Associate Professor Alexandra TKACENKO , PhD
Department of Applied Mathematics
Moldova State University
Moldova
E-mail: alexandratkacenko@gmail.com.

MULTIPLE CRITERIA FUZZY COST TRANSPORTATION MODEL OF "BOTTLENECK" TYPE

Abstract. *The multiple criteria optimization problems with fuzzy objective functions coefficients are the most important, because of their often applications in various managerial decision processes. In this paper is presented an interactive solving approach for the multi-objective transportation problem with fuzzy cost coefficients and time minimizing criterion. The approach is based on interval presentation of each cost functions coefficients. By finding of the probabilistic parameter of belonging of coefficients of objective functions to their variation intervals for every criterion, we can find iteratively the corresponding set of efficient solutions for the multiple criteria transportation model for every value of parameter, structured by the time minimizing criterion. Thus, we could solve the multi-criteria transportation problem of fuzzy type for any value of the probabilistic parameter of belonging, that is in fact one of stochastic "bottleneck" type of problem. In other words, we have obtained one significant result, that any decision-making situation described by the proposed model can be predicted by its time and cost characteristics. The proposed algorithm has proved to be quite efficient, being tested on several examples.*

Keywords: *Fuzzy programming, multi-criteria transportation model, "bottleneck" criterion, efficient solution, optimal compromise solution.*

JEL Classification: ***90C29, 90C70***

1. Introduction

The topic connected with transportation problem can be considered as old as the world is. Nevertheless the scientific research interest of this type of problems remains and continues to be endless. The reason is not due only to the attractive and clear form of these kinds of problems, but it is also due to numerous practical appliances of them. Now, it's worth pointing that problems of multi-criteria optimization including the multi-criteria transportation problems are of great interest in various research areas. This happens because is well known, the increasing of criteria number leads only to increasing of solution accuracy for multi-criteria optimization problems. The efficient solutions of these kinds of problems can be achieved using various algorithms developed in [8], [11], [13], [20] and many others. From practical applicability point of view, imposing of minimal time to realise the solution of model appears as a logical condition which would surely improve its quality. In the speciality literature the criterion of minimizing the maximum time is called a “bottleneck” criterion. A large variety of algorithms have been proposed for different kinds of multi-criteria transportation problems of “bottleneck” type. Thus, for solving the three-criteria transportation problem, including the “bottleneck” Aneja and Nair in [1] developed an efficient algorithm, but Wild and Karwan in [19] proposed an efficient algorithm for solving the generalized r-criteria transportation problem of the same type. It's important to mention, that many of economical decision problems lead to the fractional optimization models, because that a lot of important characteristics of these may be evaluated really using only some ratio relations. The ratios like the summary cost of total transportation expenditures into maximum necessary time to satisfy demands, the total benefit or production value related to the necessary time, the total depreciation into maximal using time and many other similar criteria often are essential in evaluation of several economic efficiency indexes, that lead finally to finding of some correct managerial decisions. The time-constraining criterion is, obviously, one of conditions so much important for major optimization problems. A particular case, but quite often meted is of identical denominators like the “bottleneck” time function. Moreover, we studied various cases when "bottleneck" denominator function is included as a separate criterion in the optimization model. The efficient algorithms for solving these types of models are proposed by Sharma and Swarup in [14] for one-criterion fractional transportation model of “bottleneck” type and by Tkacenko in [17] for multi-criteria fractional transportation model of the same type. I want to emphasize that all of the above mentioned algorithms were tested on various examples and proved to be quite efficient for the deterministic type of data. Unfortunately, in real life, not always be so, often, some parameters and coefficients of the optimization models are of indeterminate type [7]. That is why in the proposed work is studied the case when some of parameters of multi-criteria “bottleneck” model are of fuzzy type.

2. Problem formulation

Since for any type of mathematical optimization model, the objective function coefficients have greatest influence on both the optimal solution and the value of objective function, we propose to investigate the multi-criteria transportation model, in that these coefficients are of fuzzy type. Because of large practical appliance of the transportation models, it should be noted the certain real meanings of these coefficients. We propose to include in the model the “bottleneck” criterion separate, which is quite important for any decisional situation especially from practical point of view. The mathematical model of multi-criteria transportation problem of “bottleneck” type with fuzzy costs coefficients is the following:

$$\min Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{1} x_{ij}$$

$$\min Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{2} x_{ij}$$

$$\ldots\ldots$$

$$\min Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^{r} x_{ij} \tag{1}$$

$$\min Z_{r+1} = \max_{i,j} \{ t_{ij} \mid x_{i,j} > 0 \}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m}, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n},$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \quad x_{ij} \ge 0 \textit{ for all } i \textit{ and } j$$

where : $\tilde{c}_{ij}^{k}$ *, k=1,2…r, i=1,2,…m, j=1,2,…n are costs or other amounts corresponding to concrete interpretations of those criteria being of fuzzy type,* t_{ij} *- necessary unit transportation time from source i to destination j,* a_i *- disposal at source i,* b_j *- requirement of destination j,* x_{ij} *- amount transported from source i to destination j, that is only positive.*

We can notice, that in model (1) the firsts *-r* criteria are of fuzzy linear type, each of them being of minimal type. However, is not excluded to have in the initial model some criteria of maximum type such as, for example, maximizing of benefit, profit or more other. In fact, this doesn't not make the optimization model more complicated as using some elementary transformations the maximum types of criteria can be modified into minimal types as they appear in the model (1). Obviously, the model (1) becomes more complicate especially from solving point of view, because of the *(r+1)* criterion, which is of non-linear type.

