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AN ECONOMETRIC INSIGHT INTO PREDICTING BUCHAREST STOCK EXCHANGE MEAN-RETURN AND VOLATILITY-RETURN PROCESSES

Abstract. *Although financial time series are primarily observed in terms of asset prices, most common econometric representations refer to asset returns as a typical way to capture risks and volatility in capital markets. Accurate methods for modeling and forecasting risky asset returns and the volatility associated with them are crucially important in any investment decision and portfolio creation, as well as in risk evaluation, asset pricing, derivative securities and option pricing, especially because such financial instruments are currently involved in speculative trading actions. As an emerging capital market, Bucharest Stock Exchange provides great but highly risky investment opportunities. This paper first presents the stylized facts displayed by the Bucharest Stock Exchange BET index and then provides a state-of-the-art application of GARCH modeling approach to predicting BET index mean-return and volatility-return processes.*

Keywords: *Modeling and predicting mean-return and volatility return processes, Generalized Autoregressive Conditional Heteroscedastic (GARCH) models.*

JEL Classification: C22, C45, C51, C53, C63 G17

1. INTRODUCTION

Financial time series are primarily observed in terms of asset prices. Although the current level of such prices is an important concern, their daily fluctuations and the risks associated with them are also at the core of financial decisions. Actually, investors assess expected returns of an asset against its risk.

The underlying stochastic processes on which financial decisions are based upon generate high frequency time series. For this reason, asset returns are constructed as continuously compounded returns (or log returns), i.e., $r_t = \int_{t-1}^{t} \left(\dot{P}(t)/P(t)\right) dt = \left[\ln\left(P_t\right)\right]_{t-1}^{t} = \ln\left(P_t / P_{t-1}\right)$.

The *conditional sample mean* of daily returns $\hat{E}(r_t | I_{t-1})$, i.e., the sample mean conditional to the past information set $I_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$ is quite small, usually near zero (multiperiod log return is simply the sum of the log returns, so average daily return is the overall yearly return divided by 365 days).

Log returns on assets or portfolios are directly related to the way volatility is computed in finance. Many financial time series exhibit a time-varying conditional variance, which mean that they suffer from *heteroscedasticity* (the amplitude of fluctuations is expected to be larger for some time period than the others). This key issue typically arises in financial models where the dependent variable is the return on an asset or portfolio and the variance of the return represents the risk level of those returns.

Volatility is currently referred to as the *conditional sample variance* of daily returns $\hat{Var}(r_t | I_{t-1}) \equiv \hat{\sigma}_t^2 \approx r_t^2$, (alternatively, one can define the volatility by means of the *conditional sample standard deviation* $\hat{Std}(r_t | I_{t-1}) \equiv \hat{\sigma}_t \approx \sqrt{r_t^2}$, instead of conditional variance). It is often used to quantify the risk of the instrument over a specific time period. An instrument that is more volatile is likely to fluctuate in value more than one that is less volatile.

Volatility forecasting is an important task in financial markets. It is common knowledge that certain types of assets experience periods of high and low volatility (the so called *volatility clustering*). During some periods price variations are very frequent and have high amplitude (an extreme case is a stock market crash or bubble), while during other periods, they can seem to have little amplitude for a long time. Highly volatile markets can be exploited for speculative trading actions and are good but risky opportunities for traders to make money.

Volatility is crucially important in asset pricing. According to most asset pricing theories, risk premium is determined by the conditional covariance between the future return on the asset and one or more benchmark portfolios (e.g., the market portfolio or the growth rate in consumption). Investors and portfolio managers have certain levels of risk which they can bear. A good forecast of the volatility of asset prices over the investment holding period is a good starting point for assessing investment risk.

Unfortunately, large scale failures in risk assessment are very dangerous and may be at the origin of financial crises. Thus, accurate volatility forecasts are essential in the pricing of derivative securities, where the uncertainty associated with the future price of the underlying asset is determinant for derivatives prices. Actually, it is possible to trade volatility directly, through the use of derivative securities such as options, whose trading volume has quadrupled in only a few years before the current financial crisis. To price an option, it is necessary to know the volatility of the underlying asset until the option expires. In fact, the market convention is to list option prices in terms of volatility units, which favors a clear specification of volatility measurements in the derivative contracts, provided that the forecasts are accurate.

We can conclude that volatility modeling and forecasting has an essential role in investment, security valuation, risk management and monetary policy making. As a measure for uncertainty, it is a key input to many investment decisions and portfolio creations.

A comprehensive class of models, commonly known as ARCH (**A**uto**r**egressive **C**onditional **H**eteroscedastic) or GARCH (**G**eneralized ARCH) models, has been developed (starting from the Engle's 1982 seminal paper) to capture special characteristics of risky asset returns, such as: time-varying conditional variance, volatility clustering, leptokurtic distribution, asymmetry and mean reversion, co-movements of volatilities across assets and financial markets, and so on. Highly innovative, but still based on the classical estimation theory, this remarkable trend in financial econometrics allows for modeling both the mean-return and the volatility-return processes, i.e., the level or first moment (conditional mean) and the second moment (conditional variance) of a risky asset return process.

In what follows, we provide a state-of-the-art application of GARCH modeling approach to predicting Bucharest Stock Exchange BET index mean-return and volatility-return processes.

2. STYLIZED FACTS DISPLAYED BY THE BUCHAREST STOCK EXCHANGE BET INDEX

BET (Bucharest Exchange Trading) is the reference index for the Bucharest Stock Exchange (BSE) market. It is a free float weighted capitalization index of the 10 most liquid stocks listed on the BSE regulated market. Figure 1 displays the BET index evolution for 3170 stock-market days between September 19, 1997 and April 4, 2010 and shows a non-stationary pattern.

