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SOLVING A NEW PRIORITY M/M/C QUEUE MODEL FOR A MULTI-MODE HUB COVERING LOCATION PROBLEM BY MULTI-OBJECTIVE PARALLEL SIMULATED ANNEALING

Abstract. *In this paper, a new priority M/M/c queuing hub covering problem is presented, in which products with high priority are selected for service ahead of those with low priority. In addition, a mixed-integer nonlinear mathematical programming model is presented to find a good solution of the given problem. Due to its computational complexity, we propose a multi-objective parallel simulated annealing (MOPSA) algorithm and a new solution representation is developed to solve a number of instances. Furthermore, some computational experiments are provided to illustrate the efficiency of the presented model and proposed MOPSA. In order to show the higher performance of the proposed MOPSA, two well-known evolutionary algorithms, namely a non-dominated sorting genetic algorithm (NSGA-II) and Pareto Archive Evolution Strategy (PAES), are considered and the related results are compared and analyzed. Finally, the conclusion is presented.*

Keywords: *Hub covering problem; Multi-objective optimization; Priority queuing model; Multi-mode transportation; Parallel simulated annealing.*

JEL classification: C44, C61, L91

1.INTRODUCTION

A hub-and-spoke system has been greatly used in telecommunication and transportation areas. The main purpose of creating hub networks is to transfer commodities (e.g., goods or passengers) between origin and destination nodes through the intermediate point, which is called hub, instead of using the direct link between every origin–destination node. Moreover, the function of a hub facility is to collect, switch, sort and transfers commodity, and it decreases the number of links in the network. Therefore, hubs reduce costs and enhance the efficiency of the network.

In a *p*-hub location problem (*p*HLP), the number of hub nodes is predetermined. All types of hub models can be divided into two major parts, namely single and multiple allocation hub location problems. In single-allocation problems, each node may send and receive flows through an exactly one hub; whereas in multiple allocation problems, there is no this restriction and every node can send and receive flows through all hubs. Moreover, in hub location problems (HLPs), the objective is to minimize the total cost of locating hubs and transporting flows through the hubs. Alumur and Kara (2009) divided hub location problems in four categories based upon their objective functions: (1) Fixed-cost hub location; (2) *p*-hub median; (3) *p*-hub center and (4) *p*-hub covering problems. Interested readers are referred to comprehensive reviews proposed by Alumur and Kara (2008) and Campbell et al. (2002) and more recently Zanjirani Farahani et al. (2013) for surveys on HLPs.

Costa et al. (2008) presented a multi-objective hub location problem, in which the first objective minimizes the total travelling cost, whereas the second one minimizes the maximum service time of the hub nodes. In this problem, each non-hub node is allocated to one hub node and the number of hub nodes is predefined and denoted by *p*. Ernst et al. (2002) proposed an integer programming formulation for uncapacitated *p*-hub center problems with both single and multiple allocations. They also presented a branch-and-bound approach for solving the multiple allocation problems. Correia et al. (2010) proposed a capacitated single-allocation hub location problem that the capacity of the hub is a part of the decision making process. They also considered the balancing requirements on the network, in which the maximum and minimum number of spoke nodes allocated to the hubs are limited. Marianove and Sera (2003) introduced a new formulation of hub location in airline networks behaving as an M/D/c queue system. They enforced a probabilistic constraint that the probability of having more than a specified number of air planes in a queue should belimited. Then, they solved the model by using of the tabu search algorithm. Mohammadi et al.

(2011) presented a capacitated single-allocation *p*-hub covering problem behaving as M/M/c queues. They presented a nonlinear integer programming formulation with a new expression for arrival rate and a different flow. Then, they applied an approach for linearization of the model and solved the model by an improved meta-heuristic algorithm based on the imperialist competitive algorithm (ICA). Ishfaq and Sox (2012) designed an integrated hub operation queuing model and hub location-allocation model and studied the effect of limited hub resources on the design of inter-modal logistics networks under service time requirements.

Mohammadi et al. (2013a) developed a stochastic multi-objective multi-mode transportation model for a hub covering location problem under uncertainty. The transportation time between each pair of nodes is considered as an uncertain parameter, which is influenced by a risk factor in the network and this model is solved by a multi-objective imperialist competitive algorithm. Alumur et al. (2012) proposed a multi-modal hub location problem, which different transportation modes between hubs and different types of service time between the origin–destination nodes are considered as well as an efficient heuristic algorithm is developed for solving the model. In a new variant of HLPs, Mohammadi et al. (2013b) proposed a hub location-routing problem where each hub acts as a depot and should route the demand of its allocated nodes. They solved their multi-objective model using a new invasive weed optimization (IWO) algorithm.

