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TECHNICAL ANALYSIS AND ECONOMETRIC PREDICTION USING WAVE REFRACTION METHOD

Abstract. Technical analysis methods for determining the share price quoted on the stock markets recorded dramatic developments in recent decades. It is a common fact that in these methods, the other sciences immersion can be met. The desire to model the influence coming from behavioral economics, representing mostly qualitative variables, requires new approaches to quantify this influence.

In this paper we present a method of technical analysis to determine the future price of a share after it drew some "integer" value with a significant impact on investor's decisions. The method is based on a comparison with the analysis of related physical movement of waves after intersecting with the plane which separates two different environments.

Key words: Asset Pricing, Behavioral Finance, Qualitative Variables, Investors.

JEL Classification: G, G12

I. Research status in analyzed field

Analyzing the stock trend we may find the laws of physics. The dynamics of people behavior can be analyzed compared to the rules governing the physical world. The analysis shows the interaction of emotions and mathematics in the same time. Extracting the mathematical aspects of price movement combined to the emotional responses can help determine if a trend will continue or change. It is a common fact that in the extracting methods, the other sciences (physics, astrology etc) immersion can be met.

The Elliott Wave Principle is a form of technical analysis that traders use to analyze financial market cycles and forecast market trends by identifying extremes in

investor psychology, highs and lows in prices, and other collective factors. He proposed that market prices unfold in specific patterns, which practitioners today call Elliott waves, or simply waves. Elliott stated that "because man is subject to rhythmical procedure, calculations having to do with his activities can be projected far into the future with a justification and certainty heretofore unattainable."

The Elliot Wave Principle posits that collective investor psychology, or crowd psychology, moves between optimism and pessimism in natural sequences. These mood swings create patterns evidenced in the price movements of markets at every degree of trend or time scale.

The technical analysis is based on the supposition that asset prices move in trends and that "trends in motion tend to remain in motion unless acted upon by another force" (the analogue of the Newton's first law of motion) C. J. Neely (1997)

Silagadze (feb, 2011) determined a model of technical analyze using the George Gamow's theory of alpha decay. The alpha particle is trapped in a potential wall by the nucleus and classically has no chance to escape. However, according to quantum mechanics it has non-zero, but small probability of tunneling through the barrier and thus to escape the nucleus.

The Silagadze indicator is described by:

 S_i , i = 1, ..., n a price series for some time window. The moving mini-max of this price series, $u(S)_i$, is considered a nonlinear transformation:

$$u(S)_i = \frac{u_i}{u_1 + u_2 + ... + u_n}$$

where $u_1 = 1$ and u_i , i > 1 are defined through the recurrent relations:

$$u_i = \frac{P_{i-1,i}}{P_{i,i-1}} u_{i-1}, i=2,n$$

Evidently, the moving mini-max series satisfies the normalization condition:

$$\sum_{i=1}^{n} u(S)_i = 1$$

The transition probabilities P_{ij} , which just mimic the tunneling probabilities of a small quantum ball through narrow barriers of the price series, are determined as follows:

Pi,i+1 =
$$\frac{Q_{i,i+1}}{Q_{i,i+1} + Q_{i,i-1}}$$
, Pi,i-1 = $\frac{Q_{i,i-1}}{Q_{i,i+1} + Q_{i,i-1}}$,

where:

Qi,i+1 =
$$\sum_{k=1}^{m} \exp\left[\frac{2(S_{i+k} - S_i)}{S_{i+k} + S_i}\right]$$
, Qi,i-1 = $\sum_{k=1}^{m} \exp\left[\frac{2(S_{i-k} - S_i)}{S_{i-k} + S_i}\right]$,

m is the width of smoothing window. This parameter mimics the (inverse) mass of the quantum ball and therefore allows to govern its penetrating ability. Besides, it is assumed that $S_{i+k} = S_n$, if i + k > n, and $S_{i-k} = S_1$ if i - k < 1.

The moving mini-max $u(S)_i$ emphasizes local maximums of the primordial price series S_i .

