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MULTIMOORA-IFN: A MCDM METHOD BASED ON INTUITIONISTIC FUZZY NUMBER FOR PERFORMANCE MANAGEMENT

***Abstract**: The paper presents a multi-criteria decision making method to tackle intuitionistic fuzzy numbers, namely MULTIMOORA-IFN. Being an extension of the crisp MULTIMOORA method, the MULTIMOORA-IFN is designed to facilitate group decision making with uncertain information. The proposed methods consists of the three parts—the Ratio System, the Reference Point, and the Full Multiplicative Form—specific with different aggregation techniques. Furthermore, the robustness of the decision making process can be improved by employing ordered aggregation operators, for instance an intuitionistic fuzzy power ordered weighted average operator or an intuitionistic fuzzy power ordered weighted geometric operator. A numerical simulation exhibits the possibilities for application of the proposed method.*

***Keywords**: MCDM, MULTIMOORA, intuitionistic fuzzy number, personnel management, group decision making, IFPOWA.*

JEL Classification: C44, M12

1. Introduction

Multi–criteria decision-making methods (MCDM) methods enable to prioritize certain alternatives in terms of multiple attributes or objectives and thus find an optimal solution from a set of available alternatives. In case none of the alternatives satisfies all the objectives, a *satisfactory* decision is made instead of an *optimal* one. It is due to Roy (1996) that MODM problems can be grouped into the four wide streams: (i) *choosing* problems aim at choosing the best alternative; (ii) *sorting* problems classify alternatives into relatively homogenous groups; (iii) *ranking* problems rank alternatives from best to worst; and (iv) *describing* problems describe the alternatives in terms of their peculiarities and features. Belton and Stewart (2002) defined the three broad categories of MODM methods, namely (i) value measurement models; (ii) goal, aspiration, and reference level models; and (iii) outranking models (the French school). In this study we will extend and apply the MOORA and MULTIMOORA methods. Furthermore, the complex nature of socio-economic phenomena requires an appropriate set of MCDM methods to make sustainable management decisions. Indeed, Sadiq and Tesfamariam (2009) distinguished between the two sources of *uncertainty* in the decision making process, viz. *vagueness* related to the lack of definite or sharp distinctions, and *ambiguity* related to imprecise assessment and definition of the alternatives. It is the fuzzy logic that provides one with the means to overcome the both types of uncertainty.

Zadeh (1965), the Founder of fuzzy logic, proposed employing the fuzzy set theory to model complex systems that are hard to define in crisp numbers. Fuzzy logic hence allows coping with vague inputs and knowledge. Linguistic reasoning relying on fuzzy logics as well as interval-valued membership function was later introduced by Zadeh (1975) to tackle ambiguity in MCDM. Later on, Atanassov (1986) introduced the intuitionistic fuzzy set which takes into account the degree of indeterminacy (non-specificity). The crisp MCDM methods, therefore, are extended into fuzzy environment so that they could handle uncertain assessment (Yazdani-Chamzini et al., 2012; Razavi Hajiagha et al., 2013). In addition, the ordered weighted operators (Yager, 1988; Xu, 2011) also increase the robustness of MCDM, for they can be applied to aggregate data without subjective assessments of the importance of certain decision makers.

This study aims at updating the crisp MULTIMOORA method with intuitionistic fuzzy number (IFN) and thus offering the MULTIMOORA-IFN. The MULTIMOORA method was proposed and Developed by Brauers and Zavadskas (2006, 2010). Baležentis et al. (2012) offered the MULTIMOORA-FG for fuzzy group decision making.

The rest of paper is organized in the following manner. Section 2 brings the theoretical preliminaries for handling IFNs. Section 3 focuses on the crisp MULTIMOORA method. The following Section 4 presents the MULTIMOORA-IFN method, with its application to personnel selection given in Section 5.

2. Preliminaries

This section describes the preliminaries for IFN-based MCDM. To be specific, the first subsection presents IFNs, whereas the second one focuses on aggregation operators.