Another feature of designing a hub network is to consider the mode of transportation. There are usually different types of transportation modes in the most of hub location models (e.g., as air, ground and water). Various studies are performed on the design of an inter-modal transportation network. Bontekoning et al. (2004) and Crainic and Kim (2007) also reviewed the application of transportation.

In this paper, we study the capacitated multi-modal multi-product *p*-hub median problem with a queuing system. The major differences of this paper with the other studies are as follows:

- A single allocation strategy is considered so that each non-hub node allocated to exactly one hub;
- The number of hubs is pre-determined;
- Each hub has a certain radius to cover the non-hub nodes;
- The capacity of each hub is limited;
- Hub network is fully interconnected;
- Designing and modeling multi-product, multi-modal hub location problem;

- Incorporating a priority M/M/c queue model to address the waiting times of products in the hub nodes;
- Proposing a multi-objective model that the total cost and the maximum time of transportation should be minimized;
- Developing a multi-objective parallel simulated annealing (MOPSA) algorithm for solving this problem.

In this model, decisions made in this model consist of (1) selection of *p* hub nodes, (2) the allocation of non-hub nodes in these located hubs and (3) selection of the transportation mode between hubs. In the next section, we describe the mathematical model. Because of the complexity of hub location problems, many researchers performed meta-heuristics and found near-optimal solutions. Finally, we also propose multi-objective parallel simulated annealing (MOPSA) for solving this problem. The remainder of this paper is organized as follows. Section 2 introduces a mathematical formulation for multi-objective multi-modal hub location. Section 3 describes the proposed MOPSA to solve the problem. Section 4 presents the computational results. Conclusions are discussed in Section 5.

2. Mathematical formulation

In HLPs, there is a set of n nodes that some of them can be selected as hubs. In the proposed mathematical model, the *p* hubs are located and the remaining non-hub nodes are allocated to these located hubs in the network. Each hub can cover only a limited number of nodes. We now present a multi-objective formulation of the HLP minimizing total cost and maximum total time in the hub network. The model considers the priority M/M/c queue system in each hub. There are also different modes of transferring flow between hubs. We first present the notations; then the model presents.

2.1. Indices

n:{1,…,N} Set of nodes.
c:{1,…,C} Set of products.
m:{1,…,M} Set of modes.

2.2. Parameters

w_{ij}^{c} Flow to be sent from node i to node j for product c ($i,j \in$ N).
c_{ik}^{c} Cost of sending product c from node i to node k ($i,k \in$ N).

f_k Fixed cost of opening hub at node *k*.

t_{ik}^c Transportation time of sending product *c* from node *i* to hub *k*($i,k \in$ N).

$O_i^c = \sum_{j \in N} W_{ij}^c$ Total flow originating at node *i* for product *c*.

$D_i^c = \sum_{j \in N} W_{ji}^c$ Total flow destined for node *i* for product *c*.

P Number of hubs that should be established.

λ_k Arrival rate of product *c* at hub *k*.

μ_k Processing rate of product *c* at hub *k*.

ς_k Number of service providers at hub *k*.

C_k^m Modal connectivity cost of serving mode *m* at hub *k*.

2.3. Variables

x_{ijkl}^{cm} 1 if product *c* travels from node *i* to *j* through hub pair (*k*,*l*) using mode *m* occurred; 0, otherwise.

S_k^m is 1 if hub *k* is served by mode *m*; 0, otherwise.

z_k is 1 if node *k* is a hub; 0, otherwise.

wt_k^c Waiting time at node *k* for product *c*.

T Maximum allowed time between each pair of nodes .

2.4. Mathematical model

According to the above notations, we present a mathematical formulation for the single-allocation multi-modal hub location network design problem. The aim of this problem is to decide to locate hub nodes and allocate non-hub nodes to these located hubs. The flow of each origin–destination pair receives service within the given service time bounds and the waiting time in each hub follows the priority queuing systems. The objectives of this model are to minimize the total costs and minimize the maximum transportation time. Respect to the above descriptions, the multi-objective problem can be developed as follows.

$$\text{Min} \sum_{m} \sum_{c}^{C} \sum_{i}^{I} \sum_{k}^{K} \sum_{l}^{L} \sum_{j}^{J} W_{ij}^{c} (C_{ik}^{c} + \propto C_{kl}^{c} + C_{lj}^{c}) x_{ijkl}^{mc} + \sum_{k} f_k z_k + \sum_{m} \sum_{k} C_k^m S_k^m \tag{1}$$

$$\text{Min } T \tag{2}$$

s.t.

