0 : \mu_d = 0$, $H_1 : \mu_d \neq 0$. We apply a paired *t*-test and set the degree of freedom as 451. Table 4 shows the significant differences between AGMRF and the other methods, and thus we reject the null hypothesis.

The fitting and prediction graphs of the AGMRF for the aluminum price are also presented in Figure 4. It shows that the forecasting outputs are close to the actual observations, meaning that the results obtained with the AGMRF can reflect the current trend of aluminum prices. In addition, because the learning samples are too few, the over-fitting phenomena of BPN universally occur in the experimentation making BPN; hence, it does not perform well. Additionally, neither GM(1,1) nor LR are able to produce the desired results. These results indicate that AGMRF is an effective method for price forecasting.

Table 3: The performance of forecasting methods

Methods	MAPE (%)	SD
GM(1,1)	9.94	10.79
AGM(1,1)	9.23	9.91
AGMRF	8.52	9.04
LR	10.15	11.91
BPN	9.94	11.58

Table 4: The paired *t*-test statistics

Comparison	Paired-*t* Statistics	
AGMRF vs. GM(1,1)	$\bar{d}$	-1.422
	s_d	4.050
	t	-7.467
	p-value	**0.000**
AGMRF vs. AGM(1,1)	$\bar{d}$	-0.711
	s_d	2.428
	t	-6.223
	p-value	**0.000**
AGMRF vs. LR	$\bar{d}$	-1.632
	s_d	7.244
	t	-4.790
	p-value	**0.000**
AGMRF vs. BPN	$\bar{d}$	-1.426
	s_d	9.758
	t	-3.107
	p-value	**0.001**

Note: Boldface indicates significant difference at the 0.05 level

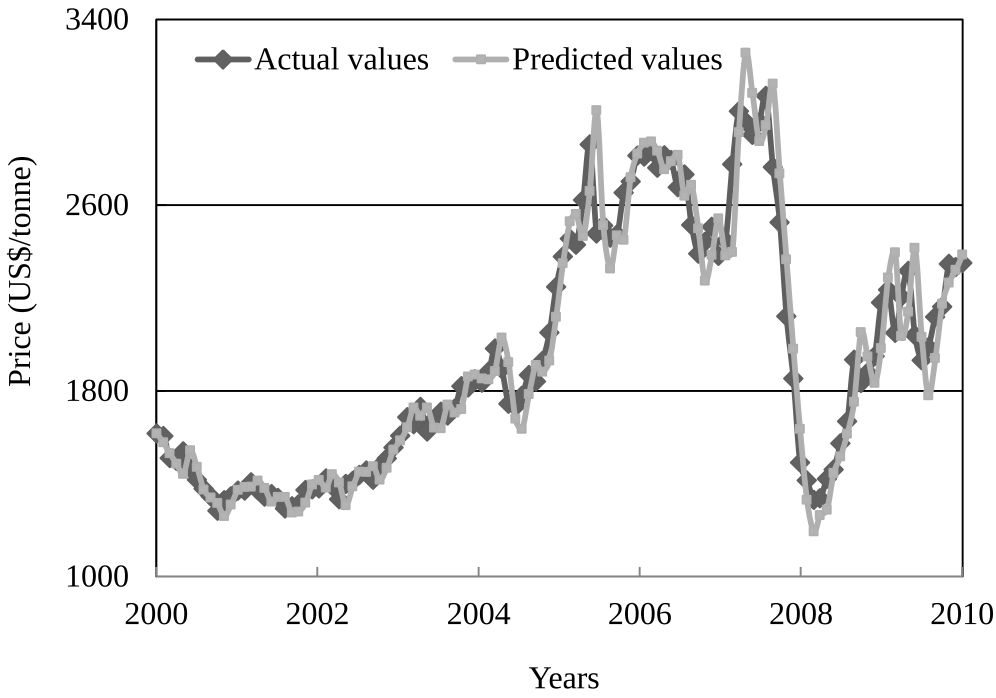

Figure 4: Actual and fitted values for aluminum price

4. Conclusions and discussion

Forecasting material prices is important for an enterprise's overall planning. However, older historical observations tend to deviate from the current situation. The specific purpose of this study is thus to determine a suitable model for price forecasting with limited new data.

Grey theory can establish models with small data sets, and so meets the requirements of this study. In this research, we presented a revised modeling procedure to address the problem of a small data set. This approach, named AGMRF, combines AGM(1,1) with a rolling framework to mitigate the drawbacks of AGM(1,1), which is only suitable for very a short-term forecasting. The demonstration shows that, with only four data, AGMRF can establish a forecasting model with a MAPE of about 8.5%, which meets the demand for small-data-set forecasting. The results from BPN are not superior to those of AGMRF, which may be because the sample size is small. Normally, a BPN needs plenty of data as the training sets to get stable learning results and to avoid the problem of over-fitting. Hence, the forecasting results of BPN could be improved by increasing the training data. The performance of AGMRF is thus the best of the models in this study for forecasting problems with small data sets, and thus is considered a suitable tool for material price forecasting problems.

In future work, AGMRF may need to be applied to other fields, such as engineering, finance, transportation, and industry to further prove its superiority. Besides, integrating some optimization methods with the grey model to further improve the forecasting performance is also an interesting study direction.