2. 1. The intuitionistic fuzzy numbers

Zadeh (1965) introduced the use of fuzzy set theory when dealing with problems involving fuzzy phenomena. Noteworthy, fuzzy sets and fuzzy logic are powerful mathematical tools for modelling uncertain systems. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership, while fuzzy sets allow partial membership. The theoretical fundaments of fuzzy set theory are overviewed by Chen (2000). In a universe of discourse *X*, a fuzzy subset $\tilde{A}$ of *X* is defined with a membership function $\mu_{\tilde{A}}(x)$ which maps each element $x \in X$ to a real number in the interval [0; 1]. The function value of $\mu_{\tilde{A}}(x)$ resembles the grade of membership of *x* in $\tilde{A}$. The higher the value of $\mu_{\tilde{A}}(x)$, the higher the degree of membership of *x* in $\tilde{A}$ (Keufmann, Gupta, 1991).

Definition 1. Let a set *X* be fixed, a fuzzy set $\tilde{A}$ in *X* is given by Zadeh (1965) as follows:

$$\tilde{A} = \left\langle \left(x, \mu_{\tilde{A}}(x)\right) \middle| x \in X \right\rangle, \tag{1}$$

where $\mu_{\tilde{A}} : X \to [0,1], x \in X \to \mu_{\tilde{A}}(x) \in [0,1]$ and $\mu_{\tilde{A}}$ denotes the degree of membership of the element *x* to the set $\tilde{A}$.

Consequently, Atanassov (1986) introduced intuitionistic fuzzy set, which is the generalization of Zadeh's (1965) fuzzy set. Intuitionistic fuzzy set is characterized by membership and non-membership functions, whereas fuzzy set is described solely by the former one. Hence, intuitionist fuzzy set can resemble imprecise or uncertain decision information. More specifically, intuitionistic fuzzy sets can describe satisfied, unsatisfied, and uncertain information (Xu, 2011). Such information can represent, for instance, voting preferences, namely support, objection, and abstention, when voting for alternatives against given criteria.

Definition 2. Let a set *X* be fixed, an intuitionistic fuzzy set $\tilde{A}$ in *X* is given by Atanassov (1986) as follows:

$$\tilde{A} = \left\langle \left(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) \right) \middle| x \in X \right\rangle, \tag{2}$$

where functions $\mu_{\tilde{A}} : X \to [0,1], x \in X \to \mu_{\tilde{A}}(x) \in [0,1]$ and $v_{\tilde{A}} : X \to [0,1], x \in X \to v_{\tilde{A}}(x) \in [0,1]$ satisfy the condition $0 \le \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1$, for all $x \in X$. Moreover, here $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ denote the degree of membership and the degree of non-membership of the element *x* to the set $\tilde{A}$, respectively.

In addition, $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$ is called the degree of indeterminacy of *x* to $\tilde{A}$ or the degree of hesitancy of *x* to $\tilde{A}$ (Xu, Yager, 2006). Noteworthy, if $\pi_{\tilde{A}}(x) = 0$ for all $x \in X$, then the intuitionistic fuzzy set $\tilde{A}$ is reduced to a fuzzy set.

Definition 3. For convenience of computation, we call $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$ an intuitionistic fuzzy number (IFN), where

$$\mu_\alpha \in [0,1], \quad v_\alpha \in [0,1], \quad \mu_\alpha + v_\alpha \le 1, \quad \pi_\alpha = 1 - \mu_\alpha - v_\alpha . \tag{3}$$

For an IFN $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$, if the value μ_α gets bigger and the value v_α gets smaller, then IFN α gets greater. Thus, we know that $\alpha^+ = (1,0,0)$ and $\alpha^- = (0,1,0)$ are the largest and the smallest IFNs, respectively.

Xu (2011) defined the following algebraic operations for any two IFNs $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1}, \pi_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2}, \pi_{\alpha_2})$:

$$\alpha_1 \oplus \alpha_2 = \left(\mu_{a_1} + \mu_{a_2} - \mu_{a_1}\mu_{a_2}, v_{a_1}v_{a_2}, (1-\mu_{a_1})(1-\mu_{a_2}) - v_{a_1}v_{a_2} \right) \tag{4}$$

$$\alpha_1 \otimes \alpha_2 = \left(\mu_{a_1}\mu_{a_2}, v_{a_1} + v_{a_2} - v_{a_1}v_{a_2}, (1-v_{a_1})(1-v_{a_2}) - \mu_{a_1}\mu_{a_2} \right), \tag{5}$$

$$\lambda\alpha_1 = \left(1-(1-\mu_{\alpha_1})^\lambda, v_{\alpha_1}^\lambda, (1-\mu_{\alpha_1})^\lambda - v_{\alpha_1}^\lambda \right), \lambda > 0, \tag{6}$$

$$\alpha_1^{\lambda} = \left(\mu_{\alpha_1}^{\lambda}, 1-(1-v_{a_1})^{\lambda}, (1-v_{a_1})^{\lambda} - \mu_{\alpha_1}^{\lambda}\right),\ \lambda > 0\,. \tag{7}$$

