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# **RELATIONSHIP BETWEEN OIL PRICES AND EXCHANGE RATES: THE CASE OF ROMANIA**

Abstract This study investigates to causality between crude oil prices and exchange rates in Romania employing monthly data from the beginning of floating exchange regime for November 2004 to December 2011. The study benefits from the recent advance in the time series econometric analysis and carries out nonlinear causality and frequency domain causality tests. According to nonlinear causality test results there is no causality between the variables. Results show that frequency domain causality results slightly differentiate from the nonlinear causality analysis and imply that there is a causality running from real exchange rate to real oil price on the mediun and long run.

Keyword: oil prices, exchange rate, Romania, frequency domain.

#### JEL Classification: F31, F41, C32

#### **1. Introduction**

The term of transition economy includes the countries located in Central and Eastern Europe. The common point of these countries is strong change in their economic structure. The defining characteristic of the transition countries is their decision to abandon central planning as the principal mode of organizing their economies and to move to market-oriented economies with significant private ownership of the means of production (IMF, 2000). Romania was included in the list of transition countries in 1989.

The country experienced different stages of development in the period including years between years 1989 and 2011. After the economic decline and recovery periods, Romania participated to European Union in 2007. International

Money Fund, World Trade Organization and European Bank for Reconstruction and Development are the other institutions which Romania joined in the last subperiod of the transition process.

By the beginning of transition period in Romania, a number of major reforms were made in the fields of privatization, price liberalization, banking and commercial law and regulations in financial system and financial institutions in order to participate global economic system. Another important reform was made in trade and foreign exchange rate system of Romania. Similar to other transition countries, Romania has started with a fixed exchange rate regime and moved gradually from an intermediate (soft peg) regime to a managed floating regime. The National Bank of Romania started to apply controlled (managed) floating regime in 1999. Finally the Bank has started to implement floating regime by the beginning of inflation targeting regime in November 2004. Transition to floating exchange rate regime allows to external factors to induce exchange rate fluctuations.

By the beginning of transition period, the Romanian economy has grown remarkably. The growing economy has induced to increase energy demand. According to Eurostat database, the primary energy demand was 34 million tonnes of oil equivalent (MTOE) in 1999, it was more than 38 MTOE in 2008. Although a contraction occured in energy consumption because of global crisis in 2008, it has increased again by the end of crisis.

Romania meets its energy demand mainly from its own sources. In this regard, its energy import dependency is lower than European Union avarage. On the other hand, a quarter of total primary energy supply is met by oil. Romania is a significant producer of oil. Domestic oil production covers a large percentage of oil needs. Oil dominates the primary energy supply of Romania with gas and solid fuels. Romania imported oil more than 4.8 MTOE, while it produced 6 MTOE in 2004 (EC, 2007). 40 % of imported energy is in the form of crude oil imported mainly from the Russian Federation and Kazakhstan.

In addition to transition to floating exchange rate regime in November 2004, increasing energy demand parallel to economic growth brings to question that "Do oil price fluctuations induce exchange rate fluctuations in Romania?". In the light of these explanations, we examine the causation linkage between real exchange rate and oil price in Romania. We employ monthly data from the beginning of floating exchange rate regime in November 2004 to December 2011. The causal relationship is identified by employing multiple testing approaches. In that respect we employ linear causality test developed by Hatemi-J (2006) and frequency domain causality test developed by Breitung and Candelon (2006).

The contribution of this study is two fold. First, the determination of causation linkage has important policy implications. The fluctuations in exchange rate impairs on economic growth (Rickne, 2009). In this regard, reducing price volatility of oil also proves exchange rate stability and hence economic growth. On the other hand, the information about possible interaction between oil prices and

exchange rate plays crucial role in making long term energy policies. By determining the causation linkage, policymakers might tend to alternative energy sources in order to reduce oil dependency and oil demand. In the light of results, we also provide information for global investors in investment decisions. By monitoring oil prices, investors may forecast exchange rate movements. Besides, financial market actors and speculators could be able to identify portfolio diversification options in exchange rate markets. Secondly, this study also attempts to compare time domain and frequency domain causality analyses approach which generates test statistics at different frequencies across spectra. By doing so, we examine the existence of causality between variables in different time frequencies. This would give opportunity to test Coudert et al.'s (2008) findings about the differences in price elasticity of oil demand and supply in different time periods.