This index posted remarkable growth rates over a long period beginning from 2001, but more consistently from June 2005 to July 2007. However, turbulent episodes have interrupted sometimes the general appreciation trend, culminating with an abrupt stock market index decline on March 28, 2005.

Unfortunately, this long-term stock market rally ended on July 24, 2007 when the BET index reached its historical maximum (10813.59 points). Starting from the last quarter of 2007 and following the lead of almost all other international financial markets affected by the global financial crisis, the Romanian financial indices have experienced a significant depression until February 25, 2009. Since that turning point, that seems to mark again the beginning of a new stock market rally, a spectacular positive appreciation of financial indices has been seen.

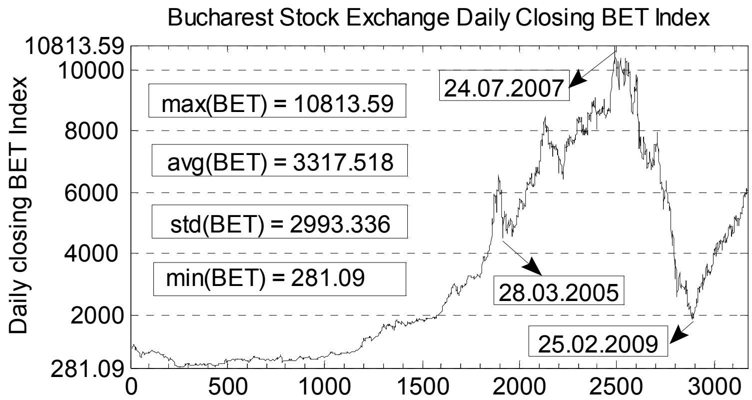

Figure 1. Bucharest Stock Exchange BET index for 3170 stock-market days between September 19, 1997 and April 4, 2010

The average value of BET index in the period under analysis was 3317.518 and its standard deviation was 2993.336.

The daily returns r_t on financial placements in the ten stocks recorded by the BET index are shown in Figure 2. The time-series plot of r_t shows a stationary or mean reverting pattern.

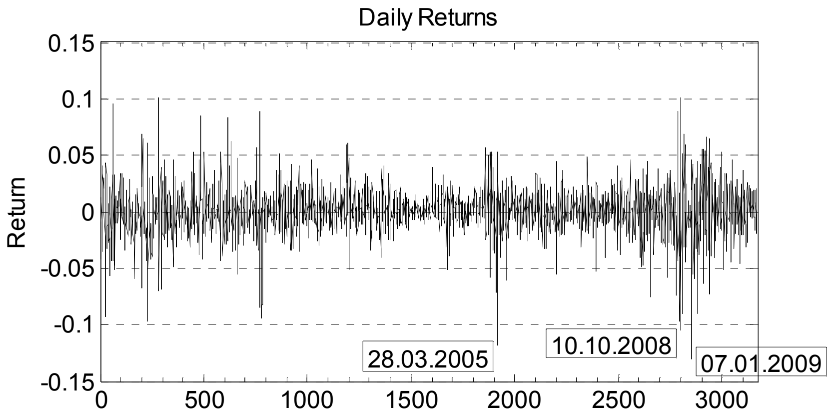

Figure 2. Daily returns: evidence of volatility clustering

For BET index, the returns averaged 0.00057, with a standard deviation measured over the entire period of 0.01888.

The volatility (squared returns) r_t^2 of BET index is displayed in Figure 3.

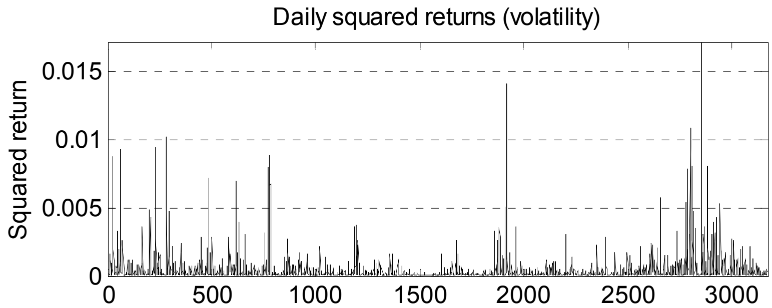

Figure 3. Daily squared returns (conditional sample variance of returns)

A visual inspection of Figures 2 and 3 reveals that the volatility varies significantly over time. This illustrates one of the most important "stylized facts" commonly observed in financial time series: *volatility clustering*, which consists of large changes (upwards or downwards) that are often followed by further large fluctuations, and small changes that tend to be followed by small fluctuations. In other words, quiet and turbulent periods of volatility tend to cluster together. The periods of high volatility are prone to outliers. Indeed, it is to be noticed that the most notable outliers occur on January 7, 2009, March 28, 2005 and October 10, 2008 (actually, around this latter date a "bucket" of outliers occur rather than a single one). After February 25, 2009, the turbulent episodes diminish in intensity and the market tends towards a quieter period.

Probability distributions for asset returns often exhibit fatter tails (excess kurtosis) than the standard normal distribution $N(0,1)$. Figure 4 shows a leptokurtic distribution of asset returns when comparing to the distribution of a Gaussian white noise process.

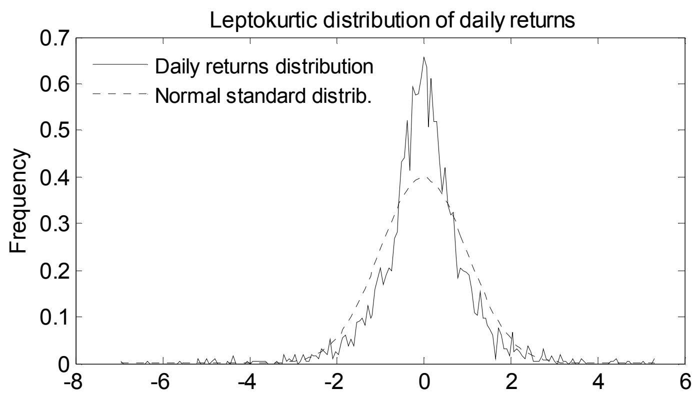

Figure 4. Leptokurtic distribution of daily returns on BET index

Implications of volatility clustering can be examined qualitatively via sample autocorrelation functions (ACF's) and sample partial autocorrelation functions (PACF's) of raw returns r_t (Figures 5 and 6).