3. Some Reasons and Statements

Since the model (1) is of multi-criteria type, as we know, these rarely admit optimal solutions. For solving these usually it builds a set of efficient solutions, known also as Pareto-optimal or non-dominated solutions, solutions of the "best compromise". In order to investigate the model of multiple criteria (1), we should propose firstly the definition of efficient solution for the deterministic type of model. We will consider the next multiple-criteria transportation model of "bottleneck" type with deterministic data, where without tying generality we assume that all of the firsts *r* criteria are of minimum type.

$$\min Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{1} x_{ij}, \ \min Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{2} x_{ij}$$

$$\ldots\ldots\ldots\ldots$$

$$\min Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, \ \min Z_{r+1} = \max_{i,j}\left\{t_{ij} \,\middle|\, x_{i,j} > 0\right\} \tag{2}$$

in conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i, \ \forall i = \overline{1,m}, \ \sum_{i=1}^{m} x_{ij} = b_j, \ \forall j = \overline{1,n}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \qquad x_{ij} \geq 0, i = \overline{1,m}, \ j = \overline{1,n}$$

with the same significance of the model parameters as in model (1), specifying that all of these are of deterministic type.

Let suppose that: $(\overline{X}, \overline{T})$ is one basic solution for the model (1), where:

$$\overline{T} = \max_{i,j}\left\{t_{ij} / \overline{x}_{ij} > 0\right\}.$$

Definition 1: The basic solution $(\overline{X},\overline{T})$ of the model (2) is a basic efficient one if and only if for any other basic solution $(X,T) \neq (\overline{X},\overline{T})$ for which exists at least one index $j_1 \in \{1,\ldots,r\}$ for which the relation $Z_{j_1}(X) < Z_{j_1}(\overline{X})$ is true, there immediately exists another, at least, one index $\exists j_2 \in \{1,\ldots,r\}$, where $j_2 \neq j_1$, for which at least, one of the both relations $Z_{j_2}(\overline{X}) < Z_{j_2}(X)$ or $\overline{T} < T$ is true. If all of these three inequalities are verified simultaneously with the equal sign, it means that the solution is not unique.

Definition: 2 *The basic solution* $(\overline{X}, \overline{T})$ *of the model* (2) *is one of the optimal (best) compromise solution for a certain time* $\overline{T}$, *if the solution* $\overline{X}$ *is located closest to the optimal solutions of each criterion.*

So, for each time level allowing placement of the basic solution for the model (2), we can determine its corresponding optimal compromise solution.

Because of the first ones *r* criteria in model (1) are of fuzzy type, we develop primarily a fuzzy technique for solving of the fuzzy multi-criteria model.

4. Fuzzy techniques

By fuzzy linear programming we mean the appliance of the fuzzy set theory to linear multi-criteria decision making problems.

Definition *An element x has a degree of membership in a set A, denoted by a membership function* $\mu_A(x)$. *The rang of the membership function is [0,1].*

In multi-criteria decision making problems, the objective functions are represented by fuzzy sets, but the decision set is defined as the intersection of all fuzzy sets and constraints. The decision rule is to select the solution having the highest membership of the decision set.

Zadeh [2] introduced the basic concepts of fuzzy set theory. Zimmermann in [22] made an innovation in the field of multi-criteria decision making. He first applied fuzzy set theory concept with suitable choices of membership functions and derived a fuzzy linear programming. He shows that obtained solution using the fuzzy linear programming is always efficient one, further it can find a optimal compromise solution.

In order to solve the model (2), firstly we propose to solve the model (3), that is obtained by excluding of the *(r+1)* “bottleneck” criterion. It is the next:

$$\min Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^1 x_{ij},\ \min Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2 x_{ij}$$

$$\ldots\ldots\ldots\ldots$$

$$\min Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij}\ , \tag{3}$$

in conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i\ ,\ \forall i = \overline{1,m}\ ,\ \sum_{i=1}^{m} x_{ij} = b_j\ ,\ \forall j = \overline{1,n}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\ ,\quad x_{ij} \ge 0\ , i = \overline{1,m}\ ,\ j = \overline{1,n}$$

with similar sense of parameters, as in the model (2).

We will apply the fuzzy linear programming technique [3] for solving the model (3). It should be noted, that in this case we apply the fuzzy approach only to the objective functions of the model. By applying of fuzzy linear programming technique to the multi-objective linear transportation model (3), we will find its optimal compromise solution .

At the first we assign for each objective function two values U_k and L_k as upper and lower bounds for the objective function Z_k :

L_k - aspired level of achievement for objective *k;*

U_k - highest acceptable level of achievement for objective *k;*

$d_k = U_k - L_k$ is obviously a degradation allowance for objective *k.*

We build the fuzzy model, because of aspiration and degradation levels for each objective have been specified. On the next step we will transform the fuzzy model into one of deterministic type model of linear programming.

