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PROPOSING AN ADAPTIVE PARTICLE SWARM OPTIMIZATION FOR A NOVEL BI-OBJECTIVE QUEUING FACILITY LOCATION MODEL

Abstract *With regards to the many decisions which are made every day in service and industrial applications, we focus on determination of the number of required facilities along with the relevant allocation process. Goal of this research is proposing a novel bi-objective facility location problem within batch arrival queuing framework under capacity, budget, and nearest-facility constraints. Two objective functions are considered which are minimizing sum of the travel time and waiting and minimizing maximum of ideal time pertinent to each facility, respectively. Second objective causes to obtain the best combination of the facilities which are more equilibrium for the proposed model solutions. Since this type of problem is strictly NP-hard, an efficient meta-heuristic algorithm namely particle swarm optimization algorithm with considering a specific representation process has been proposed. Since the output quality of the metaheuristic algorithms is severely depending on its parameters, we proposed an adaptive version of particle swarm optimization to be tuned the parameters of the algorithm. At the end, the results analysis represents the applicability of the proposed methodology.*

Keywords: *Bi-objective facility location;* $M^{[x]}/M/1$ *queue; Adaptive particle swarm optimization (APSO).*

JEL Classification: C61

1. Introduction and motivation

Expanded practical application of facility location problems (FLPs) is still getting more attractive for researchers. While dealing with the design of service networks such as health, banking, and vending machine services and so on, the location of service centers has an abundant influence on the congestion at each of them and consequently on the quality of service. In a common point, FLP was formally introduced by Alfred

Weber (1909) as an outset point in the location history. Since sensitivity of facility location is mostly related to economical resources and customer satisfaction, so decision makers (DMs) always attempt to be obtained optimal number of facilities with different constraints to be profitable for the facility lifetime and market trends evolve. In the former, several models and different solutions methodologies for FLP have proposed by Love et al. (1988); Marianov and Revelle (1995). Besides, p-median problems are one of the well-known deterministic uncapacitated problems. Since demand and service rate are as a random variable in nature these models are overly unrealistic. This research concentrates on discrete location problems (specifically on p-median problem) as determined number of possible locations. A p-median problem has two components including selecting vertices to be median locations and assigning vertices to medians which are as decision variables of p-median area (Boffey et al. (2007)).

In industrial and operational management respects, FLP is combined with other aspects such as supply chain management, pricing and revenue management, and queuing theory which received noticeable attentions of researchers. To make a model more realistic, in this article, combination of FLP with queuing approach for each facility namely queuing facility location problem (QFLP) are investigated. Queuing theory is one of the best-developed analysis techniques which are every day used in waiting line. As a main purpose of manufactures and service providers, customer satisfaction is mirrored as customer-desired characteristics (Cooper (1980)). In order to optimize decisions and to decrease waiting time, queuing approach can be utilized by service provider. Based on the service type, queuing facility location problems (QFLPs) can be divided into two types of main problems:

(I) Immobile servers which customers travel to a facility such as automated teller machines (ATMs), intercity service centers, internet mirror sites, vending machines (Wang et al. (2002)).

(II) Mobile servers which servers travel from facilities to the users such as ambulances, wireless taxis. Thorough coverage of the mobile case was provided by Berman and Krass (2001).

Moreover, multiple criteria decision making (MCDM) define as the body of methods and procedures which the concern for multiple conflicting criteria can be formally integrated into the analytical process by Ehrgott, X. Gandibleux (2000). Ohsawa (1999) has investigated a single facility bi-criteria model with quadratic Euclidean distance, continuous space, convex combination of the minisum, and minimax objectives including efficiency and equity. The first objective is a minisum cost and the second one has two forms as minisum or minimax of process time. The queuing probabilistic location set covering problem ambulances with Euclidean distances and maxisum and minisum objectives including coverage and cost for locating emergencies is proposed by Harewood (2002).