According to Eqs. 4 and 5, the following equations hold:

$$\sum_{j=1}^{n} \alpha_j = \alpha_1 \oplus \alpha_2 \oplus \cdots \oplus \alpha_n = \begin{pmatrix} 1-\prod_{j=1}^{n}(1-\mu_{\alpha_j}), \prod_{j=1}^{n} v_{\alpha_j}, \\ \prod_{j=1}^{n}(1-\mu_{\alpha_j}) - \prod_{j=1}^{n} v_{\alpha_j} \end{pmatrix}, \tag{8}$$

$$\prod_{j=1}^{n} \alpha_j = \alpha_1 \otimes \alpha_2 \otimes \cdots \otimes \alpha_n = \begin{pmatrix} \prod_{j=1}^{n} \mu_{\alpha_j}, 1-\prod_{j=1}^{n}(1-v_{\alpha_j}), \\ \prod_{j=1}^{n}(1-v_{\alpha_j}) - \prod_{j=1}^{n} \mu_{\alpha_j} \end{pmatrix}. \tag{9}$$

Definition 4. Let $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1}, \pi_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2}, \pi_{\alpha_2})$ be two IFNs, then

$$d(\alpha_1, \alpha_2) = \frac{1}{2}\left(\left|\mu_{\alpha_1} - \mu_{\alpha_2}\right| + \left|v_{\alpha_1} - v_{\alpha_2}\right| + \left|\pi_{\alpha_1} - \pi_{\alpha_2}\right|\right) \tag{10}$$

is called the distance between α_1 and α_2 (Xu, Yager, 2006).

Definition 5. Let $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$. Then a negation operator is defined in the following manner:

$$Neg(\alpha) = (v_\alpha, \mu_\alpha, \pi_\alpha). \tag{11}$$

For an IFN α, Chen and Tan (1994) introduced the score function s_α, whereas Hong and Choi (2000) defined the accuracy function h_α:

$$s_\alpha = \mu_\alpha - v_\alpha\,, \tag{12}$$

$$h_\alpha = \mu_\alpha + v_\alpha\,. \tag{13}$$

With respect to the score function and the accuracy function, Xu and Yager (2006) gave an order relation between any two IFNs:

Definition 6. Let $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1}, \pi_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2}, \pi_{\alpha_2})$ be two IFNs, then (Xu, 2011; Xu, Yager, 2006; Xu, 2007):

If $s_{\alpha_1} > s_{\alpha_2}$, then $\alpha_1 > \alpha_2$.

If $s_{\alpha_1} = s_{\alpha_2}$, then a) if $h_{\alpha_1} = h_{\alpha_2}$, then $\alpha_1 = \alpha_2$;

b) if $h_{\alpha_1} > h_{\alpha_2}$, then $\alpha_1 > \alpha_2$.

The discussed peculiarities of IFNs will provide a basis for MCDM procedure described in Section 4.

2. 2. Intuitionistic fuzzy power aggregation operators

Ambiguity related to group decision making can be tackled by assigning weights to each of decision makers. However, in certain cases these weights are hard to quantify. Yager (1988) offered a method to overcome the latter issue, namely the ordered weighted average (OWA) operator. In general, aggregation operators belonging to the family of OWA assigns the most biased ratings with the lowest weights and thus increases the robustness of decision making process. The OWA operator, indeed, is a generalization of maximin, minimax, Hurwicz, and Laplace criteria.

Actually, the OWA operator was designed to aggregate crisp variables by the means of the weighted arithmetic average. The subsequent modifications of the method were aimed at enabling aggregation of fuzzy numbers as well as application of different aggregations techniques. The power average (PA) operator was introduced by Yager (2001). PA operator takes into account the relationships between the values being fused by attributing higher weights to those values which are less deviated from the remaining ones (Xu, Yager, 2010).

It was Xu (2011) who offered the two additional aggregation methods suitable for handling IFNs, namely an intuitionistic fuzzy power ordered weighted average (IFPOWA) operator and an intuitionistic fuzzy power ordered weighted geometric (IFPOWG) operator. These operators are peculiar with features of both OWA and PA operators and thus allow tackling the uncertainty arising in the group decision making.