For individual tests, the hypotheses can be set up, for each k, as: $H_0 : \rho_k = 0$ versus $H_1 : \rho_k \neq 0$. The test statistic is calculated as $Z_k = \sqrt{T}\ \hat{\rho}_k \sim^a N(0,1)$. A $1-\alpha$ confidence interval around 0 can be constructed as $\mp z_\alpha \big/ \sqrt{T} = \mp 0.0348$, where $T = 3170$ and $z_\alpha = 1.96$ for $\alpha = 0.05$.

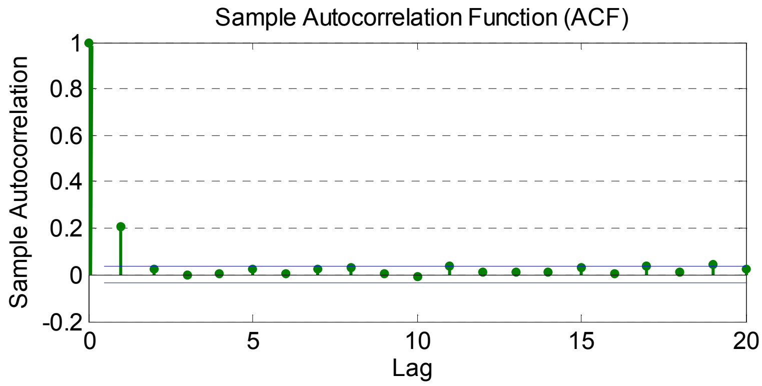

Figure 5. Sample Autocorrelation Function for r_t

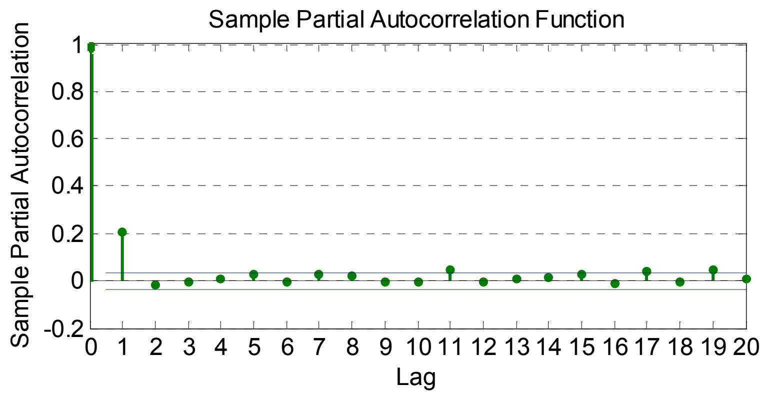

Figure 6. Sample Partial Autocorrelation Function for r_t

Similarly, the sample ACF and PACF for individual tests of squared returns (variances) r_t^2 are depicted in Figures 7 and 8.

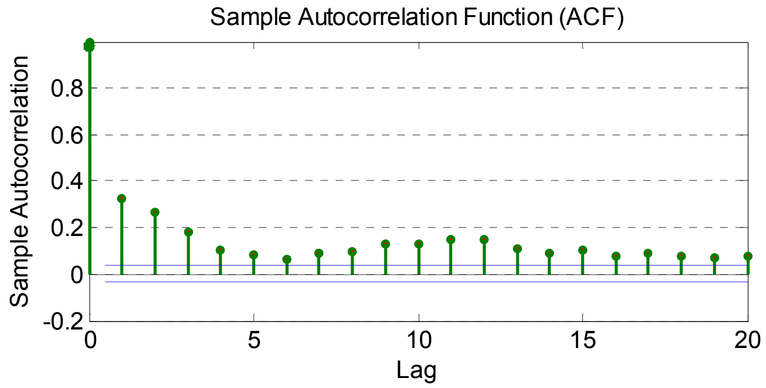

Figure 7. Sample Autocorrelation Function for r_t^2

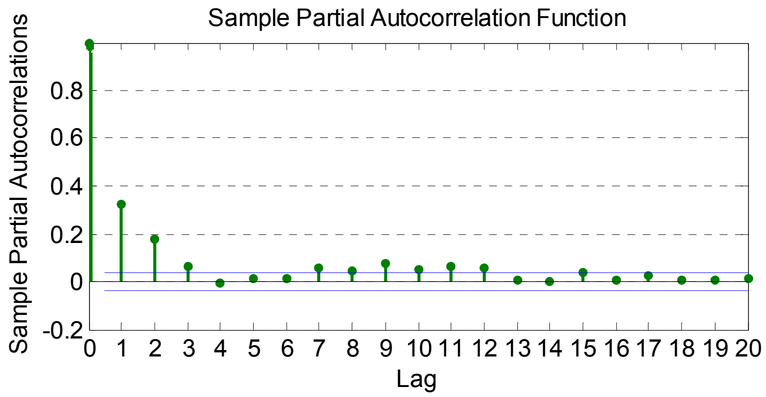

Figure 8. Sample Partial Autocorrelation Function for r_t^2

The sample autocorrelation functions (ACF) for individual tests of r_t and r_t^2, with the approximate 95% confidence level, are depicted in Figure 9. Sample autocorrelation coefficients of r_t are mostly insignificantly different from 0, except for lag k=1, for which $\hat{\rho}_1 > 1.96/\sqrt{3170}$. Those of r_t^2 are mostly significantly different from 0. The overall feature of the return series is that *returns may be mostly uncorrelated in levels, but correlated in squared terms.* Both autocorrelation functions are computed up to a lag of $k = 20$ and exhibit a typical behavior: while there is low autocorrelation in the time series of raw returns, the time series of volatility (squared returns) shows persistence, i.e. high levels of autocorrelation over long-term time scales.