The rest of paper is organized as follows. The next section devoted to summarize theoretical interpretations about the linkages between variables. The third section summarizes the existing literature investigating the relationship between variables. In the fourth and fifth sections, econometric methodology and the data are described. In the section six, empirical results are presented. We summarize and conclude empirical findings in the last section.

## 2. Theoretical Background

Theoretically there is a number of variables affect exchange rates in floating exchange rate regime. The economists have investigated the effects of possible variables on exchange rates. In this regard, real factors as well as nominal factors are examined. While Branson (1981) attributes an important role to monetarial factors, Dornbusch (1980) and Clarida and Gali (1994) find the significant effects of real variables.

Besides, Golub (1983) and Krugman (1983) attributes an important role to oil price shocks in definition of exchange rate fluctuations. While Golub (1983) explains the relationship via wealth transfer channels, Krugman (1983) argues that exchange rates differentiate due to import preferences and investment decisions of oil exporting countries in the case of oil price increases.

On the other hand, Coudert et al. (2008) suggest that a shock in U.S. dollar exchange rate also affects oil prices. They explain the relationship as follows: oil pruchases are paid in dollars. Demand depends on the domestic price for consumer countries which changes with the dollar fluctuations. Depreciation in dollar reduces the oil price in domestic currencies for countries with a floating currency (Coudert et al., 2008). This leads to an increase in real income of consumer countries and increases oil demand. So, dollar depreciation increases oil demand and this would increase oil price. Similarly, a U.S. dollar depreciation has effects on oil supply. The depreciation in U.S. dollar can also trigger inflation and reduce the income in oil producer countries, the currencies of which are linked to the

dollar. The increase in inflation and the decrease in purchasing power reduce the real disposable income and therefore the income available for drilling, everything else equals (Coudert et al., 2008). As a result, dollar depreciation may result in a reduction in oil supply, henceforth oil price. But the effects of exchange rate shocks on oil demand and supply can be observable on the long run. Because the price elasticity of both demand and supply are inelastic on the short run. So the effect of exchange rate on oil price occurs on the long run.

# 3. Literature review

In this section we focus on the literature analyzing exchange rate and oil price relation. Although there is a number studies investigating the linkage, we rank the limited number of studies in order to save the place.

We classify the literature into four groups. The first group of studies supports evidence of causality running from oil price to exchange rate. Amano and van Norden (1998) for the U.S.A. support evidence for this argument. In latter studies, Akram (2004) for Norway, Issa et al. (2006) for Canada, Chen and Chen (2007) for G7 countries, Coudert et al. (2008) for U.S.A., Narayan et al. (2008) for Fiji Islands and Mendez-Carbajo (2011) for Dominic Republic and Lizardo and Mollick (2010) for Canada, Mexico, Norway and Russia as oil exporting countries and for Denmark, Japan, Sweden, the United Kingdom and the Euro area countries as exporting countries reach similar results with Amano and van Norden (1998).

Second group of studies implies that real exchange rate shocks induce oil price fluctuations and indicates a causality runing from real exchange rates to oil prices. Indjehagopian et al. (2000) find that variation in exchange rates has an instantaneous impact on the variations in oil prices for Holland, Germany and France. In another study for developed country context, Sadorsky (2000) analyzes U.S.A. and obtains that exchange rate induces crude oil future prices. This view is supported by Zhang et al. (2008), Yousefi and Wirjanto (2004) for Indonesia, Iran, Nigeria and Saudi Arabia.