The solving fuzzy technique is the following:

Step 1. Solving of r one-criterion transportation problems.

Step 2. Building the table of values, in which are registered values of the all objective functions in the optimal solutions of every objective function.

Step 3. According to the table of values we may choose the best - L_k and the worst U_k values from the set of solutions.

The initial fuzzy model is built keeping the aspirations of each criterion, as the follows:

$$Z_k \underset{\sim}{<} L_k,\ k = \overline{1,r},$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m}, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n}, \tag{4}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad x_{ij} \geq 0 \ \ for\ all\ i\ and\ j$$

The membership function $\mu_k(X)$ is defined as the next:

$$\mu_k(X) = \begin{cases} 1, & if \quad Z^k(X) \leq L_k \\ \dfrac{U_k - Z_k}{U_k - L_k}, & if \quad L_k < Z^k(X) < U_k \\ 0, & if \quad Z^k(X) \geq U_k \end{cases}$$

Taking into account the relations (4) and the above definition of the membership function $\mu_k(X)$, the equivalent linear programming problem for the multi-objective transportation problem (3) is the following:

$$\begin{gathered}
\text{Max } \lambda \\
\text{in conditions:} \\
\lambda \le \frac{U_k - Z_k}{U_k - L_k}, \quad k = \overline{1,r}, \\
\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m}, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n}, \\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad x_{ij} \ge 0 \ \textit{for all } i \textit{ and } j, \ \lambda \ge 0.
\end{gathered} \tag{5}$$

By simplifying the model (5), we will obtain the next linear programming optimization model:

$$\begin{gathered}
\text{Max } \lambda \\
\textit{in conditions:} \\
\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + \lambda \cdot (U_k - L_k) \le U_k, \quad k = \overline{1,r}, \\
\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m}, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n}, \\
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad x_{ij} \ge 0 \ \textit{for all } i \textit{ and } j, \ \lambda \ge 0.
\end{gathered} \tag{6}$$

Thus, we can say, that using the fuzzy technique for solving the model (6), we easily find a compromise solution for the multi-objective transportation model (3).

Remark 1. The above described algorithm is applicable to all types of multi-objective transportation problems as well to the vector minimum as to the vector maximum problems.

Remark 2. The optimal compromise solution of the model (3) doesn’t necessarily to be of integer type.

In order to solve the model (2) by applying the fuzzy technique we propose the next algorithm:

Step 1. Ordering the time matrix - T according to cell values in ascending order and assigning for each cell a serial number, thus we will get all $(m \times n)$ ordered cells.

Step 2. Selecting the firsts at least *(m + n-1)* cells according to the arrangement order, until we can place the initial basic solution for the model (2), supposing that the other cells are blocked.

Step 3. By applying the algorithm of fuzzy technique for the problem with unlocked cells, we get the optimal compromise solution for the model (2) using only the unlocked cells, which corresponds to the following time: $t^* = \min \max_{i,j} \{t_{i,j} | x_{i,j} > 0\}$.

Step 4. Unlocking iteratively in increasing order of time the next matrix cell (or cells with the same time and cost values), we will return to the step 3 of the algorithm and we will find the next optimal compromise solution of model with time of its realization, obviously, higher than the previous t^* time.

The step 4 is repeated until all of cells in the matrix of time will be unblocked.

Thus, the proposed algorithm will highlight a finite set of optimal compromise solutions for the model (2), each of them corresponding to the smallest time possible of its realisation.

Because the problem has finite dimensions, the algorithm is realized in a finite number of steps.

5. Theoretical analysis of fuzzy cost multi-criteria model

We can state that the fuzzy technique is very efficient for solving the various optimization problems. Applying the fuzzy technique for solving the multi-criteria model, not only of linear type, we build a optimal compromise solution, where only the objective functions are represented in the fuzzy form. This method is applied most commonly when the parameters and coefficients of model are of deterministic type. Unfortunately in everyday life we meet not only with such cases. Because the parameters and coefficients of transportation multi-criteria models have real practical significances such as unit prices, unit costs and many other, as we know they vary quite frequently. Moreover, it was found that all of them are interconnected, i.e. they are changing simultaneously even with the same parameter of variation. For example, if electricity becomes more expensive, with one coefficient, then in a stable economy, certainly, the values of products of industries directly dependent of it will increase by the same coefficient. For other branches these may be different, but they may be calculated knowing a priori the correlation coefficients between all the branches. The last, can be calculated by applying of various statistical solving methods for certain data of production or consumption between them for selected branches. We propose to calculate the parameters for the model (1) using the formula:

$$p_{ij}^{k} = \frac{c_{ij}^{k} - \underline{c}_{ij}^{k}}{\overline{c}_{ij}^{k} - \underline{c}_{ij}^{k}}, \tag{7}$$

where: $\underline{c}_{ij}^{k}$, $\overline{c}_{ij}^{k}$ *- are the limit values of variation interval for each cost coefficient* c_{ij}^{k} *where:* $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$.