$$x_{iklj}^{mc} \leq z_k \qquad \forall i, k, l, m, c \tag{3}$$

$$x_{iklj}^{mc} \leq z_l \qquad \forall i, k, l, m, c \tag{4}$$

$$\sum_{k}^{K} z_k = P \tag{5}$$

$$\sum_{k}^{K} \sum_{l} \sum_{m} x_{iklj}^{mc} = 1 \qquad \forall i, j, c \tag{6}$$

$$x_{iklj}^{mc} \leq S_k^m \tag{7}$$

$$x_{iklj}^{mc} \leq S_l^m \tag{8}$$

$$x_{ik}^{c} \leq 1 - z_i \forall i, k, c \tag{9}$$

$$\lambda_k^c = \sum_{m} \sum_{j} \sum_{i} \sum_{l} w_{ij}^{c} . x_{iklj}^{mc} \qquad \forall k, c \tag{10}$$

$$wt_k^c = \frac{\sum_m S_k^m}{[\varsigma_k! (\varsigma_k \mu_k - \lambda_k^c)(\frac{\lambda_k^c}{\mu_k})^{\varsigma_k} \sum_{n=0}^{\varsigma_k - 1} \frac{(\frac{\lambda_k^c}{\mu_k})^n}{n!} + \varsigma_k \mu_k](1 - \sum_{n=1}^{\varsigma_k - 1} \frac{\lambda_k^c}{c_k \mu_k})(1 - \sum_{n=1}^{\varsigma_k} \frac{\lambda_k^c}{c_k \mu_k}))} \tag{11}$$

$$(t_{ik}^{c} + wt_k^c + \propto t_{kl}^{cm} + wt_l^c + t_{lj}^{c}) x_{iklj}^{c} \leq T \quad \forall i, j, k, l, c \tag{12}$$

$$x_{ik}^{c}, z_k^q, x_{iklj}^{c}, S_k^m \in \{0,1\} \tag{13}$$

In this model, the first objective function minimizes the sum of transportation and fixed costs of locating the hubs while the second one minimizes the maximum total time including transportation time and waiting time. Constraints (3) and (4) enforce that every hub pair assignment for an origin–destination pair is restricted to open hubs. Constraint (5) shows that p hub is selected. Constraint (6) ensures only one hub pair (k,l), and a specific mode m can be selected for each non-hub nodes (i,j). Constraints (7) and (8) assure that if transportation of product c from node i to j through a hub pair (k, l) by using of mode m is occurring, hubs k and l should be served by mode *m*. Constrain (9) assures that only non-hub nodes can be allocated to the hubs. Equation (10) calculates the arrival rate of each product at each hub. Equation (11) also calculates the waiting time at each hub. Constraint (12) ensures that the total transportation time and the waiting time in hub nodes should be less than the maximum allowed time, which will be determined by the second objective function. Finally, Constraints (13) show the type of variables.

3. PROPOSED MULTI-OBJECTIVE ALGORITHM

Finding a feasible solution for the problem at hand is challenging. Therefore, in this section, we apply a multi-objective parallel simulated annealing (MOPSA) algorithm to solve the considered problem. Simulated annealing (SA) is a meta-heuristic algorithm introduced by Metropolis et al. (1953) for solving continuous optimization problems. The steps of the proposed solution algorithm are described in the next subsections.

3.1. Proposed MOPSA

In this paper, a multi-objective simulated annealing (MOSA) algorithm is proposed, in which more than one solution is used for searching in the solution space to obtain Pareto-optimal solutions. Thus, one kind of the MOSA algorithm called multi-objective parallel SA (MOPSA) is proposed. Additionally, this algorithm is compared with two well-known meta-heuristic algorithms, namely NSGA-II and PAES. The better performance of the proposed MOPSA is presented for validating a higher performance of MOPSA in the HLP. Hence, the proposed MOPSA is able to search the solution space broadly and achieve to the non-dominated Pareto solutions in less computational CPU time.

3.2. Solution representation

The first step in designing a meta-heuristic algorithm is to decide how to show the solutions in an efficient way. In the other words, every HLP solution encoding procedure must show the location of hubs and the allocation of non-hub nodes to the located hubs. Solution representations in the previous papers are mostly applied in the form of a discrete representation, and integer values are used for representing the solutions. One of the great disadvantages of discrete representation is the high probability of creation the infeasible solutions during the random search. In this paper, the continuous solution representation proposed by Mohammadi et al. (2014) is used.