Let there is a collection of IFNs, $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j}, \pi_{\alpha_j})$ with $j = 1, 2, ..., n$, and the ordered set of IFNs $\alpha_{(j)} = (\mu_{\alpha_{(j)}}, v_{\alpha_{(j)}}, \pi_{\alpha_{(j)}})$ with $\alpha_{(j-1)} \geq \alpha_{(j)}$ for $j = 2, 3, ..., n$. Then the IFPOWA aggregates a set of IFNs into a single IFN in the following way:

$$IFPOWA(\alpha_1,\alpha_2,...,\alpha_n) = w_1\alpha_{(1)} \oplus w_2\alpha_{(2)} \oplus ... \oplus w_n\alpha_{(n)} = \begin{pmatrix} 1-\prod_{j=1}^{n}(1-\mu_{\alpha_{(j)}})^{w_j}, \prod_{j=1}^{n}(v_{\alpha_{(j)}})^{w_j}, \\ \prod_{j=1}^{n}(1-\mu_{\alpha_{(j)}})^{w_j} - \prod_{j=1}^{n}(v_{\alpha_{(j)}})^{w_j} \end{pmatrix}, \quad (14)$$

where $w_j = (w_1, w_2, ..., w_l, ..., w_n)$ is a set of weights such that

$$w_l = g\left(\frac{D_l}{TV}\right) - g\left(\frac{D_{l-1}}{TV}\right), \quad D_l = \sum_{j=1}^{l} V_{(j)},$$
$$TV = \sum_{j=1}^{n} V_{(j)}, \quad V_{(j)} = 1 + T(\alpha_{(j)}) \quad , \quad (15)$$

with $T(\alpha_{(j)})$ being the support of the *j*-th largest IFN by all the other IFNs:

$$T(\alpha_{(l)}) = \sum_{\substack{j=1 \\ j\neq l}}^{n} Sup\left(\alpha_{(l)}, \alpha_{(j)}\right), \quad (16)$$

where $Sup\left(\alpha_{(l)}, \alpha_{(j)}\right)$ is support of the *l*-th largest IFN by the *j*-th largest IFN, and $g:[0,1] \to [0,1]$ is a basic unit-interval monotonic (BUM) function. The following properties of BUM functions are valid: 1) $g(0) = 0$, 2) $g(1) = 1$, and 3) $g(x) \geq g(y),$ if $x > y$.

The IFPOWG operator aggregates $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j}, \pi_{\alpha_j})$ into single IFN as follows:

$$IFPOWG(\alpha_1,\alpha_2,...,\alpha_n) = (\alpha_{(1)})^{\frac{1-w_1}{n-1}} \otimes (\alpha_{(2)})^{\frac{1-w_2}{n-1}} \otimes ... \otimes (\alpha_{(n)})^{\frac{1-w_n}{n-1}} = \begin{pmatrix} \prod_{j=1}^{n}(\mu_{\alpha_{(j)}})^{\frac{1-w_j}{n-1}}, 1-\prod_{j=1}^{n}(1-v_{\alpha_{(j)}})^{\frac{1-w_j}{n-1}}, \\ \prod_{j=1}^{n}(1-v_{\alpha_{(j)}})^{\frac{1-w_j}{n-1}} - \prod_{j=1}^{n}(\mu_{\alpha_{(j)}})^{\frac{1-w_j}{n-1}} \end{pmatrix}, \quad (17)$$

where $w_j = (w_1, w_2, ..., w_l, ..., w_n)$ is a set of weights satisfying Eqs. 15 and 16.

Both IFPOWA and IFPOWG can be employed to aggregate the ratings provided by different decision makers into a single decision matrix.

3. The crisp MULTIMOORA method

As already said earlier, Multi-Objective Optimization by Ratio Analysis (MOORA) method was introduced by Brauers and Zavadskas (2006). Brauers and Zavadskas (2010) extended the method and in this way it became more robust as MULTIMOORA (MOORA plus the full multiplicative form). These methods have been applied in numerous studies focused on regional studies, international comparisons and investment management.