The third group of studies finds evidence of two way causality (the feedback relation) between oil price and real exchange rates. The initial studies imply that there is a bi-directional causality between variables in the short run. Huang and Tseng (2010) investigate the relationship between oil price and nominal exchange rate by using different kind of oil types and find bi-directional causality for U.S.A.

The fourth group of studies indicates that there is no causal relationship (the neutrality) between variables, implying that oil price and exchange rates do not provide a predictive power with respect to each other. The empirical evindence on the neutrality between oil prices and exchange rates is supported by Habib and Kalamova (2007) for Saudi Arabia, Wu et al. (2011) for U.S.A; Mohammadi and Jahan Parvar (2010) for thirteen oil exporting countries.

# 4. Econometric Methods

# 4.1. Hacker and Hatemi-J (2006) Bootstrap Process-Based Toda-Yamamoto (1995) Linear Granger Causality Test

In a standard Granger causality analysis, zero restrictions based on the Wald principle are imposed on the lagged coefficients obtained from the estimation of Vector Autoregressive (VAR) model. However, the Wald statistic may lead to nonstandard limiting distributions depending upon the co-integration properties of the VAR system that these nonstandard asymptotic properties stem from the singularity of the asymptotic distributions of the estimators (Lütkepohl, 2004). The Toda and Yamamoto (1995) (TY, hereafter) procedure overcomes this singularity problem by augmenting VAR model with the maximum integration degree of the variables. In addition to this advantage, the TY approach does not require testing for co-integration relationships and estimating the vector error correction model and is robust to the unit root and co-integration properties of the series.

The standard Granger causality analysis requires estimating a VAR (p) model in which p is the optimal lag length(s). In the TY procedure, the following VAR (p+d) model is estimated that d is the maximum integration degree of the variables.

$$y_t = v + A_1 y_{t-1} + \dots + A_p y_{t-p} + \dots + A_{p+d} y_{t-(p+d)} + \mu_t.$$
(1)

where  $y_t$  is vector of k variables, v is a vector of intercepts,  $\mu_t$  is a vector of error terms and A is the matrix of parameters. The null hypothesis of no-Granger causality against the alternative hypothesis of Granger causality is tested by imposing zero restriction on the first p parameters. The so-called modified Wald (MWALD) statistic has asymptotic chi-square distribution with p degrees of freedom irrespective of the number of unit roots and of the co-integration relations.

Hacker and Hatemi-J (2006) investigate the size properties of the MWALD test and find that the test statistic with asymptotic distribution poorly performs in small samples.  $\otimes$  the Kronecker product,  $C = p \times n(1+n(p+d))$  a selector matrix,  $S_U$  variance-covariance matrix of residuals,  $\hat{\beta} = vec(\hat{D})$  and vec is the column-stacking operator as the MWALD test statistics;

$$MWALD = (C\beta)'[C((Z'Z)^{-1} \otimes S_U)C']^{-1}(C\beta) \Box \chi_p^2$$

Monte Carlo simulation of Hacker and Hatemi-J (2006) shows that the MWALD test based on the bootstrap distribution has much smaller size distortions than those of the asymptotic distribution. Hacker and Hatemi-J (2006) extends the TY approach based on the bootstrapping method developed by Efron (1979). In this new approach that is so-called the leveraged bootstrap Granger causality test,

the MWALD statistic is compared with the bootstrap critical value instead of the asymptotic critical value.

## 4.2. Frequency domain causality test

While conventional time domain causality tests produce a single test statistic for the interaction between variables in concern, frequency domain methodology generates tests statistics at different frequencies across spectra. This is contrary to the implicit assumption of the conventional causality analysis that a single test statistic summarizes the relation between variables, which is expected to be valid at all points in the frequency distribution. Frequency domain approach to causality thereby permits to investigate causality dynamics at different frequencies. Hence, it seems to be very meaningful to carry out frequency domain causality to better understand temporary and permanent linkages between oil prices and exchange rates in the Romania. Details of frequency domain approach are given in the appendix A.