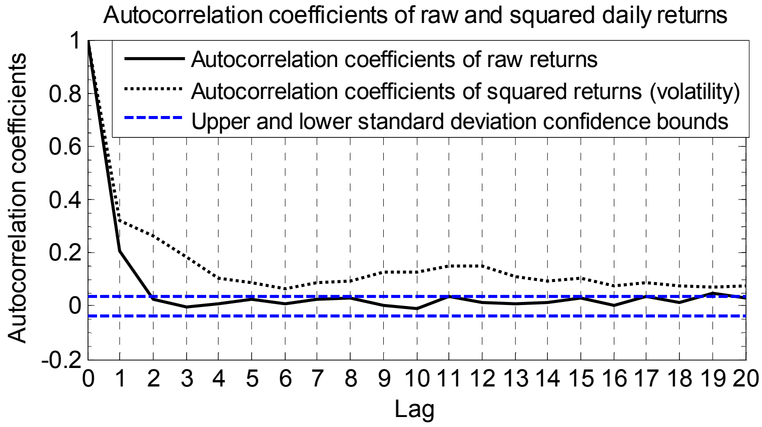

Figure 9. Autocorrelation coefficients of squared daily returns (volatility) are mostly greater than those of raw returns

For *joint tests*, the hypotheses can be set up as

$$H_0 : \rho_1 = ... = \rho_K = 0 \text{ versus } H_1 : \text{at least one } \rho_j \neq 0 \quad (j = 1,2..., K)$$

for some finite *K*. The test statistic is given by the Ljung-Box-Pierce (LBJ) or *Q(K)* statistic calculated as

$$Q(K) = n(n+2) \cdot \sum_{k=1}^{K} \left[\frac{1}{n-k} \right] \hat{\rho}_k^2 \ \sim^a \ \chi_K^2$$

We shall consider $K = 20$ and the joint test hypothesis $H_0 : \rho_1 = ... = \rho_{20} = 0$. For r_t, the *Q(K)* statistic is calculated as $Q(20) = 164.67 > \chi_{20}^2 = 31.41$ at 5% level. This leads to rejecting H_0 at 5% level. For r_t^2, the *Q(K)* statistic is calculated as $Q(20) = 1234.44 > \chi_{20}^2 = 31.41$ at 5% level. This also leads to rejecting H_0 at 5% level. The results of the joint tests are consistent with the results of individual tests. For r_t the null hypothesis is rejected because $\hat{\rho}_1$ is not significantly different from 0. Concluding that r_t are correlated is important because can guide us in choosing the right model specification for returns.

3. MEAN-RETURN AND VOLATILITY-RETURN FORECASTING

Financial decisions are a tread-off between both the asset price movement direction and the risks associated with their volatility. The analysis of Bucharest Stock Exchange market would thus be incomplete without providing a consistent method for predicting both the mean-return process and the volatility-return process on BET index. In order to simultaneously address these goals, we look for a model in class ARMA(r, m)/GARCH(p, q), which is capable to capture the two main characteristics of return data: leptokurtic distribution and volatility clustering.

The mean-return process is specified by an ARMA(r, m) model as

$$r_t = C + \sum_{i=1}^{r} AR_i \; r_{t-i} + \varepsilon_t + \sum_{j=1}^{m} MA_j \; \varepsilon_{t-j} \tag{1}$$

The volatility-return process is specified by a GARCH(p, q) model as

$$\sigma_t^2 = G_0 + \sum_{i=1}^{p} G_i \, \sigma_{t-i}^2 + \sum_{j=1}^{q} A_j \, \varepsilon_{t-j}^2 \tag{2}$$

Let $I_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$ be the past information set. The key insight of volatility modeling lies in the distinction between conditional and unconditional variances of the innovations process ε_t. The term *conditional* implies explicit dependence on a past sequence of observations. The term *unconditional* is more concerned with long-term behavior of a time series and assumes no explicit knowledge of the past. Although successive innovations are uncorrelated, they are not independent. In fact, they are serially dependent and an explicit generating mechanism for the innovation process $\{\varepsilon_t\}$ is $\varepsilon_t = \sigma_t \, e_t$, where $\sigma_t^2 = Var[\varepsilon_t | \, I_{t-1}] = E[\varepsilon_t^{\,2} | \, I_{t-1}]$ is the conditional variance of the innovations ε_t (and hence of r_t) and $e_t = \varepsilon_t / \sigma_t$ designates the standardized innovations, i.e., $e_t \sim i.i.d.N(0,1)$. Equations (1) and (2) imply grater correlation in r_t^2 than in r_t, thus allowing for significant correlation in volatility process. In the case of the GARCH(p, q) process, $Var(\varepsilon_t) = G_0 \Big/ \left(1 - \sum_{i=1}^{q} A_i - \sum_{j=1}^{p} G_j\right)$ and leads to a leptokurtic distribution.

The Maximum Likelihood (ML) procedure is commonly used to jointly estimate the mean-return process and the volatility-return process. However, a specification test of the model is necessary for choosing the orders (r, m) and (p, q) of the ARMA and GARCH models, respectively. Tests on a group of parameters can be carried out via a likelihood-ratio test $LR = 2\left(L(\hat{\theta}) - L(\tilde{\theta})\right) \sim^a \; \chi^2(r)$ under H_0, where $L(\hat{\theta})$ is the log-likelihood function of the unrestricted model; $L(\tilde{\theta})$ is the log-likelihood function of the restricted model, meaning the model with parameter restrictions under H_0 imposed; r is the number of restrictions under H_0. We start with the most parsimonious model, i.e., ARMA(0,0)/GARCH(1,1), in the position of restricted model. Subsequently, we will search for more complex models that eventually overperform it.