We can observe that $p_{ij}^{k} \in [0;1]$ $for\ \forall (i, j,k)\ where$ $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$. Moreover, any values of cost coefficients according of their variation intervals $c_{ij}^{k} \in [\underline{c}_{ij}^{k};\overline{c}_{ij}^{k}]$ for the model (1) can be calculate using the following formula:

$$c_{ij}^{k} = p_{ij}^{k} \cdot (\overline{c}_{ij}^{k} - \underline{c}_{ij}^{k}) + \underline{c}_{ij}^{k} \text{ for } i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}. \tag{8}$$

Agreeing to the formula (7), the parameters { p_{ij}^{k} } can be considered as the probabilistic parameters of belonging for every value of coefficients { c_{ij}^{k} } from their corresponding variation intervals [7]. This is successfully applied for both minimum criteria type and maximum type.

Supposing that the variables { c_{ij}^{k} *}* $for\ \forall (i, j,k)\ where$ $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$ *are continuous on theirs corresponding intervals, the parameters {* p_{ij}^{k} *} appear as the distribution functions of these variable. Therefore the functions {* p_{ij}^{k} *} enjoys all properties of distribution functions including the monotony and continuity property.*

So, when it find the increasing of prices for products, the likelihood of a higher price for each product type according of its price variation interval is greater than of a lower price.

Analogical it will occur when it is found the cheaper of products, the likelihood that the price of one product from its price interval determined priori will be lower is greater than for the higher price.

Thus are true the next relations:

- ✓ ***for the maximum type of criteria****:*

if for any two values

$c_{ij}^{m1}\ and c_{ij}^{m2}$ *, the relation:* $c_{ij}^{m2} \geq c_{ij}^{m1}$ *is true and* $(c_{ij}^{m1}, c_{ij}^{m2}) \in [\underline{c}_{ij}^{m};\overline{c}_{ij}^{m}]$

$\Rightarrow p_{ij}^{m2} \geq p_{ij}^{m1}$, *for* $i \in \{1,2,...,m\}, j \in \{1,2,...,n\}, \in \{1,2,...,r\}$ *and* m *signifies the maximum type of criterion;*

✓ ***for the minimum type of criteria**:*
if for any two values
c_{ij}^{l1} *and* c_{ij}^{l2} *, the relation:* $c_{ij}^{l2} \leq c_{ij}^{l1}$ *is true and* $\{c_{ij}^{l1}, c_{ij}^{l2}\} \subseteq [\underline{c}_{ij}^{l}; \overline{c}_{ij}^{l}]$;
$\Rightarrow p_{ij}^{l2} \leq p_{ij}^{l1}$, *for* $i \in \{1,2,...,m\}, j \in \{1,2,...,n\}, \in \{1,2,...,r\}$ *and* l
signifies the minimum type of criterion;

Particular cases of the model (1), without of the “bottleneck” criterion were analysed in [4] by Chanas and Kuchta. The authors proposed a method of interval for solving one criterion transportation model with fuzzy cost coefficients. The idea be applied to multi-criteria problem [5], but it leads to considerable increasing of the number of objective functions, which really complicates the solving process of the problem. In the papers [6],[9],[10],[12],[15],[16],[18], [21] are proposed certain analyses of various points of view about the multiple criteria transportation model with fuzzy parameters and are developed different algorithms in order to its solving. It should be noted the practical impossibility of solving these types of models using some parametric methods.

The main idea of the method, that will be developed, is the simultaneously and interconnected variation of objective functions coefficients, fact resulting directly from the practical applications of the studied models and imposing of the “bottleneck” criterion.

6. The solving Algorithm 1

1. We will suppose, that the set of all coefficients variation intervals is given by their variation limit values, they are the following: $\{[\underline{c}_{ij}^{k}; \overline{c}_{ij}^{k}]\}_{i=\overline{1,m}, j=\overline{1,n}, k=\overline{1,r}}$;
2. Let be for some indices: $i_1 \in \{1,2,...,m\}, j_1 \in \{1,2,...,n\}, k_1 \in \{1,2,...,r\}$ we know at least one coefficient value, for example of $c_{i_1 j_1}^{k_1}$, then we may calculate by applying the formula (7) the probabilistic parameter of belonging to its interval $[\underline{c}_{i_1 j_1}^{k_1}, \overline{c}_{i_1 j_1}^{k_1}]$, that is the following:

$$p_{i_1 j_1}^{k_1} = \frac{c_{i_1 j_1}^{k_1} - \underline{c}_{i_1 j_1}^{k_1}}{\overline{c}_{i_1 j_1}^{k_1} - \underline{c}_{i_1 j_1}^{k_1}},$$

Assuming that it's the same for all objective function coefficients, it will be denoted by p;

3. According of the statement about the simultaneously and interconnection of variation of objective functions coefficients, we can calculate a set of all values of objective functions coefficients: $\left\{c_{ij}^k\right\}_{i=\overline{1,m},\, j=\overline{1,n},\, k=\overline{1,r}}$ by applying the next formula: $c_{ij}^k = \underline{c}_{ij}^k + p \cdot \left(\overline{c}_{ij}^k - \underline{c}_{ij}^k\right)$ for the increasing coefficients values, and

$c_{ij}^k = \overline{c}_{ij}^k - p \cdot \left(\overline{c}_{ij}^k - \underline{c}_{ij}^k\right)$ for the decreasing coefficients values.