The MOORA method begins with matrix X where its elements x_{ij} denote ith alternative of jth objective ($i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$). MOORA method consists of two parts: the Ratio System and the Reference Point approach. The MULTIMOORA method includes internal normalization and treats originally all the objectives equally important. In principle all stakeholders interested in the issue only could give more importance to an objective. Therefore they could either multiply the dimensionless number representing the response on an objective with a significance coefficient or they could decide beforehand to split an objective into different sub-objectives.

The Ratio System of MOORA. Ratio system defines data normalization by comparing alternative of an objective to all values of the objective:

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \tag{18}$$

where x_{ij}^{*} denotes ith alternative of jth objective. Usually these numbers belong to the interval [0; 1]. These indicators are added (if desirable value of indicator is maximum) or subtracted (if desirable value is minimum), Thus, the summarizing index of each alternative is derived in this way:

$$y_i^{*} = \sum_{j=1}^{g} x_{ij}^{*} - \sum_{j=g+1}^{n} x_{ij}^{*}, \tag{19}$$

where $g = 1, \cdots, n$ denotes number of objectives to be maximized. Then every ratio is given the rank: the higher the index, the higher the rank.

The Reference Point of MOORA. Reference point approach is based on the Ratio System. The Maximal Objective Reference Point (vector) is found according to ratios found by employing Eq. 18. The j^{th} coordinate of the reference point can be described as $r_j = \max_i x_{ij}^*$ in case of maximization. Every coordinate of this vector represents maximum or minimum of certain objective (indicator). Then every element of normalized response matrix is recalculated and final rank is given according to deviation from the reference point and the Min-Max Metric of Tchebycheff:

$$\min_i \left(\max_j \left| r_j - x_{ij}^* \right| \right). \tag{20}$$

The Full Multiplicative Form and MULTIMOORA. Brauers and Zavadskas (2010) proposed MOORA to be updated by the Full Multiplicative Form method embodying maximization as well as minimization of purely multiplicative utility function. Overall utility of the i^{th} alternative can be expressed as dimensionless number:

$$U_i^{'} = \frac{A_i}{B_i}, \tag{21}$$

where $A_i = \prod_{j=1}^{g} x_{ij}$, $i = 1, 2, \cdots, m$ denotes the product of objectives of the i^{th} alternative to be maximized with $g = 1, \cdots, n$ being the number of objectives to be maximized and

where $B_i = \prod_{j=g+1}^{n} x_{ij}$ denotes the product of objectives of the i^{th} alternative to be minimized with $n - g$ being the number of objectives (indicators) to be minimized. Thus MULTIMOORA summarizes MOORA (i.e. Ratio System and Reference point) and the Full Multiplicative Form. Ameliorated Nominal Group and Delphi techniques can also be used to reduce remaining subjectivity (2010). The Dominance theory (Annex A) was proposed to summarize three ranks provided by respective parts of MULTIMOORA into a single one (Brauers, Zavadskas, 2011).

4. The MULTIMOORA–IFN method

This section describes the proposed extension of the MULTIMOORA method, namely MULTIMOORA–IFN, which enables to tackle uncertainty and vagueness in decision making. Let us assume that a group of experts (decision makers) is about to choose the most compromising alternative with respect to multiple criteria. These criteria are grouped into cost criteria ($j \in C$, C is the set thereof) and benefit criteria ($j \in B$).

Step 1. Each expert evaluates the alternatives under consideration in terms of the defined criteria. The assessments are expressed in IFNs α_{ij}^{k} , where $i = 1,2,...,m$ stands for respective alternatives, $j = 1,2,...,n$ denotes the *j*-th criterion, and $k = 1,2,...,K$ indicates the *k*-th decision maker. Thus *K* response matrices are defined.

Step 2. Either IFPOWA or IFPOWG aggregation operator is employed to aggregate the expert decision matrices into a single response matrix:

$$IFPOWA(a_{ij}^{1}, a_{ij}^{2}, ..., a_{ij}^{K}) = \alpha_{ij}, \forall i, j, \tag{22}$$

where α_{ij} denotes the value given for the *i*-th alternative according to the *j*-th criterion. Similarly:

$$IFPOWG(a_{ij}^{1}, a_{ij}^{2}, ..., a_{ij}^{K}) = \alpha_{ij}, \forall i, j, \tag{23}$$

Step 3. The transformed response matrix $\beta_{m \times n}$ is defined by transforming cost criteria into benefit ones. Subsequently, a negation operator is used in accordance to Eq. 11 to transform cost criteria into benefit ones:

$$\beta_{ij} = \begin{cases} \alpha_{ij}, j \in B \\ Neg(\alpha_{ij}), j \in C \end{cases}. \tag{24}$$

These values are bounded to the interval of [0, 1] and therefore do not require an additional normalization.