Currently, multi-criteria location problems have had an abundant expansion. Due to in the literature multi-objective QFLP models are much less than the mono one, the

QFLP model with immobile servers, stochastic customer demand as a bi-objective model has been investigated. Wang et al. (2002) proposed a facility location model as M/M/1 queuing system which customers visit the closest facility and a maximum expected waiting time is considered as a constraint. The proposed model by Wang et al. (2002) was developed by Berman and Drezner (2007) within M/M/m queuing framework which more than one server can be located at each facility. Berman et al. (2006) provided a similar model with more constraints on lost demand and minimizing the number of facilities has provided. In the later, a bi objective facility location problem within M/M/1 queuing framework on p-median problems was proposed by Pasandideh and Niaki (2010a). Two objective functions were including: (I) minimizing summation of travel time and waiting time and (II) minimizing average facility idle time percentage. Besides, a genetic algorithm and desirability function technique have utilized to solve the model.

To make a model more realistic, we consider customer arrival as a batch system. In fact, pervious models will be faced with a problem when a batch of primary customers arrives into the system. So, as a research question, we follow to develop a novel bi objective QFLP within batch arrival queuing framework under capacity, budget, and nearest-facility constraints. It requires to be mentioned that nearest-facility constraint causes to meet customers possible nearest facility. Furthermore, in order to determine more efficacious solutions, the second objective has been provided which causes to be constructed equilibrium outputs for the proposed model. Two another aspects are provided in modelling area consisting: (I) each part of the first objective function involve specific weight and (II) increasing service quality level by considering the coefficient in capacity constraint. To determine the number of required facilities and allocation of the customers to the facilities, an efficient meta-heuristic algorithm namely particle swarm optimization (PSO) algorithm has been developed to solve the problem. Due to the output quality of the metaheuristic algorithms is severely depending on its parameters, the adaptive version of PSO (APSO) algorithm has been considered to enhance accuracy and precision of the proposed model solutions. Thus, these gaps lead to be made research questions of this research.

Problem definition associate with assumptions, parameters, decision variables, and mathematical model are presented in Section 2. In Section 3, the L_P-metric technique is proposed to combine the objectives. The proposed APSO specifications are described in Section 4. Finally, the results, and the conclusions along with suggestions for future research are presented in Sections 5 and 6, respectively.

2. The proposed bi-objective QFLP model

In the former, one of the assumptions of the capacitated facility location problem is that demand is known and fixed. Since the condition of locating facilities and assigning demand points to those facilities is as uncertainty, we consider demand as stochastic and model each facility as an independent queue. In this regard, the numbers of required facilities along with allocation of the customers to the facilities are usually two main questions which answer to them is requisite in most service and industrial application. In the discrete FLP literature, the most problems with immobile servers and stochastic demand are considered with an objective function. Besides, many realistic problems involve simultaneous optimization of several objectives. In this sort of optimization problems, there is usually no single optimal solution. All objectives are considered when a set of alternative solutions are optimal in the wider sense which no other solutions in the search space are superior to them as Pareto-optimal solutions (Deb (2001)). The general multi-objective problem is defined as Eq. (1).

$$\begin{aligned} &Minimize\ f_1(\\ &Minimize\ f_2(x) \\ &. \\ &. \\ &. \\ &Minimize\ f_O(x) \\ &S \\ &x \in \\ &X \subseteq \square^Q \end{aligned} \quad (1)$$

where $x = \{x_1, x_2, \ldots,$ is set of *p*-dimensional decision variables. In order to make model more realistic, we propose a bi-objective facility location model which two considered objective functions are: (I) minimizing the aggregate travel time of customers per unit time plus the aggregate waiting time of customers per unit time and (II) minimizing maximum of ideal time pertinent to each facility. The second objective has been proposed to obtain more equilibrium and efficacious solutions. Furthermore, the available budget, capacity, and nearest-facility constraints have been considered. As one of main contribution, we consider to act each facility as a batch arrival queuing system. A variant of the $M^{[x]}/M/1$ queuing model occurs when one assumes batch arrivals. Each arrival epoch now corresponds to the arrival of a batch of customers where the batch sizes are independent, identically distributed, and random variables (Cooper (1980)). The customers within a batch are served one by one at a time, and, as before, the service times of the customers are independent, identically distributed, and random variables, with Poisson distribution function. Fig. 1 represents queuing facility location problem schematically.