The parameter estimates of the model ARMA(0, 0)/GARCH(1, 1), along with their standard errors and t statistics are given in Table 1.

Table 1

Parameter	Value	Standard error	t statistic
C	0.00120137648	0.0002457	4.88900
G_0	1.383841168583e-05	1.3475e-06	10.2700
G_1	0.75723927129	0.0089234	84.8596
A_1	0.22347032540	0.0110620	20.2021

Figure 10 shows the contour plot of the Log-likelihood function and the point $(G_1, A_1) = (0.757239, 0.2234703)$ where it reaches its maximum value, i.e. $LLF_{1,1} = 8587.2917729$.

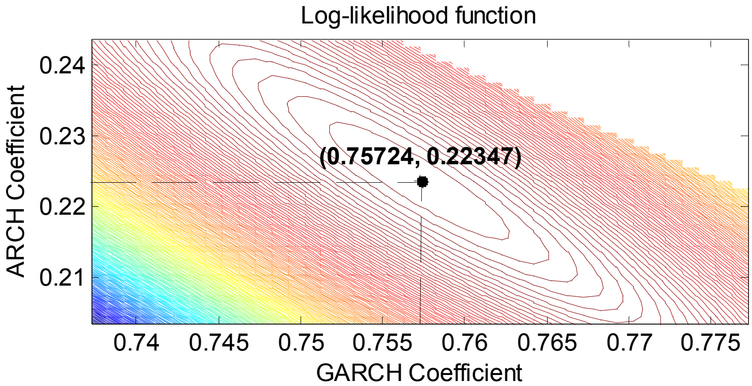

Figure 10. Contour plot of the Log-likelihood function.

We are now going to check whether the basic (most parsimonious) model ARMA(0,0)/GARCH(1,1) is the best model for the return series, or we must specify another model which is more consistent with the data generating process. Helpfully, simple diagnostic tests can be performed at this stage. The idea is that if the ARMA(0,0)/GARCH(1,1) model fits return volatility best, then standardized residuals or innovations $\hat{\varepsilon}_t / \hat{\sigma}_t = \hat{e}_t$ should be approximately a white noise process. Thus, its sample ACF should be mostly in-significantly different from 0.

However, we can see that there exist several autocorrelation coefficients at different lags that exceed the confidence interval bounds $\mp 1.96/\sqrt{3169} = \mp 0.034817289$, such as $\hat{\rho}_{\hat{e}}(1) = 0.188$, $\hat{\rho}_{\hat{e}}(2) = 0.05654$. The squared standardized innovations $\hat{e}_t^2$ approach better a white noise process; however, there still exist some autocorrelation coefficients slightly exceeding the confidence interval bounds. These results are easily confirmed by visual inspection on Figures 11 and 12.

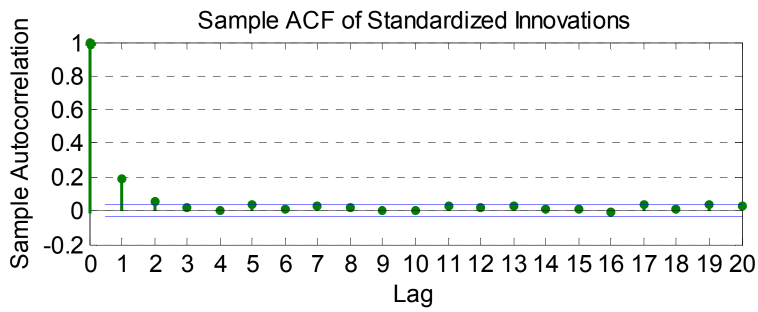

Figure 11. Sample Autocorrelation Function of standardized innovations

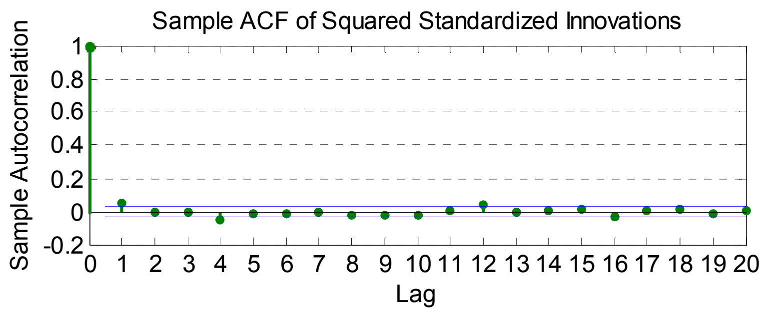

Figure 12. Sample Autocorrelation Function of squared standardized innovations

The model selection procedure is based on the likelihood-ratio test and is attempted to find a more complex model that eventually overperforms the basic ARMA(0,0)/GARCH(1,1) one. If H_0 is rejected then it suggests that the restricted model is preferred. If H_0 is not rejected it suggests a more complex (unrestricted) model is preferred. The results of specification tests are given in Table 2.

Table 2

Model Specification Test: Log-likelihood ratio	LR test Statistic	Critical Value	Accepted Hypothesis	LLF_r vs. LLF_u
ARMA(0,0)/GARCH(2,1) vs. ARMA(0,0)/GARCH(1,1)	30.108768	3.8414588	H1	8602.34616 vs. 8587.29177
ARMA(0,0)/GARCH(1,2) vs. ARMA(0,0)/GARCH(1,1)	-1.74114e-08	3.8414588	H0	8587.29177 vs. 8587.29177
ARMA(1,0)/GARCH(1,1) vs. ARMA(0,0)/GARCH(1,1)	86.359005	3.8414588	H1	8630.47127 vs. 8587.29177
ARMA(0,1)/GARCH(1,1) vs.	83.658214	3.8414588	H1	8629.12088 vs.