Step 4. *The Ratio System of MULTIMOORA–IFN.* The assessments of a certain alternative are aggregated across the criteria by employing Eq. 8:

$$y_i = \sum_{j=1}^{n} \beta_{ij} . \tag{25}$$

Accordingly, alternatives with higher values of $y_i = (\mu_{y_i}, v_{y_i}, \pi_{y_i})$ are attributed with higher ranks (cf. Definition 6).

Step 5. *The Reference Point of MULTIMOORA–IFN.* Generally, the two types of the reference point might be chosen (Brauers, 2004): (i) the Maximal Objective Reference Point, and (ii) the Utopian Reference Point. In case of the Maximal Objective Reference Point, the maximum for every criterion is defined according to Definition 6: $\beta_j = \max_i \beta_{ij}$. In case of the Utopian Reference Point one may set $\beta_j = (1,0,0)$. Then, Chebychev distances from the reference point are calculated for each of the alternatives (Eq. 10):

$$\max_j \{d(\beta_{ij}, \beta_j)\}. \tag{26}$$

The alternatives with larger distances from the Reference are attributed with lower ranks.

Step 6. *The Full Multiplicative Form of MULTIMOORA–2T.* The overall utility of a certain alternative is determined by employing Eq. 9:

$$U_i = \prod_{j=1}^{n} \beta_{ij} \ . \tag{27}$$

The alternatives are ranked in descending order of the overall utility.

Step 7. The three ranks obtained in Steps 4–6 are summarized by applying the Dominance theory (Brauers, Zavadskas, 2011), see Annex A.

5. Personnel selection: a numerical example

Step 1. A personnel selection problem will illustrate the group decision-making procedure according to MULTIMOORA–IFNs. The enterprise has formed an executive committee consisting of four decision-makers (DM_1, DM_2, DM_3, and DM_4); adopted from Baležentis et al. (2012). The committee is about to choose the best candidate from another four participants (A_1, A_2, A_3, and A_4) to fill the vacancy. The committee has decided to consider the following eight attributes: (1) Creativity, innovation (C_1); (2) Leadership (C_2); (3) Strategic planning (C_3); (4) Communication skills (C_4); (5) Team management (C_5); (6) Emotional steadiness (C_6); (7) Educational background (C_7); (8) Professional experience (C_8). More specifically, these criteria are expressed in IFNs. All the criteria, hence, are subjective as well as benefit ones. Table 1 presents the initial assessment.

Step 2. In this example, we assume $g(x) = x$ and employ the IFPOWA operator by the virtue of Eq. 14 to aggregate the expert decision matrices into a single response matrix (Table 2). Indeed, one may choose $g(x) = x^k, k > 0$ in order to obtain a different weight vector.

Step 3. All of the criteria are benefit ones, therefore we do not need to employ the negation operator (cf. Eq. 24).

Table 2. The aggregated decision matrix

	C_1	C_2	C_3	C_4
A_1	(0.42,0.47,0.11)	(0.53,0.27,0.2)	(0.4,0.41,0.19)	(0.38,0.44,0.18)
A_2	(0.64,0.3,0.06)	(0.64,0.25,0.11)	(0.49,0.33,0.18)	(0.48,0.47,0.05)
A_3	(0.48,0.45,0.07)	(0.39,0.58,0.03)	(0.6,0.22,0.18)	(0.46,0.4,0.14)
A_4	(0.77,0,0.23)	(0.69,0.24,0.07)	(0.4,0.49,0.11)	(0.45,0.32,0.23)

	C_5	C_6	C_7	C_8
A_1	(0.39,0.51,0.1)	(0.5,0.32,0.18)	(0.61,0.22,0.17)	(0.41,0.54,0.05)
A_2	(0.64,0.3,0.06)	(0.59,0.27,0.14)	(0.65,0.31,0.04)	(0.42,0.58,0)
A_3	(0.34,0.59,0.07)	(0.55,0.43,0.02)	(0.48,0.39,0.13)	(0.46,0.43,0.11)
A_4	(0.77,0,0.23)	(0.66,0.25,0.09)	(0.52,0.38,0.1)	(0.48,0.43,0.09)

Step 4. The four candidates are ranked according to the Ratio System, cf. Eq. 25. Table 3 presents the results.