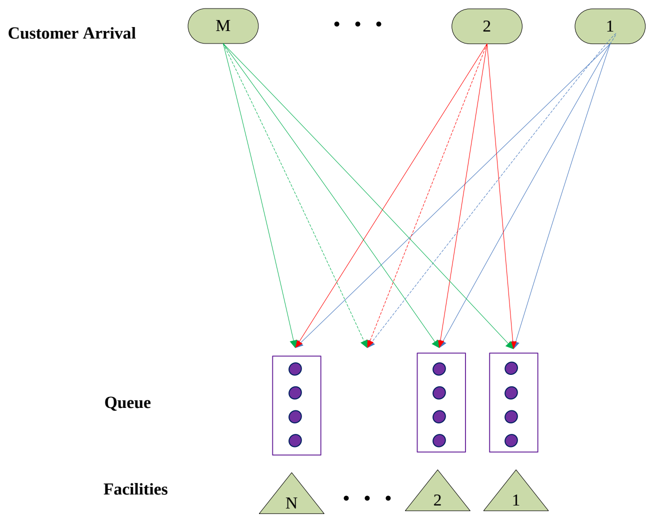

Figure 1. Scheme of queuing facility location problem

Customer nodes can be arrived as a batch which number of customers in each batch follows a geometric process. It should be also mentioned that customers are assumed to visit the closest open facility and each facility have definite capacity. As other contributions of this paper, two conditions including considering the weight for each part of the first objective function and choosing the coefficient to increase service quality level have been investigated. In particular, assumptions, parameters, and variables of the model are first defined. Then, the non-linear mixed integer programming model is illustrated. In this research, below assumptions is considered:

(I) Each customer batch can only be assigned to a facility.

(II) Customers batch travel to each facility to receive the service as immobile servers

(III) The service request of each customer batch follows an independent Poisson distribution and each open facility has only one server with exponential service times. Actually, an open facility behaves as an $M^{[x]}/M/1$ queue.

(IV) Batch size is considered as a random variable.

(V) Each customer batch can only be assigned to a facility.

To formulate this problem, we define the indices, parameters, and decision variables as follows:

M : The total number of customer batch nodes; $i = 1, 2, ..., M$

N : The total number of potential facility nodes; $j = 1, 2, ..., N$

P : Maximum member of servers which can be on-duty; ($P \leq N$)

t_{ij} : The travelling time from costumer batch *i* to facility node *j*; ($i \in M, j \in N$)

w_j : The expected waiting time of customer batches assigned to facility node *j*; ($j \in N$)

λ_i : The demand rate of service requests from customer batch node *i*; ($i \in M$)

μ_j : The service rate for server *j*; ($j \in N$)

τ_j : The demand rate at open facility node *j*; ($j \in N$)

c_j : Fixed cost of establishing a facility at potential node *j*; ($j \in N$)

B : Available budget for establishing facilities

α : Weight ratio for each part of first objective

β : The coefficient of service quality level

U : A large positive number

S : Batch size as random variable

$E(s)$: Average of batch size

π_{0j} : Idle probability of open facility *j*

ρ_j : Utilization factor of facility *j*

$$x_{ij} = \begin{cases} 1 & ; \text{ if Costomer } i \text{ is assigned to facility } j \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & ; \text{ if facility } j \text{ is opened} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