Alternatively, the moving mini-max $d(S)_i$ which will emphasize local minimums can be construct. All what is needed is to change $Q_{i,i-1}$ and $Q_{i,i+1}$ in the above formulas with $Q'_{i,i-1}$ respectively with $Q'_{i,i+1}$ defined as follows:

$$Q'_{i,i-1} = \sum_{k=1}^{m} \exp[-2\frac{S_{i-k} - S_i}{S_{i-k} + S_i}], \ Q'_{i,i+1} = \sum_{k=1}^{m} \exp[-2\frac{S_{i+k} - S_i}{S_{i+k} + S_i}],$$

Moldovan, Darie (2011) pointed that the data mining methods are considered to be a smart choice for selecting the right technical indicators, allowing tests on very large datasets (an essential condition, regarding the large volume of financial data available) and many combinations of parameters' values, combining daily, weekly or monthly prices for tests

Price movement is due to the phenomenon of mass action, in which most of the present actors on the capital market, trading in that title, choose a level crossing, called the psychological threshold, and accelerate the daily yields obtained or the return to a price below that level, declining at a calculable rate.

Investors' perception of certain values, for example EUR 10 for a share whose price has been in the last six months below that level, results in a change of environment through which a share price trend passes. Thus the passage of some value in a specific value is similar to the passage of light from an optical medium to another. Environmental change is due exclusively to the psychological factor of investors. If holders of more than half of the shares traded believe that the specific amount is a value that can not be overcome in the short term, then this value will act as a medium less dense and will return to trend that worked to the passing point (time). Investors assign greater importance to certain values due to:

1. There are values that stand out by reason of having integer values and many investors enter the analysis.

2. Want to transform the profit in cash.

3. They think that many market players will mark the profits and the price decreases.

4. Other factors driving this perception.

It is a certainty that we have many orders to trade related to these "reference integer values".

II. Classical methods for evaluating stocks trend

According to Choy (2011, June) technical analysis use certain price indicators of the stock market to facilitate the buy and sell decisions. The techniques require the user to look for patterns in the market which would either signal a buy or sell situation.

According to Irwin (2008), many researches have indicated that it is possible to generate positive returns through the use of technical analysis.

They pointed that the most common and efficient classical methods for evaluating stocks trend for medium terms are:

1.Relative Strength Index

The relative strength index is a common measure used in technical analysis. It is first developed by J.Welles Wilder in 1978 in his book *New Concepts in Technical Trading Systems*. The index attempts to measure how much of the stock is being sold and thus whether the stock is oversold or undersold.

The method first calculates the upward change (denoted by U) and the downward change (denoted by D) $% \left(\frac{1}{2}\right) =0$

Let Change = Close(today) – Close(yesterday)

1. If Change > 0 then D = 0 and U = Change

2. If Change < 0 then D = Change and U = 0

3. If Change = 0 then D = 0 and U = 0

After we have calculated the values of U and D, we will then calculated the average U and D through the use of exponential moving average. After we have calculated the moving averages, we then calculate the relative strength by using the formula given below,

$$RS = \frac{EMA[N]ofU}{EMA[N]ofD}$$

The relative strength index is then calculated using the formula below.

$$RSI = 100 - 100 * \frac{1}{1 + RS}$$

The suggested smoothing period is 27 days which using formula will yield a smoothing period of 14. The rules to using the RSI are much simpler than MACD. If the RSI goes above the upper limit, then the stock is undersold and over bought. Conversely, if the RSI goes below the lower limit, then the stock is oversold and under bought. To summarize the implementation, below is the rule table for implementation. Rule for RSI:

- 1. If RSI > upper limits, sell the stock.
- 2. If RSI < lower limits, buy the stock.
- 3. If lower limits < RSI <upper limits, hold the stock.

2. Moving Average Convergence/Divergence

MACD is developed in the 1960s by Gerald Appel. The concept behind MACD is to find the difference between a fast exponential moving average and a slow exponential moving average. The points of intersection between the 2 exponential moving averages indicate the buy and sell signals depending on individual interpretation. The formula suggested by Gerald Appel in 1960s are the 12 days and 26 days exponential moving averages difference which is formulated as below:

$$MACD = EMA[12] - EMA[26]$$

A trigger line is also constructed by the use of an additional 9 days exponential moving average on the MACD which is suggested by Gerald Appel. The formula is shown below,

Signal =
$$EMA[9]$$
 of MACD

To determine the weight for use in MACD, the following formula is used:

$$\alpha = \frac{2}{N+1}$$

The above 3 formula forms the basis for MACD investment strategy. There are a few implementations of the above MACD with varying rules. The two implementations that will be used in this case are the pure MACD method and the MACD-signal method.