4. Application of algorithm of fuzzy technique.

So, for any value of probabilistic parameter of belonging we have obtained the multi-criteria transportation deterministic model of "bottleneck" type. For its solving we may apply the above proposed algorithm like as for the model (2). In this case for every time level we can apply the fuzzy technique, thus obtaining the corresponding optimal compromise solution. By modifying the time level, we can obtain the set of all compromise solutions corresponding to all time levels. Thus, for every time level we will have the corresponding optimal compromise solution.

It's necessary to remark, that for a new value of probabilistic parameter of belonging, we will solve again one deterministic model of type (2).

We can solve the model (1) also by finding of a set of efficient solutions for each value of the probabilistic parameter of belonging. This algorithm is more difficult, but for each value of this parameter, it offers for the decider one large set of efficient solutions, which are very important in order to elaborate one correct decisional strategy.

7. The Solving Algorithm 2

Procedure 1.

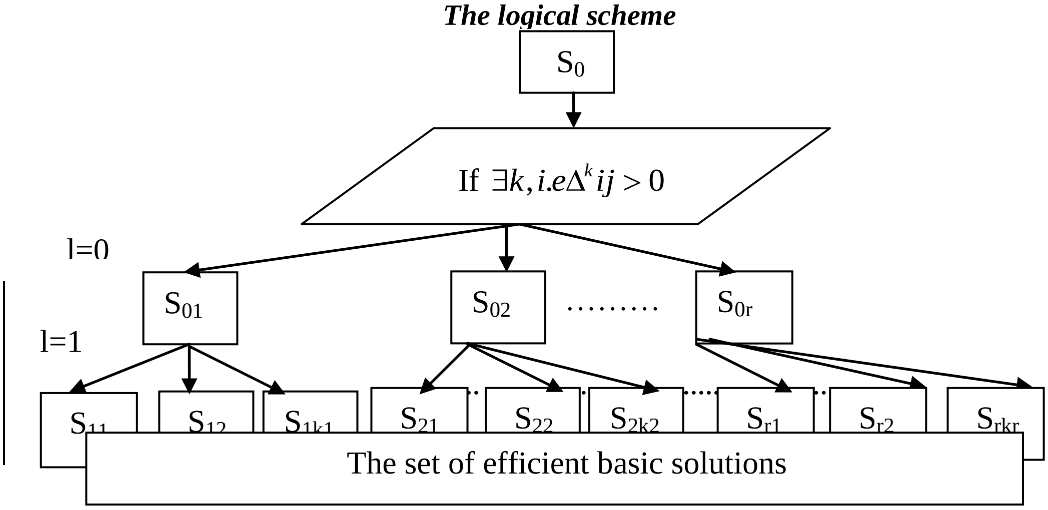
Primarily we will perform the first three steps of Algorithm 1.

Procedure 2.

Procedure 2 is one iterative. Each iteration supposes a deeper search level and finding of a new efficient basic solutions and completing of the multitude of efficient basic solutions with other new, according of the new unblocked cells. The exploration procedure of each time level is finite in depth and ends on every branch, when the same solutions have been found at one upper level of any other branches or when all possibilities of improvement have been exhausted at a certain level. In this case it will unblock a new value of time in the increasing order, therefore, the cost of cells in all the matrices, for which it will be resumed the Procedure 2. Unblocking procedure continues until all cells of time and those that correspond to matrices of cost become available. So, finally we get the set of all sets of efficient solutions of the transportation multi-criteria problem of "bottleneck" type, each of them corresponding to a certain time level.

The solving procedure is performing according to the next logical scheme:

The logical scheme



where: $\Delta_{ij} = (u_i + v_j) - c_{ij}, \; k \le m * n$,

m, n are defined by the problem dimension, k is an ordering index of cells, according with the time table of data

The explicit presentation of the solving Procedure 2

The first step. We arrange the values $\{t_{ij}\}$ from the matrix T in increasing order using for this an ordering index, let it be k. We outline that in the model (1) there are n supplies and m demands and $(r+1)$ objective-functions (including time criterion).

The second step. We try to find an initial basic solution using one of the criteria, namely the matrix of this criterion, from that we will use only the cells according to ordering increasing *p*-index. Obviously, the initial basic solution will be placed in at least $(m+n-1)$ cells. Thus, in the matrix in which we placed the initial basic solution will be unblocked p_0 cells, where $p_0 \geq m+n$-1. The obtained solution at this iteration will mark the 0-level of the logical tree of efficient solutions. We will consider that the following cells with ordering indexes greater than p_0 are blocked. For the 0-level we will calculate the corresponding T_0 according to the following formula: $T_0 = \max_{i,j} \{ t_{ij} \,|\, x_{i,j} > 0 \}$.