First objective function is divided into two parts. The first part represents aggregate travel time of customers per unit time and the second one indicates aggregate waiting time of customers per unit time. Due to the demand rate at each opened facility site requires to be calculated, we defined parameter. Since each open facility behaves as an $M^{[x]}/M/1$ queue (Gross and Harris (1998)), the expected waiting time at open facility site *j* is also as Eq. (2).

$$\tau_j = \sum_{\forall i} \mathrm{E(S)}\lambda_i x_{ij} \quad \text{and} \quad \mathrm{w_j} = \frac{\dfrac{\rho_j}{1-\rho_j} + \dfrac{\rho_j(\frac{E(S^2)}{E(S)} - 1)}{2(1-\rho_j)}}{\tau_j} \qquad ; \mathrm{j} = 1,2,\dots,n \qquad (2)$$

Where is utilization coefficient for facility j which obtains as $\rho_i = {}^{\tau_j}/_{}$ (Gross and Harris (1998)). So the first objective is the summation of traveling and waiting time which should be minimized. As stated by characteristics of an $M^{[x]}/M/1$ queue, the second objective considers other aspect of system provider's goal which indicates as idle probability at each facility. In this regard, previous researches represented the average of idle probability but we make an important contribution to the idle probability as objective function. In fact, providing average of idle probability will decrease this probability for each facility; however the facility with maximum idle probability is more sensitive and considerable. Thus, the second objective function is as $\pi_{0i} = 1 -$ which cause to be minimized idle probability of facility which is maximum one (Gross and Harris (1998)). Since the idle probability of the inactive facility should not to be computed, we multiply by second objective. To do so, the proposed mathematical model will be as follows:

$$Min \;\; T_1 = \alpha \sum_{\forall i}\sum_{\forall j} \lambda_i t_{ij} x_{ij} + \; (1-\alpha)\sum_{\forall i}\sum_{\forall j} \lambda_i \frac{\dfrac{\rho_j}{1-\rho_j} + \dfrac{\rho_j\left(\frac{E(S^2)}{E(S)} - 1\right)}{2\left(1-\rho_j\right)}}{\tau_j} x_{ij} \qquad (3)$$

$$Min \; T_2 = max_j\left\{\left(1 - \frac{\tau_j}{\mu_j}\right) y_j\right\} \qquad (4)$$

S.t.

$$\sum_{\forall j} x_{ij} = 1 \qquad ; \;\; i = 1,2,\dots,m \qquad (5)$$

$$x_{ij} \le y_j \qquad ; i = 1,2,\dots,m, \;\; ; j = 1,2,\dots,n \qquad (6)$$

$$\sum_{\forall j} \mathrm{y}_j \;\le\; \mathrm{P} \qquad ; \;\; i = 1,2,\dots,m \qquad (7)$$

$$\mathrm{E(S)} \sum_{\forall i} \lambda_i x_{ij} < \beta\, \mu_j \qquad ; \; j = 1,2, \dots, n \tag{8}$$

$$\sum_{\forall j} c_j y_j < B \tag{9}$$

$$\sum_{\forall k \in N} t_{ik} x_{ik} \leq \left(t_{ij} - U\right) y_j + U \qquad ; \; i = 1,2, \dots, m \;, \quad j = 1,2, \dots, n \tag{10}$$

$$x_{ij} \in \{0,1\} \;, \quad y_j \in \{0,1\} \qquad ; \; i = 1,2, \dots, m \;, \quad j = 1,2, \dots, n \tag{11}$$

$$\tau_j = \sum_{\forall i} \mathrm{E(S)} \lambda_i x_{ij} \;, \; \rho_j = {}^{\tau_j}/_{\mu_j} \qquad ; \; j = 1,2, \dots, n$$

The concept of each part of the proposed model is illustrated as follows:

- Eq. (3) represents the first objective function which the aggregate travel time of customers per unit time plus the aggregate waiting time of customers per unit time must be minimized.
- The second objective function which minimizes the maximum of ideal time pertinent to each facility is represented in Eq. (4).
- Eq. (5) ensures that each customer must be assigned only to a facility.
- When a facility not to be selected, any customer can not be assigned towards it which is ensured by Eq. (6).
- Eq. (7) ensures maximum number of opened facilities.
- Eq. (8) considers capacity constraint for each server which the input to each server must be less than its capacity. Moreover, for increasing the service quality, we multiply a coefficient by service rate.
- Eq. (9) insures fixed cost of establishing the opened facilities must be less than available budget.
- Eq. (10) ensures that this assignment is curried out to the closest facility.
- Finally, the binary restrictions on the decision variables are stated in Eq. (11).

3. The MODM technique

Since applicability of multi objective optimization in engineering sciences grew over the recent decades, we utilize transforming a multi objective problem into a single which is one of the most well known methods for solving multi objective optimization problems. Due to the weights must be determined by decision maker (DM), we choose a weighted method namely L_P-metric ($P \in ([1, \infty) \cup \{)$ to be authorized selection of the weights by DM (Stadler (1984)). The difference between each objective function with its optimum value is minimized which is the goal of L_P-Metric technique. Moreover, this method chooses a desired point $f_i^* \in$ and looks for an optimal solution which is as near as possible to this point (Ehrgott and Gandibleux (2003)). It should be mentioned that when an appropriate collect of solutions is obtained by the single-objective optimization problem, the solutions approximate as a Pareto front or Pareto surface in objectives space (Stadler (1984)). This section investigated the weighted L_P-Metric technique to transform two objective functions to a single one as Eq. (12).

$$\text{Minimize} \quad \left(\sum_{i=1}^{Q} \left[\varpi_i \left| \frac{f_i^* - f_i(x)}{f_i^*} \right| \right]^P \right)^{1/P}$$

$$S.t.: \qquad (12)$$

$$x \in X \subset \square^Q$$

Where ϖ_i is the weight of objective function i which is stated by DM. Parameter Q indicates number of investigated objective functions. Particularly, for P = 1, the definition yields the so called Manhattan metric, L_2 is the Euclidean metric, and L_∞ is the Tchebycheff metric. Next section represents characteristics of the proposed metaheuristics.

4. The proposed adaptive PSO

As observed in previous section, the proposed mathematical model is as a constrained non-linear integer programming problem. Due to the nature of the proposed model is NP-hard, the model is hard enough to be solved by an exact method (Wang (2002)). Regards to the complicated calculation procedures and time consuming optimization models, the use of metaheuristic algorithms is very common and efficient. Since the population-based metaheuristics have been prosperous in solving models similar to our proposed model (Pasandideh and Niaki (2010a); Farahani et al. (2010)), an adaptive PSO (APSO) has been investigated. To enhance the performance of the APSO, we develop a new chromosome representation which three constraints of the proposed model have been feasible. In order to justify proposed metaheuristic, a PSO has been considered to represent efficiency and intelligence of the proposed APSO. As mentioned above, since metaheuristics are so sensitive than its parameters, to tune the parameters, adaptive version of PSO has been proposed to obtain more appropriate outputs for our model. In following subsections, general steps involved in the proposed APSO algorithm are illustrated:

4.1. Primary concepts

Kennedy and Eberhart (1995) have proposed PSO algorithm as an evolutionary computation technique which based on the movement and intelligence of swarms looking for the most fertile feeding location. In fact, PSO algorithm simulates the behavior of flying birds (particles) and their means of information exchange to solve optimization problems. Each particle flies in the search space with specific velocity which is dynamically adjusted according to its own flying experience and its companions' flying experience. The rate of the position change for each particle is depicted by velocity vector ($\vec{V}$). At each iteration, the particles are updated by $x^{i,best}[iteration]$ and $x^{g,best}[iteration]$ which are local best solution that the particle has achieved so far and global best solution obtained in the swarm up to this point, respectively. Each particle is updated by Eq. (13), (Clerc and Kennedy 2002):