The pure MACD has only 2 rules for trading.

- 1. If 12 periods EMA goes above 26 periods EMA, the rule is to buy.
- 2. If 12 periods EMA goes below 26 periods EMA, the rule is to sell.
- The MACD-signal method has 2 rules for trading.
- 1. If MACD goes above signal line, the rule is to buy.

2. If MACD goes below signal line, the rule is to sell.

III. Evaluating the stocks trend using wave refraction method

1. The trend seen as a wave motion

By comparison with the physical, price trend can be seen in an action similar to optical waves, sound, liquids and other types of wave motion. The only difference, that makes us less receptive, is that the speed of price adjustment is much smaller than the other waves, in particular the optical ones. Any behavior where the crossing point from one medium to another is as follows:

- One part is reflected (the share price trend and its meaning will change back to lower values than the reference value, with an angle of reflection equal to the angle of incidence).

- Some will be absorbed, the phenomenon of total reflection (stationary course will be around for a period of several days).

- Some will pass and the speed will be changed due to move into a new environment (the share price will continue its upward trend, but the angle of refraction will be different from the angle of incidence).

To facilitate comprehension of the concept and have a good starting point in this document, we believe that the trend action, after reaching the "integer" reference value will be in one of the two situations:

- Will return to the environment they came, and a share price will fall by a certain speed.

- Will continue to rise, the price will increase by a calculable speed.

In the case of geometrical optics, some light is reflected and the other part changes its direction when entering in the second medium.



Figure 1: Refractory light from passing to another optical medium

source: http://www.physicstutorials.org

where:

IO - is the radius of incidence, determined by the line which shows the direction of light;

O - the point of incidence where the ray of light emanating from one medium encounters a new environment;

N - normal, perpendicular line to the surface separation in the point of incidence;

OR- refracted ray is the line that shows the direction of light that has passed the second environment;

I - incidence angle is the angle between the refracted ray with the normal line;

R - angle of refraction, the angle formed by the refracted ray with the normal line. The situation of refraction of a wave passing from one medium to another (the

deviation from the initial trajectory) is due to the change of the travel speed for the new medium, which is more dense or less dense compared to the original environment.

The laws of refraction are:

- Line of incidence, the refracted ray and normal line are in the same plane.

- Snell's Law: for two given envoirments, the ratio of the sine of the angle of incidence and the angle of refraction is constant.

 $\frac{\sin i}{\sin r} = m$, m is the relative refractive index of the second to the first environment.

 $m = \frac{n_2}{n_1}$, where n_1 and n_2 are the refractive index for the two environments.

For optics, refractive indices depend on the following factors:

- Nature of the environment;

- The color of incident light.

Moving waves behave differently when passing from one medium to another. The sound waves increase their speed in a dense environment (speed of sound in water is greater than air), while the optical waves decrease the speed in a dense environment (speed of light in glass is smaller than in air).

Physically, refraction is a consequence of the Huygens-Fresnel principle; he states that a wave propagates little by little, the points on the wavefront are some secundary "sources" - the sum of the wave fronts of these sources will be the new wave front. Considering the boundary between the two environments as a place of forming secondary sources, we can see that the new wave front will travel at an angle different from the angle of incidence of the original wavefront.

We believe that the trend is acting as a wave of actions to move with a similar optical activity. After comparison with natural waves, we have the following assumptions:

1. The line of incidence is calculated from the closing prices of the share taken in the last 10 days before point (day) of incidence.

2. The point of impact (breaking point) is when the share comes at a price equal to the "full value".

3. Incidence angle (i) is the angle between the incidence line with the normal.

4. n_1 is the refractive index of the environment, formed in the previous period to achieve "full value". We consider this index as equal to 1.