The third step *(exploration of the deep branch).* We shall try to improve the solution from the actual level, using for this only the unblocked cells. For this purpose we shall calculate the values: $\Delta_{ij} = (u_i + v_j) - c_{ij}$. All configurations of basic solutions can be recorded at the next level *l=1*. Thus the logical tree will contain on level 1 no more than p_1 branches, where $p_1 \geq p_0 - (m+n-1)$. The procedure from the 3^{rd} step is iterative one and explores the possibility to increase the number of logical branches on the next level using every the branches from the previous level. If all possibilities of placement have been explored as to improve at least one of criteria using for this purpose just the p_0 cells (according the described ordering) , then one can go to the 4^{th} step.

The fourth step. We will unblock the next *(* p_0 *+1)* cell and will obtain a new achievement time for a new efficient solution, which will be obviously greater or equal than the previous time. I’d like to outline that, after each unblocking iteratively procedure there are again $(m \times n - p_0)$ blocked cells, because after every unlocking we consider: $p_0 = p_0 + 1$. If the relation $\Delta_{i,j} \geq 0$ is true at least for one criterion, for this cell, we will repeat the procedure of the 3^{rd} step, otherwise we shall continue to unblock the next cell, according to the ordering index k until we will get $p_0 = m \times n$. After finishing up the 4^{th} step, the set of all basic solutions of model (1) will have been recorded , out of which we can easily select the ones that are basic and efficient.

One can see that the logical solving tree iteratively increase its branches by exploration of a new configurations of the basic solutions on every level. The increasing of both the numerous of branches of each level as well as the number of levels is constrained by the fact that the problem is of finite dimensions on the one hand and on the other hand by the request that the new solution configuration should not be repeated.

The correctness of the above algorithm is based from the following theorem.

Theorem. The set of all efficient basic solutions for the multiple criteria transportation problem with fuzzy cost coefficients and “bottleneck” criterion (1) is found by applying the above Algorithm1 and 2.

Proof. Let L_T be a list of basic efficient solutions of model (1) being found by applying the above algorithm 1 and 2 for one value of probabilistic parameter of belonging p . We suppose, that exists one basic efficient solution S_{j1} for the model (1), that was found using another algorithm different of the above one, so it results that $S_{j1} \notin L_T$. Let S_{j1} corresponds to T_{j1}. We will fix it on the branch that corresponds to the T_{j1} beginning with the level 0. Wide exploration of the fixed branch leads to the registration of all basic solutions of the branch T_{j1}. So, all the basic solutions that correspond to time T_{j1} belong to this set. We will separate in the set $L_{T_{jl}}$ the efficient basic solutions, that correspond to time T_{j1}. It is obvious that $L_{j1} \subset L_T$. As a result, if $S_{j1} \in L_{T_{j1}}$, then S_{j1} is a basic efficient solution found by applying the above algorithm or if $S_{j1} \notin L_{T_{j1}}$, then S_{j1} is not a basic solution and moreover, it is not one basic efficient. So, is true the following: either S_{j1} is a basic efficient solution and it belongs in the list L_T or it is not a basic efficient solution. We proved that for one value of probabilistic parameter of belonging p we obtained the set of all corresponding efficient solutions for the model (1). Therefore, by modifying this parameter, we get another lot of effective solutions for the model (1). Building the set of efficient solutions for model (1) for any value of the parameter p in the interval [0,1], in fact, we fully solve the proposed model. The theorem is proved.

8. Conclusions

In this paper is developed an integrate multistage procedure to solve the multi-objective transportation problem of "bottleneck" type with fuzzy objective functions coefficients. By applying the hypothesis about the interconnection and similarly variation of the model's objective functions coefficients, we reduce the model to one of deterministic type. After, for each of possible time level we construct its corresponding set of efficient solutions. I would like to emphasize, that at this stage we may apply the fuzzy technique for finding the optimal compromise solution, corresponding to the early established time level. However, as it's known, the set of efficient solutions offers several options for developing optimal management strategies. By modifying of the time level, we can obtain all sets of efficient solutions, each of them corresponding to its time of realization, but it should be noted for only one value of probabilistic parameter of belonging. In dependence of the economic stability, the parameter may following different laws of distribution. Finally, we conclude, that these kind of models are very actually and utile especially from the decisional and managerial point of view, therefore these deserve to be further investigated and studied.

Example:

Let be the following 3-criteria problem with 3 supplies and 4 demands. Supposing that we know the set of variation intervals of objective cost coefficients, that is the follow:

$[\underline{c}^1_{11};\overline{c}^1_{11}] = [0.5; 1.5];$ $[\underline{c}^1_{21};\overline{c}^1_{21}] = [0.8; 1.2];$ $[\underline{c}^1_{31};\overline{c}^1_{31}] = [6; 10];$

$[\underline{c}^1_{12};\overline{c}^1_{12}] = [1; 3];$ $[\underline{c}^1_{22};\overline{c}^1_{22}] = [5; 13];$ $[\underline{c}^1_{32};\overline{c}^1_{32}] = [7; 11];$