3.3. Initial solution

The initial solution of the proposed PSA, unlike the classical SA, consists of more than one solution shown in Figure 1. *NPop* is a number of initial solutions and is an input parameter of PSA. The initial solutions are appeared randomly based on the previous section.

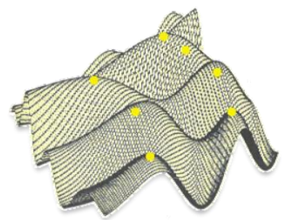

Figure 1. Search space with more than one initial solution

3.4. Neighborhood search

The proposed algorithm uses three steps for searching the initial solution neighborhood for achieving to better solutions in the each iteration. These three steps include mutation, assimilation and crossover that will be described in details as follows.

3.4.1. Mutation step

Mutation of initial solution is the first step in searching the neighborhood solutions and is done based on the predefined number *nMutate.* The mutation consists of two parts: 1) mutation for the continuous part of the solution and 2) mutation for discrete part of the solution. In the first part, there are three different procedures for mutating continues part including: a) swap, b) reversion and c) inversion mutation (Figure 2). In the swap mutation, two random bits are determined and the places of them are exchanged. In the reversion mutation, permutation of the random part of the solution is reversed. In the inversion mutation, one bit is selected randomly and its value is replaced by a new random value. Furthermore, two different strategies are also proposed for mutating the discrete part of each solution. First, the discrete part maintains unchanged. Second, one bit is randomly chosen and replaced with another random number which is not equal to previous one.

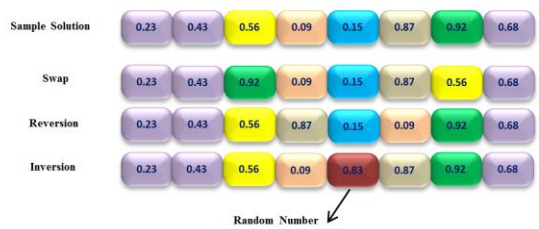

Figure 2. Three steps of mutation

3.4.2. Assimilation step

After mutation, the *nPop* group of mutated solution with a number of *mutational* is existed. For instance, if *nPop* is equal to 5 and *nMutate* is equal to 10, then we have 50 new mutated solutions consisting of 5 groups and 10 solutions in each group. Then, the mutated solutions called colonies start their moving toward the best solution called imperialist in each group by the use of an assimilation operator in imperialist competitive algorithm as shown in Figure 3, where *d* shows the distance between the colony and the imperialist, *x* shows the amount of moving of colony toward the imperialist, which is uniformly distributed between 0 to $\beta \times d$, where β is a number between 1 and 2. The colony moves toward its imperialist in other direction with an θ angle, which is uniformly distributed between $-\gamma$ and γ. It has been considered that $\pi/4$ is the most suitable value of γ. Finally, the best solution in the each iteration is determined by the use of non-dominance strategy and crowding distances.

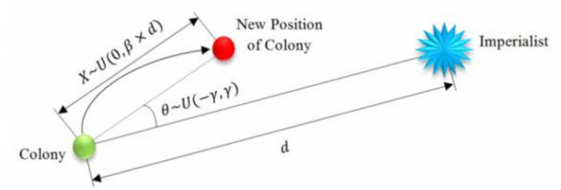

Figure 3. Moving colonies toward the imperialist with a random angle θ

3.4.3. Crossover step

Now, all solutions created in the two previous steps combined by the use of GA crossover. Three different crossover operators (i.e., included one-point crossover, two-point crossover and uniform crossover) are applied for both continuous and discrete parts, separately (Figure 4). It should be mentioned that operators *a* and *b* are used for both continues and discrete parts, whereas operator *c* is used differently for these parts. In this paper, we apply a binary tournament selection for choosing parents.

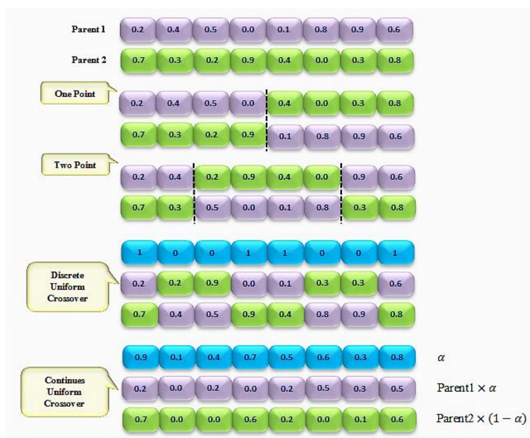

Figure 4. Three steps of crossover

3.5. Next population

The proposed MOPSA should have initial solutions to apply three main neighborhood search strategies to achieve non-dominated Pareto solutions. We use the non-dominance strategy and crowding distance metric for choosing the *nPop* better solutions among all the created solutions for applying in the next iteration's initial solutions. While two corresponding solutions are being compared to choose the next iteration initial solutions, three different cases may occur:

1. If one solution dominates another one, it will be chosen.
2. If two solutions cannot dominate each other, a solution with higher crowding distance metric will be selected for the next iteration.
3. If the old solution dominates the new one, the probable acceptance function of simulated annealing will be accepted as stated in the next section.