5. n_2 is the refractive index of the environment formed in the next period to achieve "full value" for higher prices than this share. This index differs from the one solely because of investor sentiment on the value that separates the two areas.

6. Normal line is the perpendicular line on the time axe to the point given by the reference value.

For example we will look at Ubisoft shares, the largest developer of games in Europe. The reference date is 08/11/2010 and "full value" under discussion is the price of 10 eur per share.



Figure 2. 10 eur threshold, Ubisoft Action.

Source: http://www.google.com/finance

The problem we propose to solve is the one when we know the angle of incidence and refractive index to determine future price of action under consideration. To do this, it is necessary to determine the angle of refraction (angle of reflection is equal to that of incidence),

$$r = \sin^{-1}\left(\frac{n_1}{n_2}\sin i\right),\,$$

 n_1 and n_2 are the refractive indices of the two averages.

Since in our case the refractive index of the first medium is 1, the formula becomes:

$$r = \sin^{-1}(\frac{1}{n_2}\sin i)$$

If a high incidence angle ("ray" falls very obliquely), the phenomenon of total reflection can occur, practically the trend will not continue the upward

trajectory. This occurs for $i > i_{critic}$, where $i_{critic} = \sin^{-1}(\frac{n_2}{n_1})$.

This means that we can be in one of the following situations:

1. if $n_2 > 1$, the angle of refraction will be lower than the incidence and prices will increase by a higher return than in the past 10 days.

2. if $n_2 < 1$, the angle of refraction will be greater:

a. i < i $_{critic}$; prices will rise with a return lower than in the past 10 days.

b. $i = i_{critic}$; we have the phenomenon of total reflection and the prices will hover around the "full value".

c. $i > i_{critic}$; we have a phenomenon of reflection and prices fall, the trend changed its trajectory. The reflection angle will be equal to or very near to the angle of incidence.

In fact, if we determine the angle made by the trend with the normal line and we find the second environment index (index of refraction) determined by the perception of investors in any of the two cases (refraction or reflection) can lead to the line angle where the future prices can be found.

2. Determining the incidence line

Through the speed of a moving trend (we call it speed compared with the waves of motion), we understand the trajectory of prices given by the prices previous to the reference level. The proposed model, valid for short periods only, consider closing prices obtained in a period of 10 trading days prior to the point of intersection with the new environment.

The relationship between profitability and the angle of incidence is as follows:

- If the daily profitability increases during this period, the incidence power increases, by reducing the angle of incidence.

- If the daily profitability decreases, angle of incidence increases and, depending on the density of the second medium, there is a trend more likely to change their meaning.

- If constant, then we find the incidence line.

To determine the effect of refraction or reflection, we need to determine a line of incidence. This will be achieved using mcmmp to get a linear trend of the 10 daily closing prices.

Differently from mcmmp is that there is an additional condition: the line is forced to pass through the point of incidence.

The function of incidence line is

$$y(t) = a + b * t$$

where a and b are parameters of the line and t is the right time.

Using the method of least squares and taking into account the additional restriction we must solve the following system:

(1)
$$10a + b \sum_{t=1}^{10} t = \sum_{t=1}^{10} y(t)$$

(2) $a \sum_{t=1}^{10} t + b \sum_{t=1}^{10} t^2 = \sum_{t=1}^{10} t * y(t)$
(3) $a + b * 10 = \text{integer value}$

3. Determination of the refractive index n 2

This index takes values different from 1 only because of investors' expectations about the "integer value" analysis.

The formula proposed in this paper:

 $\mathbf{n}_2 = \mathbf{l} \cdot \mathbf{k} \cdot \mathbf{p}$, where:

- 1 examines the evolution of prices once the action has previously reached the "integer value" and the time point of incidence.

$$l = \frac{NZC}{NZS}$$
, where:

- NZC - represents the number of days with positive daily returns over the period.

- NZS - the number of days with negative daily returns over the period.

- k is the ratio of average trading volume over the last 10 days and average volume of trading days with positive returns in the period under review.

K=
$$\frac{VMT10}{VMTC}$$
, where:
- $VMT10$ is the average trading volume over last 10 days;
- $VMTC$ is the average trading volume day with positive earnings
the period under review.

in

-p is the ratio of average trading price in the last 10 days and "integer value".

$$p = \frac{PM10}{VI}$$
, where:

- PM10 is the average trading price in the last 10 days.