ARMA(0,0)/GARCH(1,1)				8587.29177
ARMA(1,0)/GARCH(2,1) vs. ARMA(0,0)/GARCH(1,1)	105.45147	5.9914645	H1	**8640.01751** vs. 8587.29177
ARMA(1,0)/GARCH(2,1) vs. ARMA(1,0)/GARCH(1,1)	19.092466	3.8414588	H1	**8640.01751** vs. 8630.47127

The maximum value of log-likelihood function (LLF) of returns is obtained for ARMA(1,0)/GARCH(2,1), i.e., $LLF = 8640.01751$, which clearly overperforms the other models. The parameter estimates, their standard errors and t statistics are given in Table 3.

Table 3

Parameter	Value	Standard error	t statistic
C	0.001018082	0.0002348	4.3359
AR_1	0.175328820	0.017922	9.7827
G_0	1.567828e-05	1.6879e-06	9.2888
G_1	0.341508083	0.064553	5.2903
G_2	0.363368675	0.056117	6.4752
A_1	0.270212576	0.015571	17.3537

First, let us note that the unconditional mean of the return process is $\hat{r} = C/(1 - AR_1) = 0.001234531$, whereas the unconditional standard deviation of the return process is $\hat{\sigma} = \sqrt{G_0/(1 - G_1 - G_2 - A_1)} = 0.02508745$.

Figure 13 plots the conditional standard deviations derived from the fitted returns. The plot reveals that most values of $\hat{\sigma}_t$ (including the most recent ones) fall below the unconditional standard deviation $\hat{\sigma}$.

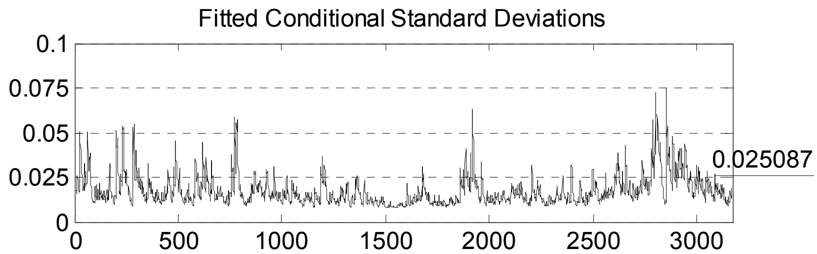

Figure 13. Fitted conditional standard deviations when comparing with the unconditional standard deviation $\hat{\sigma} = 0.02508745$.

The resulting ARMA(1,0)/GARCH(2,1) model can be now used to simulate sample paths for return series, innovations, and conditional standard deviation processes (Figure 14). The simulation procedure acts as a filter allowing to generate a (possibly) correlated return series $\{r_t\}$ from a white noise input series $\{\varepsilon_t\}$. Reversely, one can use the generated return series $\{r_t\}$, from which innovations $\{\varepsilon_t\}$ and conditional standard deviations $\{\sigma_t\}$ processes can be inferred. This inferring procedure is based on calling the log-likelihood objective function and acts as a whitening filter associated with the simulation, because it infers the white noise process from the return series. By comparing the simulated and inferred innovations we can see (Figure 15) that the difference is insignificant, dropping to about zero after the first compared pair. The cause for the initial non-zero difference is the transient effect introduced by the inference.

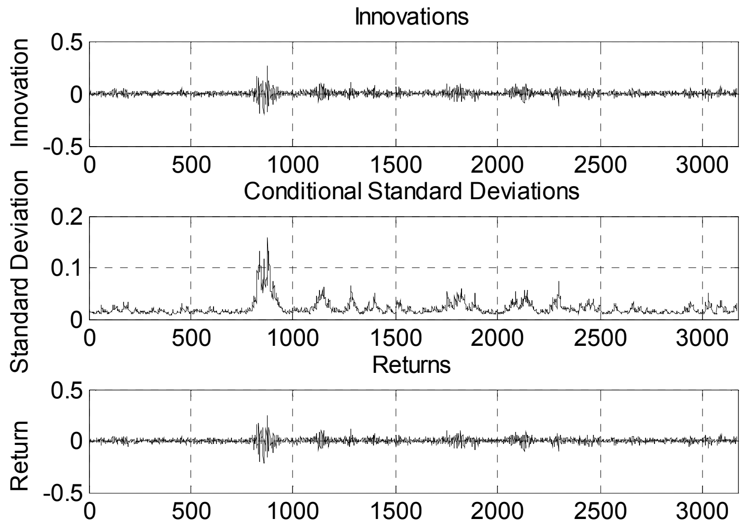

Figure 14. Fitted innovations, conditional standard deviations and returns

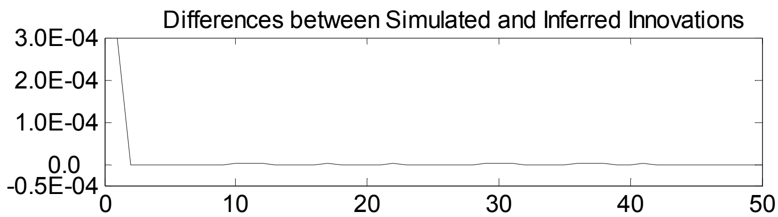

Figure 15. Comparing simulated and inferred innovations

Although plots of both the fitted innovations $\hat{\varepsilon}_t$ and the fitted returns $\hat{r}_t$ in Figure 14 display volatility clustering, plot of standardized residuals $\hat{e}_t = \hat{\varepsilon}_t / \hat{\sigma}_t$ in Figure 16 shows a stable pattern with little clustering and is approximately a white noise process.