#### 5. Data

The exchange rate is defined as the foreign currency price of the U.S. dollar, concluding that the dollar appreciates as the nominal value of exchange rate rises. Real exchange rates (RER) are constructed using consumer price indices. The world price of oil, quoted in U.S. dollars, is chosen as representative of the general movement in oil prices over the period. To obtain the real oil price (ROP), the U.S. dollar price of oil was converted to domestic prices using the U.S. dollar exchange rate and then deflated by the domestic consumer price index. We employ monthly data belonging o period between November 2004 and December 2011. All the data is obtained from the International Financial Statistics published by IMF. The descriptive statistics are reported in table 1. It seems that data characteristics are slightly different in each series. First of all, as expected, the mean of the real exchange rate is higher than real oil price is higher than real exchange rate. However, kurtosis value of real exchange rate is smaller than real oil price.

	Table 1 Descriptive Statistics						
	Time Span	Variable	Mean	Std. Dev.	Coef. of var.	Skewness	Kurtosis
	November 2004- December 2011	RER	4.665	0.057	0.012	-0.062	3.675
		ROP	0.538	0.202	0.375	-0.28	2.737

**Fable 1 Descriptive Statistics** 

Source: Author's computations using Eviews 6.

Notes: Coefficient of variation is the ratio of standard deviation to mean. Descriptive statistics are for log series. ROP: real oil prices, RER: real exchange rates.

### 6. Empirical Findings

Prior to the identification of possible causality between the real exchange rates and the real oil prices, it is necessary to determine integration degree of variables. In that respect, we employ a battery of the unit root tests Augmented Dickey and Fuller test (henceforth ADF), Phillips and Perron (henceforth PP), Dickey Fuller GLS (henceforth DF-GLS), and Kwaitkowski et al. (henceforth KPSS). The results from the unit root tests in table 2 show that ADF, PP and DF-GLS test do not reject the null of a unit root for the levels of real exchange rate and real oil price. When the ADF, PP and DF-GLS tests are applied to the first differences of the variables, the results indicate that all variables are stationary. Consistent with these results, the KPSS test for the null hypothesis also shows that the variables are stationary in their first differences. The unit root analyses thereby imply that the variables are integrated of order one. Accordingly, the maximum integration order (d) of the variables equal to one in the Toda Yamamoto linear Granger causality analysis.

		ADF	DF-GLS	PP	KPSS
Levels					
Intercept	RER	-2.800 (1)*	-0.803 (1)	-3.205 (3)**	0.223*
1	ROP	-3.039 (2)**	-2.236 (2)**	-2.247 (4)	0.416
Intercept and Trend	RER	-2.690 (1)	-1.340 (1)	-2.961 (1)	0.222
	ROP	-3.508 (2)**	-3.489 (2)**	-2.661 (4)	0.078**
First-differences					
	RERR	-6.701 (0)***	-3.873 (0)***	- 6.618(4)***	0.373***
Intercept	ROPR	-7.557 (0)***	-2.351 (1)***	- 7.710(3)***	0.043***
	RERR	-6.832 (0)***	-5.886 (0)***	- 6.741(4)***	0.115***
Intercept and Trend	ROPR	-7.505 (0)***	-3.588 (1)***	- 7.662(3)***	0.039***

 Table 2 Results of the Unit Root Test

Notes: For the KPSS test: \* The asymptotic critical values of LM statistic for intercept 0.739, 0.463 at the %1 and %5 levels, \*\* the asymptotic critical values of LM statistic for trend and intercept 0.216, 0.146 at the %1 and %5 levels.

For the DF-GLS test: \*The asymptotic critical values for without trend -2.591, -1.944 at the %1 and %5 levels. \*\* The asymptotic critical values for with trend -3.602, -3.177 at the %1 and %5 levels. The figures in parenthesis denote the number of lags in the tests that ensure white noise residuals. They were estimated through the Schwarz criterion.