3.6. Probability acceptance function

It is possible that a new solution with the worsen objective function value is accepted with a small probability determined by the Boltzmann function, $\exp(-\Delta/kT)$, where Δ is the difference of objective function values between the current and new solution, *k* is a constant and *T* is the current temperature. The worse solution will be applied for the next iteration if a created random value *r* is less than probability value *P*. The Boltzmann function and Δ in MOPSA are as bellows:

$$P = e^{\frac{-\Delta f}{T}} \tag{14}$$

$$\Delta f = \left| \frac{(f_1(x) - f_1(y))}{f_1(x)} + \frac{(f_2(x) - f_2(y))}{f_2(x)} \right| \tag{15}$$

where *x* is the new solution, *y* is the old solution and f_1 and f_2 are the first and second objective functions, respectively. In the Boltzmann function, *T* shows the current temperature that is calculated by:

$$T_{n+1} = \alpha . T_n \tag{16}$$

where α is a constant that takes a value between 0 and 1 and it is used for decreasing the temperature at each next iteration.

3.7. Stopping criteria

In this paper, the number of function calls (NFCs) is used as the stopping criteria. The Pseudo code of the proposed MOPSA is illustrated in Figures 5.

```
NFC ← 0
set the parameters of PSA (nPop, nMutate, pCrossover, α, β, T0)
create initial solution ← nPop
terminate ← false
while (terminate = false) do
        mutate each initial solution ← nMutate
        find the best solution (imperialist) ← non-dominated sorting & crowding distance
        update NFC
        assimilate mutated solutions (colonies) toward imperialist ← β
        merge whole new created solution
        apply crossover ← pCrossover
        update NFC
        find better new solutions ← nPop
        update NFC
        if (new solution dominates old solution) then
            accept the new solution
        else if (no one dominate the other one) then
            calculate crowding distance- CD of each solution
```

```
            if (CD of new solution > CD of old solution) then
                accept the new solution
            end if
        else
            apply probable acceptance function ← P = e^(−Δf/T) ← Δ & T
            create random value r
            if (r < P) then
                accept new solution
            else
                accept old solution
            end if
        end if
        update NFC
        T ← α × T
        if (NFC = predefined value) then
            terminate = true
        end if
end while
```

Figure 5. The Pseudo code of proposed MOPSA

NSGA-II assumptions and parameters value

- The initial population is randomly generated.
- The crossover operator is exerted on random selected solutions using one of these operators, namely one-point crossover, two-point crossover and uniform crossover, randomly.
- The mutation operator is executed on random selected solution using one of these operators, namely inversion, swap and reversion, randomly.
- The crossover and mutation rates are set to 0.75 and 0.3, respectively, using the RSM method.
- The number of the initial population is set to 200 and 300 for small and large-sized problems, respectively.
- The NFCs stopping criteria is set on 40000 and 120000 for small and large-sized problems, respectively.

PAES assumptions and parameters value

- The size of archive is 200.
- One of these operators, namely inversion, swap and reversion, are selected randomly as revolution.
- The NFCs stopping criteria is set on 40000 and 120000 for small and large-sized problems, respectively.

3.8. Comparison metrics

To certify the performance of the proposed MOPSA, four comparison metrics are applied as follows.

1. *Quality metric* (*QM*): The contribution of each algorithm from Pareto solutions is shown by this index, in which simply measured by putting together the non-dominated solutions found by the algorithms and calculating the ratios between non-dominated solutions of each algorithm, where an algorithm with a higher value of the QM has better performance.

2. *Mean ideal distance* (*MID*): The closeness between Pareto solutions and ideal point (f_1^{best}, f_2^{best}) is determined using *MID* index. The *MID* index is calculated by:

$$MID = \frac{\sum_{i=1}^{n} \sqrt{\left(\frac{f_{1i} - f_1^{best}}{f_{1,total}^{max} - f_{1,total}^{min}}\right)^2 + \left(\frac{f_{2i} - f_2^{best}}{f_{2,total}^{max} - f_{2,total}^{min}}\right)^2}}{n} \quad (17)$$

where *n* is the number of non-dominated solutions and $f_{i,total}^{max}$ and $f_{i,total}^{min}$ are the maximum and minimum values of each fitness functions among the all n on-dominated solutions obtained by the algorithms, respectively. Regarding to this definition, the algorithm with a lower value of *MID* has a better performance.