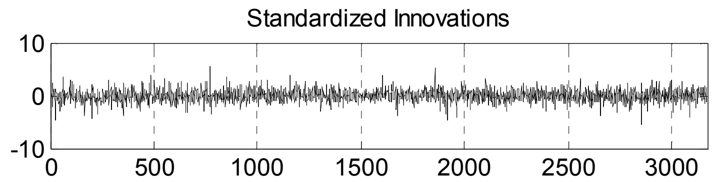

Figure 16. Standardized innovations across time

Figures 17 and 18 display the sample ACFs of standardized innovations and squared standardized innovations, respectively, with the approximate 95% confidence level and indicate that all sample autocorrelations do not exceed significantly the confidence interval bounds of $\mp 1.96/\sqrt{3169} = \mp 0.034817$.

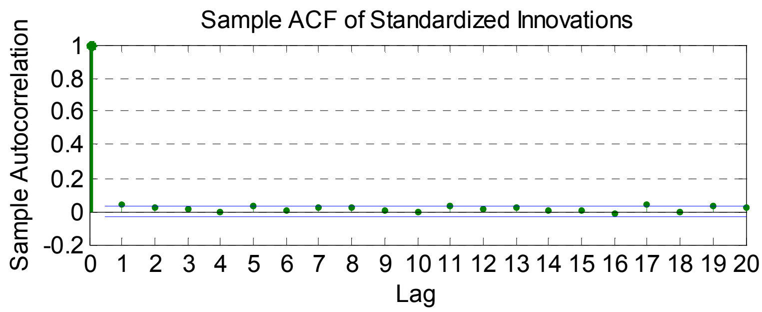

Figure 17. Sample Autocorrelation Function of standardized innovations

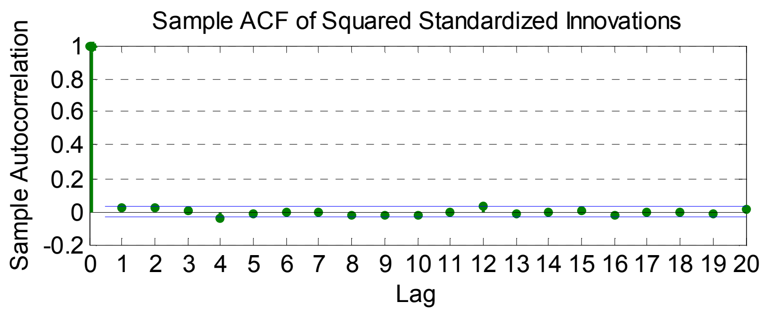

Figure 18. Sample Autocorrelation Function of squared standardized innovations

Both the Q-test and the ARCH test also confirm the explanatory power of the selected model by indicating the acceptance (with highly significant pValues) of their respective null hypotheses for a lag $K = 20$ (Table 4).

Table 4

Test name	p Value	Test statistic	Critical value	H_0 hypothesis
Q-test	0.47297863	9.76081298	31.4104328	acceptance
ARCH test	0.44182367	20.2592068	31.4104328	acceptance

There are two types of forecast we can construct. The first one is an ex-post forecast in which we obtain term-structure of volatility recursively for $t = 2, \dots, T$, treat term-structure of volatility for a sub-period, say $t = 2, \dots, T_1 < T$ as historical returns, forecast volatility over the remaining subperiod of $t = T_1 + 1, \dots, T$, and compare forecasted volatility with realized volatility over $t = T_1 + 1, \dots, T$. Thus, the ex-post forecast provides a way to assess how good the forecasting equation is.

The second one is known as ex-ante forecast in which we obtain term-structure of volatility recursively for $t = 2, \dots, T$, treat term-structure of volatility for the whole period as historical returns, and forecast volatility into future periods, say $t = T + 1, \dots, T + h$.

The computed Minimum Mean Square Error (MMSE) forecasts of the conditional mean and the conditional standard deviation of the return series for a h-period horizon, using the model parameter estimates, allow to obtain information about the asymptotic behavior of the stochastic processes.

Thus, we can see that the ARMA(1,0) process for r_t is stationary (asymptotically stable) and the conditional mean forecasts $\hat{r}_t$ decay rapidly to their equilibrium (steady-state) value $\hat{r} = C/(1 - AR_1) = 0.001234531$, which represents the unconditional mean (Figure 19).

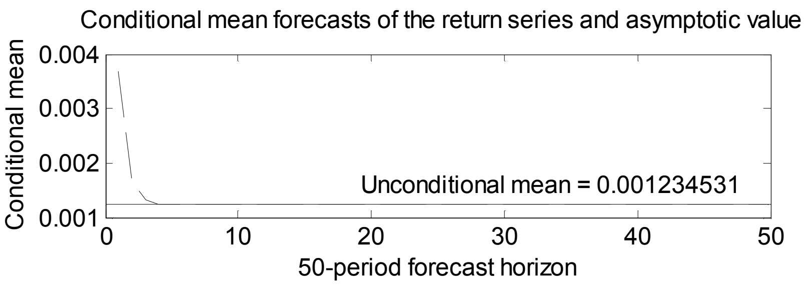

Figure 19. Asymptotic behavior of the conditional mean forecasts $\hat{r}_t$

The forecasts of return volatilities are constructed based on two equations: the GARCH(2,1) conditional variance process, i.e., $\sigma_t^2 = G_0 + G_1\sigma_{t-1}^2 + G_2\sigma_{t-2}^2 + A_1\,\hat{\varepsilon}_{t-1}^2$, and the unconditional variance of $\hat{\varepsilon}_t$ from the GARCH(2,1) model, i.e., $\sigma^2 = G_0/(1 - G_1 - G_2 - A_1)$. The expression for σ^2