For the ADF test: \* shows the results of Dickey Fuller test in the case of zero lag length and lag length choosen due to SIC criteria.\*\* For the ADF test, the Mac Kinnon(1996) critical values for with constant -.3.485, -2.885, -2.579 at the 1 %, 5 % and 10 % levels. The critical values for with constant and trend -4.035, -3.447 ve -3.148 at the 1 %, 5 % and 10 % levels, respectively.

For the PP test: \*Values in the paranthesis show bandwiths obtained according to Newey-West using Bartlett Kernel criteria. \*\* For the PP test Mac Kinnon(1996) critical values for with constant -3.483, -2.884, -2.579 at the 1 %, 5 % and 10 % levels.the critical values for with constant and trend -4.033, -3.446 and -3.148 at the 1 % 5 % and 10 % levels, respectively.

Source: Author's computations using Eviews 6.

In the first step we employ bootstrap process based Toda Yamamoto (1995) linear causality (TY hereafter) test. Table 2 shows the results of bootstrap process based linear TY Granger type causality test results. As indicated in the table 2 statistics for both tests are higher than critical values obtained by replication for 10.000 times. According to test results, there is a uni-directional causality running from real oil price to real exchange rate.

Real oil prices to real exchange rates							
		Bootstrap critical values					
MWALD	p-value	1%	5%	10%			
11.369	0.000	12.499	8.240	6.444			
Real exchange rates to real oil prices							
		Boot	Bootstrap critical values				
MWALD	p-value	1%	5%	10%			
0.743	0.862	12.265	8.335	6. 674			

 Table 3 Results of Linear Granger Causality Test

Note: p-value denotes asymptotic chi-sqaure distribution. The AIC was used to determine the optimal lag lengths for VAR(p+d) models. Numbers in brackets are p-values. The number of replication is 10000.

Source: Author's computations using Eviews 6.

In the second step, we employ Breitung and Candelon's (2006) analysis which permits to decompose the causality test statistic into different frequencies. We calculate the test statistics at a high frequency of  $\omega_i = 2.5$  and  $\omega_i = 2.0$  to examine short term causality,  $\omega = 1.00$  and  $\omega = 1.50$  to examine medium term causality and finally  $\omega = .01$  and  $\omega = .05$  to investigate long term causality. By doing so, we are able to learn both temporary and permanent relations between variables. According to results of frequency domain causality test, we imply that there is no effect of real oil prices on real exchange rate in any time period. Results show that the causal

relationship running from real exchange rate to real oil price is valid for long and medium term.

Real exchange rate to real oil price								
	Long	Term	Medium	Medium Term		Short Term		
$\omega_{i}$	0.01	0.05	1.00	1.50	2.0	2.50		
	8.311*	15.474*	2.703	8.159*	5.864*	4.503*		
Real oil price to real oil price								
	Long Term		Medium	Ferm Short Term		rm		
$\omega_i$	0.01	0.05	1.00	1.50	2.00	2.50		
	1.563	3.089	0.391	0.380	1.273	0.157		

Table 4 Results for frequency domain causality test

Notes: The lag lengths for the VAR models are determined by SIC. F- distribution with (2, T-2p) degrees of freedom equals 5.99. Source: Author's computations using Gauss.

While linear causality analysis results imply existence of causality running from real oil price to real exchange rate in Romania, frequency domain causality results find evidence the causality running from real exchange rate to real oil price on the longer periods. As can been, both the time domain and frequency domain causality tests imply different results.

## 7. Conclusions

In this study, we examine monthly data belonging November 2004-December 2011 period in order to find whether there is an interaction between oil price fluctuations and exchange rate volatilities in the Romanian economy. In this regard, we employ linear causality test developed by Hatemi-J (2006) and frequency domain causality test developed by Candelon and Breitung (2006).