Step 5. We define the Reference Point $\beta_j = (1,0,0)$ and thus rank the alternatives in terms of their distance from it (Eq. 26): those with smaller distances are attributed with higher ranks (Table 4).

Table 1. The ratings provided by the decision makers (DM_1–DM_4) to the candidates (A_1–A_4) in terms of the multiple criteria (C_1–C_8).

		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
DM_1	A_1	(0.5,0.4,0.1)	(0.5,0.3,0.2)	(0.2,0.6,0.2)	(0.4,0.4,0.2)	(0.4,0.5,0.1)	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.3,0.5,0.2)
	A_2	(0.7,0.3,0)	(0.7,0.2,0.1)	(0.6,0.2,0.2)	(0.6,0.4,0)	(0.7,0.3,0)	(0.2,0.7,0.1)	(0.5,0.5,0)	(0.6,0.4,0)
	A_3	(0.5,0.4,0.1)	(0.4,0.6,0)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.2,0.7,0.1)	(0.4,0.6,0)	(0.4,0.4,0.2)	(0.5,0.3,0.2)
	A_4	(0.7,0.2,0.1)	(0.7,0.2,0.1)	(0.2,0.7,0.1)	(0.5,0.2,0.3)	(0.7,0.2,0.1)	(0.6,0.2,0.2)	(0.3,0.6,0.1)	(0.5,0.5,0)
DM_2	A_1	(0.2,0.7,0.1)	(0.5,0.3,0.2)	(0.5,0.3,0.2)	(0.3,0.5,0.2)	(0.2,0.7,0.1)	(0.5,0.4,0.1)	(0.5,0.3,0.2)	(0.3,0.7,0)
	A_2	(0.6,0.3,0.1)	(0.7,0.2,0.1)	(0.4,0.4,0.2)	(0.1,0.9,0)	(0.6,0.3,0.1)	(0.4,0.4,0.2)	(0.4,0.5,0.1)	(0.1,0.9,0)
	A_3	(0.4,0.5,0.1)	(0.2,0.8,0)	(0.6,0.3,0.1)	(0.5,0.3,0.2)	(0.4,0.5,0.1)	(0.8,0.2,0)	(0.6,0.3,0.1)	(0.5,0.4,0.1)
	A_4	(0.7,0.2,0.1)	(0.8,0.2,0)	(0.5,0.4,0.1)	(0.5,0.4,0.1)	(0.7,0.1,0.2)	(0.8,0.2,0)	(0.4,0.4,0.2)	(0.6,0.3,0.1)
DM_3	A_1	(0.6,0.3,0.1)	(0.5,0.3,0.2)	(0.2,0.7,0.1)	(0.4,0.4,0.2)	(0.6,0.3,0.1)	(0.5,0.3,0.2)	(0.7,0.2,0.1)	(0.6,0.4,0)
	A_2	(0.5,0.3,0.2)	(0.7,0.2,0.1)	(0.6,0.3,0.1)	(0.4,0.4,0.2)	(0.5,0.3,0.2)	(0.7,0.2,0.1)	(0.6,0.3,0.1)	(0.2,0.8,0)
	A_3	(0.5,0.5,0)	(0.4,0.5,0.1)	(0.6,0.2,0.2)	(0.3,0.6,0.1)	(0.5,0.5,0)	(0.4,0.5,0.1)	(0.2,0.7,0.1)	(0.3,0.6,0.1)
	A_4	(0.9,0,0.1)	(0.7,0.2,0.1)	(0.6,0.3,0.1)	(0.5,0.2,0.3)	(0.9,0,0.1)	(0.7,0.2,0.1)	(0.6,0.4,0)	(0.5,0.3,0.2)
DM_4	A_1	(0.3,0.6,0.1)	(0.6,0.2,0.2)	(0.6,0.2,0.2)	(0.4,0.5,0.1)	(0.3,0.6,0.1)	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.4,0.6,0)
	A_2	(0.7,0.3,0)	(0.2,0.7,0.1)	(0.3,0.5,0.2)	(0.6,0.4,0)	(0.7,0.3,0)	(0.8,0.1,0.1)	(0.9,0.1,0)	(0.6,0.4,0)
	A_3	(0.5,0.4,0.1)	(0.5,0.5,0)	(0.6,0.2,0.2)	(0.5,0.5,0)	(0.2,0.7,0.1)	(0.5,0.5,0)	(0.6,0.3,0.1)	(0.5,0.5,0)
	A_4	(0.7,0.1,0.2)	(0.5,0.4,0.1)	(0.2,0.7,0.1)	(0.2,0.8,0)	(0.7,0.1,0.2)	(0.4,0.5,0.1)	(0.7,0.2,0.1)	(0.2,0.8,0)