3. *Diversification metric* (*DM*): This metric reports the spread of the Pareto solutions set and is measured by Equation (18). Regarding to this definition, the algorithm with a higher value of *DM* has a better performance.

$$DM = \sqrt{\left(\frac{\max f_{1i} - \min f_{1i}}{f_{1.total}^{max} - f_{1.total}^{min}}\right)^2 + \left(\frac{\max f_{2i} - \min f_{2i}}{f_{2.total}^{max} - f_{2.total}^{min}}\right)^2} \quad (18)$$

4. *Spacing metric* (*SM*): This metric measure the uniformity of the spread of the non-dominated set solutions. This metric is defined according to Equation (19). Regarding to this definition, the algorithm with a lower value of *SM* has a better performance.

$$SM = \frac{\sum_{i=1}^{n-1} |\bar{d} - d_i|}{(n-1)\bar{d}} \quad (19)$$

where, d_i is the Euclidean distance between consecutive solutions in the obtained non-dominated set of solutions and $\bar{d}$ is the average of these distances.

3.9. Data generation

The required data for the proposed problem consists of a number of nodes, number of hubs, transportation cost, transportation time, risk factor, fixed costs, capacity of vehicles and hubs, distance between nodes and flow and covering radius. The value and distribution of the above-mentioned input parameters are shown in Table 1. Some special number of hubs has been considered for each number of nodes. Also, each problem instance is shown as "*Number of nodes # number of hubs*" (e.g., 50#8 means 50 nodes and 8 hubs).

Table 1. Value and distribution of input parameters

	Parameter				
	n	*n*	*P*	*F*	*W*
Value & Distribution	10	40	U ~ (10,20)	U ~ (200,600)	U ~ (1,10)
	15	50			
	20	70	*C*	*r*	
	25	100	U ~ (1,10)	U ~ (1,10)	
	30				

4. COMPUTATIONAL RESULTS

The proposed MOPSA is used on a number of test problems and the performance of this algorithm is compared with NSGA-II and PAES. Table 2 shows the above four comparison metrics for small-sized problems. Tables 3 and 4 list them for medium-sized problems. Tables 5 and 6 show that the proposed MOPSA outperforms the NSGA-II and PAES in large test problems.

Table 2. Comparison metrics for small-sized problems

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
10#3	0	0.205	0.795	0.525	0.727	0.741
10#4	0	0.354	0.646	0.372	0.744	0.778
15#3	0	0.200	0.800	0.788	0.791	0.920
15#4	0	0.133	0.867	0.695	0.738	0.823
15#5	0	0	1	1.012	0.571	0.914
20#3	0	0	1	1.084	0.499	0.900
20#4	0	0.272	0.727	1.354	0.793	0.970
20#5	0.179	0.388	0.433	0.827	0.583	0.892
20#6	0	0.362	0. 638	1.131	0.853	0.984
25#3	0.203	0.200	0.597	1.026	1.101	1.245

25#4	0	0.263	0.737	0.987	1.311	0.851
25#5	0	0.165	0.835	1.151	0.945	1.238
25#6	0	0	1	0.997	0.964	1.395
30#3	0	0	1	1.105	0.731	1.176
30#4	0.246	0.159	0.595	1.002	1.089	1.103
30#5	0	0	1	0.993	0.984	0.998
30#6	0.043	0.207	0.750	0.858	0.570	0.899
30#7	0.097	0.138	0.765	1.080	0.939	1.094
30#8	0	0	1	0.567	0.536	0.645