The results imply that time domain and frequency domain causality approaches imply different results. While time domain causality analysis indicates causality running from real oil price to real exchange rate, frequency domain causality analysis implies reverse causality in all time frequencies.

According to frequency domain causality analysis results, it is clear that oil price shocks do not affect exchange rate in the Romanian economy. This result is consistent with Romanian's energy dependency level. Low energy dependency prevents the effects of oil price shocks. Also relatively low share of oil in total energy consumption helps to explain the absence of the relationship running from

oil price to exchange rate. On the other hand, existence of causality running from exchange rate to oil price on the medium and long run supports the Coudert et al.'s (2008) implications about the price elasticity of oil demand and oil supply. The effect of exchange rate fluctuations appears on the longer periods due to inelasticity of oil demand and supply on the short run.

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### Appendix A

To test for causality based on frequency domain, Geweke (1982) and Hosoya (1991) defined causality;

$$M_{y \to x}(\omega) = \log \left[ \frac{2\pi f_x(\omega)}{|\psi_{11}(e^{-i\omega})|^2} \right] = \log \left[ 1 + \frac{|\psi_{12}(e^{-i\omega})|^2}{|\psi_{11}(e^{-i\omega})|^2} \right]$$
(2)

if  $|\psi_{12}(e^{-i\omega})|^2 = 0$  that y does not cause x at frequency  $\omega$ . If components of  $z_t$  are I(1) and cointegrated,  $\Theta(L)$  has a unit root. Breitung and Candelon (2006) investigate the causal effect of  $M_{y\to x}(\omega) = 0$  if  $|\psi_{12}(e^{-i\omega})|^2 = 0$ . The null hypothesis is equivalent to a linear restriction on the VAR coefficients.  $\psi(L) = \Theta(L)^{-1}G^{-1}$  and  $\psi_{12}(L) = -\frac{g^{22}\Theta_{12}(L)}{|\Theta(L)|}$ , with  $g^{22}$  as the lower diagonal

element of  $G^{-1}$  and  $|\Theta(L)|$  as the determinant of  $\Theta(L)$ , it follows y does not cause at frequency  $\omega$  if

$$|\Theta_{12}(e^{-i\omega})| = \left|\sum_{k=1}^{p} \theta_{12,k} \cos(k\omega) - \sum_{k=1}^{p} \theta_{12,k} \sin(k\omega)i\right| = 0$$
(3)

with  $\theta_{12,k}$  denoting the (1,2)-element of  $\Theta_k$ . Thus for  $|\Theta_{12}(e^{-i\omega})|=0$ ,

$$\sum_{k=1}^{p} \theta_{12,k} \cos(k\omega) = 0 \tag{4}$$

$$\sum_{k=1}^{p} \theta_{12,k} \sin(k\omega) = 0$$
(5)

Breitung and Condelon's (2006) applied to linear restrictions (4) and (5) for  $\alpha_j = \theta_{11,j}$  and  $\beta_j = \theta_{12,j}$ . Then the VAR equation for  $x_t$  can be implied as

$$x_{t} = \alpha_{1}x_{t-1} + \dots + \alpha_{p}x_{t-p} + \beta_{1}y_{t-1} + \dots + \beta_{p}y_{t-p} + \varepsilon_{1t}$$
(6)

and the null hypothesis  $M_{y\to x}(\omega) = 0$  is equivalent to the linear restriction with  $\beta = [\beta_1, ..., \beta_p]'$ 

$$H_0: \quad R(\omega)\beta = 0 \tag{7}$$

and

$$R(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \dots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \dots & \sin(p\omega) \end{bmatrix}$$
(8)

The causality measure for  $\omega \in (0, \pi)$  can be tested a Standard F-test for the linear restrictions imposed by Eq.(4) and Eq. (5). The test procedure follows an F- distribution with (2, T-2p) degrees of freedom.