- VI is the integer value in the analysis.

4. Validating the used statistical indicators

All indicators are analyzed according with 3 properties: sensitivity, noncatastrophic, noncompensatory.

L, K and P (the indicators used in this paper) are given by the ratio between two values (A and B):

I = A/B

This means that all three of them will act the same when speaking about their properties.

a) Sensitivity is the basic property which shows that when we have modifications of the independent variables values the value of the dependent variable will be modified also. Considering de dependent variable Y and the independent variables X1, X2, ..., Xn:

Y = f(X1, X2, ..., Xn)

An indicator I having the f function form, is sensitive if:

 $Y - Y \neq 0$, where Y'= f(X1+x1, X2+x2, ..., Xn+xn)

Where: x1, x2, ..., xn, are variations different by "0".

Actually an indicator I is sensitive if the f function is injective.

In our case, all of them (L, K, P) are sensitive.

b) Noncompensatory is the property not to obtain the same value of the dependent variable using different values for the independent variables. This property is very important because it shows if a situation is unique or not.

For example:

I = x + y

Where: x, y – independent variables

I is compensatory because when we have Δx and Δy , with the same size but different direction, for x and y variables we get the same value for I:

$$I' = (x + \Delta x) + (y + \Delta y) = x + y + (\Delta x + \Delta y) = I + (\Delta x + \Delta y)$$

If $\Delta x = -\Delta y$:

 $I' = (x + \Delta x) + (y - \Delta x) = x + y = I$

L, K and P are compensatory, but considering two different populations the probability to obtain the same value is less than 5%.

c) An indicator is catastrophic if there are particular values for the independent variables that makes impossible to obtain a value for the dependent variable.

For example if I = A/B, when B = 0 it is impossible to obtain a certain value for I.

K and P look like catastrophic indicators but VMTC (the average trading volume day with positive earnings in the period under review) and VI (the integer value in the analysis) are strictly positive for any population. This means that K and P are non catastrophic.

In L case, NZS (the number of days with negative daily returns over the period) may be 0, but the probability is less than 10%.

Considering these three properties and the behaviour of each indicator for any statistical population we obtain the following values for the validation indicator as follows: for K and P the validation value is 0,974 and for L si 0,925.

We may conclude that all of them are very good indicators as these values are greater than 0,92, Ivan (2005).

5. Determining the angle of refraction or reflection

Angle of refraction is: $r = \sin^{-1}(\frac{1}{n_2}\sin i)$

IV. Numerical illustration of the proposed method and comparisons with other methods

1. Determining the incidence line

For our example (Ubisoft company, listed on the stock exchange in France), analyzed data from 26 October 2010 to 8 November 2010 (10 trading sessions) are:

Table 1: Evolution of UBI in the period 10/26/2010 to 11/08/2010

day	26.10	27.10	28.10	29.10	01.11	02.11	03.11	04.11	05.11	08.11
t	1	2	3	4	5	6	7	8	9	10
Share	8,937	8,969	9,244	9,349	9,244	9,297	9,289	9,331	9,739	10
price										
v(t)										

Given that in this case "full value" is 10, equation 3 becomes:

 $a+b*10=10 \Leftrightarrow a=10(1-b)$

Using SPSS software, regression modeling of the function y(t) = 10(1-b)+b*t we get: b=0.13 eur.

Angle of incidence $i = \operatorname{arccotangent}(b) = \operatorname{arccotangent}(0,13) = 82.5$ degrees.

Since this angle is very large, if n_2 is smaller than 1, then we have the phenomenon of reflection and the trend will change the sign resulting in a decrease in prices.

2. Testing the significance of the b parameter

We used a nonlinear regression with 4 model evaluations and 2 derivative evaluations.

Solving the above model with SPSS we obtained the following data considering the "b" parameter:

Table 2: testing "b" parameter

Parameter	Estimate	Std. Error	Confidence Inter	val 95 %
			Lower	Upper
В	,130933334	,012247922	,103226610	,158640057

In order to see if "b" parameter is significant from statistic point of view we need to calculate the Student test value "calculated t" and to compare it with the critical value "critical t".