Problem No.	**Diversity Metric (DM)**			**Mean Ideal Distance (MID)**		
	PAES	**NSGA-II**	**MOPSA**	**PAES**	**NSGA-II**	**MOPSA**
10#3	0.990	1.289	1.682	0.695	0.653	0.573
10#4	1.058	1.245	1.837	0.587	0.614	0.546
15#3	0.346	0.831	1.568	0.762	0.838	0.431
15#4	1.431	0.532	0.959	0.851	0.699	0.329
15#5	1.158	0.463	0.944	0.741	0.396	0.361
20#3	1.270	1.323	1.087	0.783	0.716	0.537
20#4	1.269	0.831	0.903	0.751	0.548	0.342
20#5	1.150	0.971	1.319	0.893	0.529	0.518
20#6	1.051	1.100	1.188	0.743	0.685	0.623
25#3	0.538	1.391	1.456	0.535	0.681	0.258
25#4	0.947	0.995	1.288	0.783	0.653	0.710
25#5	0.739	1.148	1.329	0.593	0.555	0.521
25#6	0.957	0.537	1.463	0.582	0.474	0.263
30#3	1.043	0.513	1.224	0.734	0.684	0.633
30#4	0.929	1.179	1.188	0.564	0.737	0.459
30#5	0.863	1.146	1.354	0.723	0.800	0.513
30#6	1.324	0.683	1.130	0.469	0.286	0.357
30#7	1.236	0.743	0.945	0.638	0.458	0.281
30#8	0.632	0.789	1.263	0.853	0.588	0.513

Table 3. Comparison metrics for a large test problem with 40 nodes

Problem No.	**Quality Metric (QM)**			**Spacing Metric (SM)**		
	PAES	**NSGA-II**	**MOPSA**	**PAES**	**NSGA-II**	**MOPSA**
40#3	0	0.083	0.916	1.210	1.412	1.488
40#4	0.198	0.382	0.419	1.272	0.819	1.362
40#5	0	0.398	0.602	1.198	1.671	1.694
40#6	0	0.039	0.960	1.372	1.164	1.327
40#7	0	0.393	0.607	1.258	0.931	1.452
40#8	0.143	0.298	0.558	1.146	1.057	1.034
40#9	0	0.298	0.701	1.139	1.177	1.439
40#10	0.104	0.184	0.711	1.319	0.973	1.134

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
40#3	1.147	1.125	1.3690	0.692	0.735	0.385
40#4	1.003	1.137	1.183	0.591	0.631	0.442
40#5	0.993	1.175	1.198	0.603	0.778	0.452
40#6	1.058	1.083	1.198	0.589	0.683	0.412
40#7	0.696	1.201	1.087	0.675	0.853	0.418
40#8	0.512	1.189	0.731	0.425	0.738	0.337
40#9	1.258	0.914	1.148	0.653	0.568	0.426
40#10	1.148	1.153	1.408	0.658	0.724	0.375

Table 4. Comparison metrics for a large test problem with 50 nodes

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
50#3	0.082	0.372	0.536	1.485	1.345	1.473
50#4	0	0	1	1.298	1.352	1.255
50#5	0	0.135	0.865	1.583	1.451	1.473
50#6	0	0	1	1.249	1.045	0.854
50#7	0	0.160	0.840	1.168	1.132	1.416
50#8	0	0.95	0.915	1.142	1.365	0.983
50#9	0	0	1	1.142	0.583	1.362
50#10	0.970	0	0.903	1.067	1.023	0.855
50#11	0	0	1	0.934	0.972	0.819
50#12	0	0	1	1.121	1.001	1.133

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
50#3	1.179	0.802	1.264	0.635	0.784	0.603
50#4	0.723	0.738	1.285	0.593	0.684	0.391
50#5	1.184	1.245	1.273	0.658	0.511	0.495
50#6	1.106	0.895	0.937	0.684	0.783	0.425
50#7	0.953	0.673	1.319	0.584	0.615	0.398
50#8	1.054	1.035	0.812	0.313	0.598	0.249
50#9	0.851	0.979	1.178	0.570	0.790	0.330
50#10	0.489	1.068	1.294	0.791	0.561	0.389
50#11	1.362	0.894	1.481	0.573	0.660	0.650
50#12	0.983	0.715	1.108	0.689	0.546	0.239

Table 5. Comparison metrics for a large test problem with 70 nodes

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
70#3	0	0.250	0.750	0.481	0.652	1.005
70#4	0	0	1	1.197	0.618	0.893
70#5	0	0	1	0.694	0.593	0.894
70#6	0	0.473	0.627	0.389	0.627	0.768
70#7	0	0	1	0.538	0.874	0.917
70#8	0	0	1	1.067	0.493	0.458
70#9	0.056	0	0.944	0.654	1.184	0.898
70#10	0	0	1	0.773	0.963	0.873
70#11	0	0	1	1.089	0.756	0.973
70#12	0	0	1	0.564	0.695	0.694
70#13	0	0	1	1.046	1.083	1.175
70#14	0	0.452	0.547	0.921	0.795	0.823
70#15	0	0	1	1.057	0.593	1.094
70#16	0	0	1	0.498	1.034	0.876