REFERENCES

[1]**Badescu, A.V., Smeureanu, I., Cristea, R.N., Asimit, A.V. (2010),** ***Portfolio Assets Selection through Grey Numbers Implementation***. *Economic Computation and Economic Cybernetics Studies and Research*, 44, 23-36;

[2]**Bermudez, J.D., Corberan-Vallet, A., Vercher, E. (2011),** ***Forecasting Correlated Time Series with Exponential Smoothing Models***. *International Journal of Forecasting*, 27, 252-265;

[3]**Chen, C.I., Chen, H.L., Chen, S.P. (2008),** ***Forecasting of Foreign Exchange Rates of Taiwan's major Trading Partners by Novel Nonlinear Grey Bernoulli Model NGBM(1,1).*** *Communications in Nonlinear Science and Numerical Simulation*, 13, 1194-1204;

[4]**Chui, F., Elkamel, A., Surit, R., Croiset, E., Douglas, P.L. (2009),** ***Long-term Electricity Demand Forecasting for Power System Planning Using Economic, Demographic and Climatic Variables***. *European Journal of Industrial Engineering*, 3, 277-304;

[5]**Costa, A.M., Franca, P.M., Lyra, C. (2011),** ***Two-level Network Design with Intermediate Facilities: An Application to Electrical Distribution Systems*****.** *Omega-International Journal of Management Science*, 39, 3-13;

[6]**Delcea, C., Scarlat, E. (2010),** ***Finding Companies' Bankruptcy Causes Using a Hybrid Grey-Fuzzy Model***. *Economic Computation and Economic Cybernetics Studies and Research*, 44, 77-94;

[7]**Deng, J.L. (1982),** ***Control Problems of Grey Systems.*** *Systems & Control Letters*, 1, 288-294;

[8]**Deng, J.L. (1989),** ***Introduction to Grey System Theory***. *Journal of Grey System*, 1, 1-24;

[9]**Gu, J.R., Zhu, M.C., Jiang, L.G.Y. (2011),** ***Housing Price Forecasting Based on Genetic Algorithm and Support Vector Machine***. *Expert Systems with Applications*, 38, 3383-3386;

[10]**Hsu, L.C. (2011),** ***Using Improved Grey Forecasting Models to Forecast the Output of Opto-electronics Industry***. *Expert Systems with Applications*, 38, 13879-13885;

[11]**Hu, M.Y., Zhang, G.Q., Jiang, C.Z., Patuwo, B.E. (1999),** ***A Cross-validation Analysis of Neural Network out-of-sample Performance in Exchange Rate Forecasting***. *Decision Sciences*, 30, 197-216;

[12]**Jin, M., Zhou, X., Zhang, Z.M., Tentzeris, M.M. (2012),** ***Short-term Power Load Forecasting Using Grey Correlation Contest Modeling***. *Expert Systems with Applications*, 39, 773-779;

[13]**Kayacan, E., Ulutas, B., Kaynak, O. (2010),** ***Grey System Theory-based Models in Time Series Prediction***. *Expert Systems with Applications*, 37, 1784-1789;

[14]**Lauret, P., Fock, E., Randrianarivony, R.N., Manicom-Ramasamy, J.F. (2008),** ***Bayesian Neural Network Approach to Short Time Load Forecasting***. *Energy Conversion and Management*, 49, 1156-1166;

[15]**Li, D.C., Chang, C.J., Chen, C.C., Chen, C.S. (2010),** ***Using Non-equigap Grey Model for Small Data Set Forecasting - A Color Filter Manufacturing Example***. *Journal of Grey System*, 22, 375-382;

[16]**Li, D.C., Chang, C.J., Chen, W.C., Chen, C.C. (2011),** ***An Extended Grey Forecasting Model for Omnidirectional Forecasting Considering Data Gap Difference***. *Applied Mathematical Modelling*, 35, 5051-5058;

[17]**Li, D.C., Yeh, C.W. (2008),** ***A Non-parametric Learning Algorithm for Small Manufacturing Data Sets*****.** *Expert Systems with Applications*, 34, 391-398;

[18]**Li, D.C., Yeh, C.W., Chang, C.J. (2009),** ***An Improved Grey-based Approach for Early Manufacturing Data Forecasting***. *Computers & Industrial Engineering*, 57, 1161-1167;

[19]**Liu, S.F., Lin, Y. (2006),** ***Grey Information: Theory and Practical Applications***. *Springer:* London, Britain;

[20]**Totten, G.E., Mackenzie, D.S. (2003),** ***Handbook of Aluminum: Volume1, Physical Metallurgy and Processes***. Marcel Dekker, Inc.: New York, United States;

[21]**Tsaur, R.C., Liao, Y.C. (2007),** ***Forecasting LCD TV Demand Using the Fuzzy Grey Model GM(1,1).*** *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, 15, 753-767;

[22]**Yamaguchi, D., Li, G.D., Nagai, M. (2007),** ***A Grey-based Rough Approximation Model for Interval Data Processing***. *Information Sciences*, 177, 4727-4744;

[23]**Yokum, J.T., Armstrong, J.S. (1995),** ***Beyond Accuracy: Comparison of Criteria Used to Select Forecasting Methods***. *International Journal of Forecasting*, 11, 591-597.