- a. "calculated t" = Estimated parameter/Standard Error = 0,1309/0,01224= 10,7
- b. "critical t"

Lower limit of the confidence interval = Estimated value of "b" – Standard Error*"critical t".

"critical t" = (0,1309 - 0,1032)/0,01224 = 2,26

Considering that "calculate t" is greater than "critical t" (10,7 > 2,26) we may conclude that the "b" parameter is significantly different by "0" value and therefore it has statistical significance.

3. Determination of the refractive index n₂

In our example, the period in question is April 26, 2010 to November 8, 2010. EUR 10 UBI was last touched on 26 April. Under these conditions the values of indicators are presented below.



Figure 3 Evolution of Ubisoft action in a full cycle of value

a) calculating the relative size of 1: The number of trading days during the period analyzed is 141.

NZC = 62 daysNZS = 79 days

1 = 62/79 = 0.7848

b) calculating the relative size k Volumes traded in the last 10 days are:

Table 3: Trading volume in the period 10/26/2010 to 11/08/2010

Day	26.1	27.1	28.1	29.1	01.1	02.1	03.1	04.1	05.1	08.11
	0	0	0	0	1	1	1	1	1	
Т	1	2	3	4	5	6	7	8	9	10
Trading volume per share(thousands of shares)	429	523	800	1024	477	300	723	594	1093	1153

Under these conditions VMT10 = 711,600 shares.

Average trading volume for the 62 days of positive returns VMT = 1,028,400 shares.

K = 711.6 / 1028.4 = 0.6919

c) calculating the relative size p

The average price of the last 10 days is eur 9.4 and eur 10 is the integer value.

P = 9.4 / 10 = 0.94

Refractive index of the second environment is:

 $n_2 = l*k*p= 0.7848*0,6919*0,94 = 0,5104$

This value is much smaller than the refractive index of the first environment. This will cause a larger angle of refraction angle of incidence. If the angle of incidence is lower than the critical angle, we will record price declines immediately after reaching the "full value" of 10 eur.

4. Determining the angle of refraction or reflection

In our example n_2 is 0.5104 and I is 82.5 degrees. The angle of refraction is sin r = 0.73/0.51 = 1.43. Because sin r is greater than 1, then that means we find the phenomenon of reflection.

This happens primarily because the incidence angle is 82.5 degrees higher than the critical angle for the low density of the second environment, namely:

$$i_{critic} = \sin^{-1}(\frac{n_2}{n_1}) = 30$$
 degrees.

If the angle of incidence was below 30 degrees, taking into account that the refractive index of the second environment is lower than the one of the first environment, we still had an upward trend but with lower returns.

In these circumstances, the trend changes meaning with an angle equal to the angle of incidence. Thus, in the period ahead, the trend is downward with a negative return on average equal in modulus with the return on the incidence line.

Thus, we expect the next period after November 3, prices will fall by an average of 0.13 eur per day, so that today, December 10, we can easily notice this.



Figure 4: Ubisoft price action in the first 4 trading days after November 8

In the previous figure, we see that after four days of trading on 12 November, the closing price was EUR 9.44.

Considering an average decrease of 0.13 eur per day, we expect the price on November 12 to be 10-0,13*4=9,48 eur, very close to the actual value.

In fact the results are consisting with the methods presented in this paper. By applying the other methods we obtained the same signal of selling because a decrease of price is going to occur.

The interesting point is that the refraction method is giving the smallest difference between the expected price and the real value for short term compared to classic technical methods for determining the trend.

V. Conclusions

This technical analysis causes, with a high probability, share price developments for the coming days, from the time it is reached the "integer value".

There are many attempts to overcome a certain value of price due to changes in investors' perception, so that the density of the medium, given by values greater than or equal to that price, changes. It is possible that by successive changes, that are

computable, to have a penetration of the current environment. These cases will be presented in subsequent communications.

This paper represents the beginning of the impact analysis of psychological factors on the price action. We chose these "integers" as they have visible influence on investors' perceptions.

This method is efficient only in situations where, during the period analyzed, there are no special events (announcements related to turnover, etc.).

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