$[\underline{c}^1_{13};\overline{c}^1_{13}] = [4; 10];$ $[\underline{c}^1_{23};\overline{c}^1_{23}] = [2; 4];$ $[\underline{c}^1_{33};\overline{c}^1_{33}] = [3; 5];$

$[\underline{c}^1_{14};\overline{c}^1_{14}] = [4; 10];$ $[\underline{c}^1_{24};\overline{c}^1_{24}] = [2; 6];$ $[\underline{c}^1_{34};\overline{c}^1_{34}] = [4; 8];$

$[\underline{c}^2_{11};\overline{c}^2_{11}] = [2; 6];$ $[\underline{c}^2_{21};\overline{c}^2_{21}] = [3; 7];$ $[\underline{c}^2_{31};\overline{c}^2_{31}] = [4; 8];$

$[\underline{c}^2_{12};\overline{c}^2_{12}] = [3; 5];$ $[\underline{c}^2_{22};\overline{c}^2_{22}] = [7; 9];$ $[\underline{c}^2_{32};\overline{c}^2_{32}] = [1; 3];$

$[\underline{c}^2_{13};\overline{c}^2_{13}] = [2; 4];$ $[\underline{c}^2_{23};\overline{c}^2_{23}] = [3; 10];$ $[\underline{c}^2_{33};\overline{c}^2_{33}] = [3; 7];$

$[\underline{c}^2_{14};\overline{c}^2_{14}] = [1; 7];$ $[\underline{c}^2_{24};\overline{c}^2_{24}] = [3; 12];$ $[\underline{c}^2_{34};\overline{c}^2_{34}] = [0.6; 1.4];$

Let's assume that we have one value, for example c^1_{12} =2;
We want to build the set of all efficient solutions, knowing the time matrix, that is the next:

Time=

10	95	73	52	8
68	66	30	21	19
37	63	19	23	17
11	3	14	16	$b_j \backslash a_i$

Solution procedure:

Knowing the value c^1_{12} =2 , we apply formula: $p^k_{ij} = \dfrac{c^k_{ij} - \underline{c}^k_{ij}}{\overline{c}^k_{ij} - \underline{c}^k_{ij}}$ in order to determine the probabilistic parameter of belonging, that is: $p^k_{ij} = \dfrac{1}{2}$. Let suppose that it is the same for all cost coefficients. Because in the variation interval $[\underline{c}^1_{12};\overline{c}^1_{12}]$ for the value c^1_{12} =2 we found the increase in cost, to determine the other cost coefficients, we apply the next formula:

$c_{ij}^k = p \cdot \left(\overline{c}_{ij}^k - \underline{c}_{ij}^k\right) + \underline{c}_{ij}^k$. We obtain the next data for the objective functions coefficients:

Cost 1, 2 =

1 4	2 4	7 3	7 4
1 5	9 8	3 9	4 10
8 6	9 2	4 5	6 1

By using the above proposed ***Algorithm 2*** we have found the following 11 efficient basic solutions:

$X^1 = \left(x_{11} = 8, x_{21} = 3, x_{22} = 2, x_{23} = 14, x_{33} = 1, x_{34} = 16\right),\ S^1 = \left(76,207,68\right);$

$X^2 = \left(x_{11} = 8, x_{21} = 3, x_{22} = 3, x_{24} = 13, x_{33} = 14, x_{34} = 3\right), S^2 = \left(64,276,68\right);$

$X^3 = \left(x_{11} = 6, x_{14} = 2, x_{21} = 5, x_{23} = 14, x_{32} = 3, x_{34} = 14\right),\ S^3 = \left(78,203,68\right);$

$X^4 = \left(x_{11} = 8, x_{21} = 3, x_{23} = 14, x_{24} = 2, x_{32} = 3, x_{34} = 14\right),\ S^4 = \left(72,213,68\right);$

$X^5 = \left(x_{11} = 8, x_{21} = 3, x_{24} = 16, x_{32} = 3, x_{33} = 14\right),\ S^5 = \left(58,283,68\right);$

$X^6 = \left(x_{13} = 8, x_{21} = 11, x_{22} = 2, x_{23} = 6, x_{32} = 1, x_{34} = 16\right),\ S^6 = \left(208,167,73\right);$

$X^7 = \left(x_{13} = 6, x_{14} = 2, x_{21} = 11, x_{23} = 8, x_{32} = 3, x_{34} = 14\right),\ S^7 = \left(202,173,73\right);$

$X^8 = \left(x_{12} = 2, x_{13} = 6, x_{21} = 11, x_{23} = 8, x_{32} = 1, x_{34} = 16\right),\ S^8 = \left(186,171,95\right);$

$X^9 = \left(x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16\right),\ S^9 = \left(176,175,95\right);$

$X^{10} = \left(x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{24} = 13, x_{33} = 14, x_{34} = 3\right),$

$S^{10} = \left(143,265,95\right);$

$X^{11} = \left(x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{23} = 13, x_{33} = 1, x_{34} = 16\right),\ S^{11} = \left(156,200,95\right).$

Finally, I'd like to mention that we solved the problem only for the probabilistic parameter of belonging: $p = \frac{1}{2}$. We observe, that for this parameter value the data of problem coincide with the data from the example of Aneja and Nair article [1]. We can mention that, using the proposed ***Algorithm 2,*** we obtained with 2 efficient basic solutions more compared as the authors' results from this article.