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
70#3	0.189	0.578	1.060	1.001	0.489	0.130
70#4	1.060	0.201	0.465	0.727	0.525	0.264
70#5	0.236	0.431	1.075	0.876	0.961	0.366
70#6	0.464	0.827	1.348	0.523	0.686	0.180
70#7	0.499	1.070	0.912	0.841	0.702	0.350
70#8	1.173	0.721	0.750	0.851	0.511	0.371
70#9	0.500	1.200	1.055	0.571	0.651	0.261
70#10	0.610	1.090	0.590	0.761	0.701	0.231
70#11	1.200	0.339	0.901	0.671	0.281	0.264
70#12	1.100	0.570	0.701	0.851	0.651	0.131
70#13	1.250	0.610	0.835	0.741	0.523	0.235
70#14	1.140	0.920	0.783	0.764	0.498	0.460
70#15	1.011	0.181	1.061	0.730	0.462	0.262
70#16	0.601	0.751	0.915	1.030	0.853	0.213

- The proposed MOPSA has more contribution of obtaining Pareto optimal solutions with considerably higher qualities in comparison with both NSGA-II and PAES.
- The proposed MOPSA provides non-dominated solutions that have less average values of the spacing metric. These data reveal that non-dominated solutions obtained by the proposed MOPSA are more uniformly distributed in comparison with both NSGA-II and PAES.

- The average values of the diversification metric in the proposed MOPSA are greater than NSGA-II (i.e., MOPSA finds non-dominated solutions that have more diversity), and in most of the test problems, the MID values in the proposed MOPSA are smaller than those of NSGA-II and PAES.

Table 6. Comparison metrics for a large test problem with 100 nodes

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
100#3	0	0	1	1.240	0.471	0.661
100#4	0	0.43	0.57	0.511	1.052	1.370
100#5	0	0	1	0.201	0.011	1.181
100#6	0.346	0	0.654	0.357	0.501	0.581
100#7	0	0	1	0.801	0.662	1.053
100#8	0	0	1	0.291	1.713	0.553
100#9	0	0	1	1.473	0.901	0.454
100#10	0	0	1	0.853	1.042	0.971
100#11	0	0	1	1.001	0.709	0.594
100#12	0	0	1	0.072	1.072	0.731
100#13	0	0	1	0.973	0.653	0.401
100#14	0.083	0.347	0.5697	0.363	1.236	1.059
100#15	0	0	1	0.858	0.920	1.368
100#16	0	0	1	1.054	0.631	1.123
100#17	0	0	1	0.894	1.138	0.775
100#18	0	0	1	1.118	0.981	1.043

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	PAES	NSGA-II	MOPSA	PAES	NSGA-II	MOPSA
100#3	0.651	0.240	0.666	0.840	1.351	0.239
100#4	0.553	1.042	1.084	0.453	0.551	0.438
100#5	0.436	1.044	1.172	0.801	0.864	0.109
100#6	1.301	0.871	0.845	0.736	0.758	0.403
100#7	1.050	0.430	1.020	0.311	0.371	0.225
100#8	0.255	1.174	0.367	0.684	0.598	0.149
100#9	1.110	0.191	0.552	0.401	0.883	0.031
100#10	0.861	1.136	0.961	0.653	0.590	0.275
100#11	1.151	0.291	0.679	0.471	0.8251	0.281
100#12	0.240	1.050	1.188	0.589	0.207	0.340
100#13	0.961	0.578	0.662	0.569	0.883	0.181
100#14	0.736	1.041	0.905	0.634	0.535	0.215
100#15	0.563	1.214	1.120	0.688	0.741	0.479
100#16	0.758	0.883	1.163	0.967	0.789	0.397
100#17	1.053	0.652	0.901	0.761	0.921	0.179
100#18	1.199	0.453	0.951	0.821	0.771	0.272

5. CONCLUSION

In this paper, we studied the capacitated multi-mode *p*-hub covering problem, in which the mode of the inter-hubs transportation is a part of the decision making process and for each potential hub a set of modes was available. The transportation mode depends on travelling time. This model follows the single-allocation principles for each product in order to each product can receive and send flow only through one hub. Since the priority of products is different, we extended the M/M/c queue model to priority M/M/c one, in which the product with high priority was selected for service ahead of that with low priority. For solving this problem, we proposed a multi-objective parallel simulated annealing (MOPSA). The performance of the proposed MOPSA was compared with GA and PAES algorithms. The results showed that the proposed MOPSA provided non-dominated solutions that had less average values of the spacing metric and its Pareto-optimal solutions considerably had higher qualities in comparison with NSGA-II and PAES.

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