allows G_0 be expressed in terms of G_1, G_2, A_1, and σ_t^2. So, substituting and re-arranging yields $\sigma_t^2 = \sigma^2 + G_1\left(\sigma_{t-1}^2 - \sigma^2\right) + G_2\left(\sigma_{t-2}^2 - \sigma^2\right) + A_1\left(\hat{\varepsilon}_{t-1}^2 - \sigma^2\right)$. For $t+h$, it is $\sigma_{t+h}^2 = \sigma^2 + G_1\left(\sigma_{t+h-1}^2 - \sigma^2\right) + G_2\left(\sigma_{t+h-2}^2 - \sigma^2\right) + A_1\left(\hat{\varepsilon}_{t+h-1}^2 - \sigma^2\right)$. So, the best forecast of return volatility *h* periods ahead is:

$$E\left(\sigma_{t+h}^2 \mid I_t\right) = E\left(\sigma^2 + G_1\left(\sigma_{t+h-1}^2 - \sigma^2\right) + G_2\left(\sigma_{t+h-2}^2 - \sigma^2\right) + A_1\left(\hat{\varepsilon}_{t+h-1}^2 - \sigma^2\right) \mid I_t\right),$$

or

$$E\left(\sigma_{t+h}^2 \mid I_t\right) = \sigma^2 + \left(G_1 + G_2 + A_1\right)^{h-1}\left(\sigma_{t+1}^2 - \sigma^2\right), \text{ with } h = 1,\ldots,H.$$

$G_1 + G_2 + A_1$ measures the speed of decay of shocks to volatility; as $G_1 + G_2 + A_1 \to 1$ from below, volatility shocks become more persistent. The asymptotic behavior is defined by $\lim_{h\to\infty} E\left(\sigma_{t+h}^2 \mid I_t\right) = \sigma^2$, i.e., long-term forecast of return volatility converges to its unconditional variance. Furthermore, $\lim_{h\to 1} E\left(\sigma_{t+h}^2 \mid I_t\right) = \sigma_{t+1}^2$, i.e., short-term forecast of return volatility is the next period conditional variance of the return.

According to the GARCH(2,1) model for volatility estimated for the Bucharest Stock Exchange BET index, the conditional standard deviation forecasts $\hat{\sigma}_t$ approach the unconditional standard deviation $\hat{\sigma}$ slowly from below, where $\hat{\sigma} = \sqrt{G_0/\left(1 - G_1 - G_2 - A_1\right)} =$ 0.02508745085 (Figure 20). The volatility is highly persistent since $G_1 + G_2 + A_1 =$ 0.975089 is not so far from 1 (the limit of stationarity).

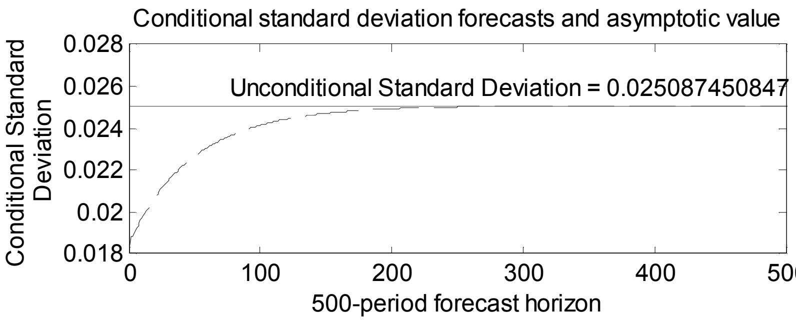

Figure 20. Asymptotic behavior of the conditional standard deviation forecasts $\hat{\sigma}_t$

4. CONCLUSION

Bucharest Stock Exchange BET index obeys all the “stylized facts” about financial time series documented in practice, that clearly do not conform to standard assumptions required when using the Box-Jenkins approach (i.e., constant conditional variance and normal conditional distribution). The BET index displays volatility clustering and time-varying conditional variance, instead. It behaves like a stochastic process that follows a heavy-tailed (leptokurtic) and skewed probability distribution and thus more outliers are expected to occur than from a normal distribution. Volatility (i.e., variation in squared returns) is mostly caused by changes in risk.

The ARCH/GARCH approach to simultaneously forecasting mean-return and volatility-return processes is crucial in situation where the risk is the central issue. We first addressed the model specification problem and used a likelihood-ratio test for choosing the orders (r, m) and (p, q) of the two-equation model in the class ARMA(r, m)/GARCH(p, q). We started with the most parsimonious model, i.e., ARMA(0,0)/GARCH(1,1), in the position of restricted model. Subsequently, we searched for more complex (unrestricted) models that eventually overperform it. Several high order (unrestricted) models qualified as better competitors (i.e., having the null hypothesis of the likelihood-ratio test rejected). Among them we found that ARMA(1,0)/GARCH(2,1) reached the maximum value of log-likelihood function and thus overperformed the other models.

Once the model structure has been identified and the parameters have been estimated, the two-equation model can be used for simulation and forecasting. The simulation procedure acts as a filter allowing to generate a (possibly) correlated return series $\{r_t\}$ from a white noise input series $\{\varepsilon_t\}$ and reversely, to infer the innovations $\{\varepsilon_t\}$ and the conditional standard deviations $\{\sigma_t\}$ processes from the generated return series $\{r_t\}$. Forecasts for mean and the variance can also be computed: - the conditional mean forecasts $\hat{r}_t$, which decay rapidly to their equilibrium (steady-state) value $\hat{r}$, representing the unconditional mean; - the conditional standard deviation forecasts $\hat{\sigma}_t$, which approach asymptotically the unconditional standard deviation $\hat{\sigma}$, slowly from below.

Finally, our econometric insight into predicting Bucharest Stock Exchange mean-return and volatility-return processes revealed the centrality of the econometric analysis of risk for making financial decisions and supporting critical trading actions such as asset pricing, portfolio optimization, derivative securities and option pricing.

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