REFERENCES

[1] **Aneja, Y.P. K.P.K. Nair (1979),** ***Bicriteria Transportation Problem;*** *Management Sci.*, V.25, N.1, p.[73-79] ;

[2] **Belmann, R., L. Zadeh (1970),** ***Decision Making in a Fuzzy Environment***; *Management Sci*.,17B, p.[141–164];

[3] **Bit, A.K.,M.P. Biswal, S.S. Alam (1992),** ***Fuzzy Programming Approach to Multicriteria Decision Making Transportation Problem***; *Fuzzy Sets and Systems* 50, p.[35–41];

[4] **Chanas, St., Dorota Kuchta (1996),** ***A Concept of the Optimal Solution of the Transportation Problem with Fuzzy Cost Coefficients***; *Fuzzy Sets and Systems* 82 , p.[299-305];

[5] **Das, S.K.,A. Goswami, S.S. Alam (1999),** ***Multiobjective Transportation Problem with Interval Cost, Source and Destination Parameters;*** *European Journal of Operational Research,* Volume 117, Issue 1, p.[100–112];

[6] **Debashis Dutta, A. Satyanarayana Myrthy (2010),** ***Multi-Choice Goal Programming Approach for a Fuzzy Transportation Problem****; IJRRAS*, 2(2), 2010, 132-139;

[7] **Geetha, S., K.P.K. Nair ,** ***A Stochastic Bottleneck Transportation Problem****;* *Journal of Operational Research Soc*. V.45, No.5, p.[583-588];

[8] **Gupta, B.,R. Gupta (1983),** ***Multi-Criteria Simplex Method for a Linear Multiple-objective Transportation Problem***. *Indian Jour. Pure, Appl.Math.*14(2), p.[222-232];

[9] **Guzel Nuram(2010),** ***Fuzzy Transportation Problem with the Fuzzy Amounts and the Fuzzy Costs*** *;World Applied Sciences Journal* 8(5), p.[543-549];

[10] **Han, S. Li, X. (2004),** ***Fuzzy Programming Approach Solution for Multi-objective Solid Transportation Problem****; Journal of Southeast University* (English Edition) 20 (1), p.[102-107];

[11] **Isermann H.(1979),** ***The Enumeration of all Efficient Solutions for a Linear Multiple-objective Transportation Problem***; N.R.L.Q., V.26, N1, p.[123-139];

[12] **Liang, T.F. (2008),** ***Fuzzy Multi-objective Production/Distribution Planning Decisions with Multi-product and Multi-time Period in a Supply Chain;*** *Computers and Industrial Engineering* 55 (3), p.[676-694];

[13] **Ringuest, J.L.,D.B. Rinks (1987),** ***Interactive Solutions for the Linear Multiobjective Transportation Problems***. *European Journal of Operational Research* ,32, p.[96–106];

[14] **Sharma,S.R., K. Swarup (1977),** ***Transportation Fractional Programming with Respect to Time****;* Ricerca Operativa, 7(3), p.[49-58];

[15] **Shiang-Tai Liu (2006),** ***Fuzzy Total Transportation Cost Measures for Fuzzy Solid Transportation Problem****; Applied Mathematics and Computation* 174 , p.[927–941];

[16] **Surapati, P., Roy (2008),** ***Multiobjective Transportation Model with Fuzzy Parameters: A Priority Based Fuzzy Goal Programming Approach****; Journal of Transportation Systems Engineering and Information Technology* 8 (3), p.[40-48];

[17] **Tkacenko, A.I. (2004),** ***The Multiobjective Transportation Fractional Programming Model****; Computer Science Journal of Moldova*, V12, N3(36), Chisinau, p.[397-405];

[18] **Waiel F. Abd El-Waheda, Sang M. Lee (2006),** ***Interactive Fuzzy Goal Programming for Multi-objective Transportation Problems****; The International Journal of Management Sci., Omega* 34 , p.[158 – 166];

[19] **Wild, Bill.Y., K. R. Karwan, M.H. Karwan(1994),** ***The Multiple Bottleneck Transportation Problem****; Computer Ops. Res*., Vol.20, No.3, p.[261-274];

[20] **Zaki, Sayed A. , Abd Allax A. Mousa, Hamdy M. Geneedi, Adel Y. Elmekawy (2012),** ***Efficient Multiobjective Genetic Algorithm for Solving Transportation, Assignment and Transsipment Problems***; *Applied Mathematics*, 3, p.[92-99];

[21] **Zangiabadi, M.,H. R. Maleki (2007),** ***Fuzzy Goal Programming for Multiobjective Transportation Problems;*** *Journal of Applied Mathematics and Computing*, V. 24, Issue 1-2, p.[449-460];

[22] **Zimmeramann, H.J. (1978),** ***Fuzzy Programming with Several Objective Functions****; Fuzzy Sets and Systems* 1, p.[45-55].