Step 6. Eq. 27 is employed to obtain ranks for each of alternatives according to the Full Multiplicative Form (Table 5).

Table 3. The Ratio System.

	y_i	s_α	Rank
A_1	(0.9929,0.0004,0.0067)	0.9925	4
A_2	(0.9990,0.0001,0.0009)	0.9989	2
A_3	(0.9943,0.0010,0.0047)	0.9933	3
A_4	(0.9995,0,0.0005)	0.9995	1

Table 4. The Reference Point.

	$\max_j \{d(\beta_{ij}, \beta_j)\}$	Rank
A_1	0.62	3
A_2	0.58	1
A_3	0.66	4
A_4	0.6	2

Table 5. The Full Multiplicative Form

	y_i	s_α	Rank
A_1	(0.0017,0.9847,0.01777)	-0.983	3
A_2	(0.0099,0.9724,0.01777)	-0.9625	2
A_3	(0.0021,0.9912,0.0067)	-0.9891	4
A_4	(0.0121,0.9302,0.0577)	-0.9181	1

Step 7. Then by using the Ratio System, the Reference Point and the Full Multiplicative Form to rank the candidates, we have the following results (Table 6). The Dominance theory (Annex A) is employed to summarize the three different ranks provided by respective parts of MULTIMOORA-IFN. The last column in Table 6 presents the final ranking.

Table 6. Ranking of the candidates according to MULTIMOORA–IFN

	The Ratio System	The Reference Point	The Full Multiplicative Form	MULTIMOORA-IFN (Final rank)
A_1	4	3	3	3
A_2	2	1	2	2
A_3	3	4	4	4
A_4	1	2	1	1

According to the multi-criteria evaluation, the fourth candidate (A_4) should be recruited, whereas the second candidate (A_2) is the second-best option. At the other end of spectrum, candidates A_1 and A_3 are the last two.

6. Conclusion

The proposed method MULTIMOORA-IFN enables to tackle vague and ambiguous ratings express in IFNs thanks to the power ordered weighted average operators employed when aggregating the opinions of the decision makers. Furthermore, the three parts of MULTIMOORA-IFN prioritizes the alternatives in terms of different techniques and thus provides one with a more robust ranking. A decision maker can choose the BUM function for the aggregation operator as well as the type of the operator and thus test the sensitivity of the results. Furthermore, the reference point can also be defined in various ways.

The numerical simulation of personnel selection was implemented to test the MULTIMOORA-IFN. Further extensions of the MULTIMOORA method aimed at facilitating soft computing with various types of information would contribute to the area of the imprecise decision making.

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Annex A. The Dominance Theory

Absolute Dominance means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1–1–1). *General Dominance in two of the three methods* is of the form with a < b < c <d:

(d–a–a) is generally dominating (c–b–b);
(a–d–a) is generally dominating (b–c–b);
(a–a–d) is generally dominating (b–b–c);
and further transitiveness plays fully.

Transitiveness. If *a* dominates *b* and *b* dominates *c* than also *a* will dominate *c*. *Overall Dominance of one alternative on the next one.* For instance (a–a–a) is overall dominating (b–b–b) which is overall being dominated, with (b–b–b) following immediately (a–a–a) in rank (transitiveness is not playing). *Absolute Equability* has the form**:** for instance (e–e–e) for 2 alternatives. *Partial Equability* of 2 on 3 exists e. g. (5–e–7) and (6–e–3). Despite all distinctions in classification some contradictions remain possible in a kind of *Circular Reasoning*. We can cite the case of:

Object A (11–20–14) $\succ$ Object B. (14–16–15);
Object B (14–16–15) $\succ$ Object C (15–19–12); but
Object C (15–19–12) $\succ$ Object A (11–20–14).

Here, the operator $\succ$ represents a *General Dominance*. In such a case the same ranking is given to the three objects.