$$x^i[I+1] = x^i[I] + V^i[I+1]\;;$$

$$V^i[I+1] = \underbrace{\gamma \times V^i[I+1]}_{\text{Momentum Part}} + \underbrace{C_1 \times rand() \times \left(x^{i,best}[I] - x^i[I]\right)}_{\text{Cognitive Part}} + \underbrace{C_2 \times rand() \times \left(x^{g,best}[I] - x^i[I]\right)}_{\text{Social Part}} \qquad (13)$$

The used parameters are defined as follows:

γ: Inertia factor which is used to control the amount of the previous velocity between $x^{i,best}$ and $x^{g,best}$.

C_1: Personal learning coefficient which represents weight of the stochastic acceleration terms that pulls each particle toward $x^{i,best}$.

C_2: Global learning coefficient which represents weight of the stochastic acceleration terms that pulls each particle toward $x^{g,best}$.

I: Number of iteration.

$rand$ (): Random number in the range [0, 1].

As indicated in Eq. (13), momentum part represents the influence of the last velocity towards the current velocity, cognitive part indicates the primitive thinking by itself; and social part shows the cooperation among the particles. At each iteration, when current $x^{i,best}$ is better than previous position, the new position is selected as local best solution. Similarly, if current $x^{g,best}$ is better than previous best global position, the new best global position will be chosen. Therefore, PSO algorithm possesses high search efficiency by combining local search and global search by self experience and neighboring experience, respectively. As defined above, to generate initial population, random generation policy has been considered. In order to increase feasibility of chromosomes and satisfy more constraint, we develop a new type of chromosome which will be explained in following subsection.

4.2. Chromosome representation

Since most key features for representing a chromosome is its feasibility, we propose sort of chromosome which caused to be feasible constraints (5), (6), and (7). To do so, numbers of required facilities along with allocation of the customers to the facilities are decision variables which must be provided in chromosome representation. To satisfy some constraints, chromosome representation is formed as three parts which are illustrated as follows: First part is consisted number of customer nodes (*M*) as a 1×*M* vector. Each member is involved a random number between zero and one. Second part is indicated number of facility nodes (*N*) as a 1×*N* vector which similarly each member is a random number between zero and one. Third part consisted a random number between one and maximum member of servers which can be on-duty (*P*). The structure of proposed chromosome is depicted in Fig. 2

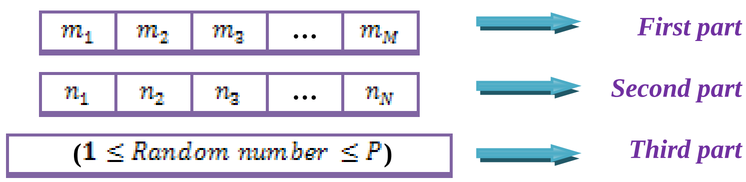

Figure 2. Chromosome Scheme

4.3. Decoding process

When the chromosome is represented, it required to illustrate its decoding process. The proposed decoding process of the chromosome is as a backward order. To increase clarity of the chromosome function, the coding process including encoding and decoding schemes have been described as a numerical example. Consider $M = 7$, $N = 5$, and $P = 4$. Therefore, all steps of the coding process are implemented as following subsections:

4.3.1. As regards to the third part of the chromosome, a random number (p) between one and four is generated. Consider $p = 3$.

4.3.2. The second part of chromosome (vector $\vec{n}$) with five genes is generated which each gene indicates a random number between zero and one in Fig. 3.

0.43	0.18	0.58	0.93	0.29

Figure 3. Generated vector n

4.3.3. The generated number as ascending order should be sorted and it requires being reserved the position of numbers as Fig. 4.

0.18 (2)	**0**.29 (5)	**0**.43 (1)	**0**.58 (3)	**0**.93 (4)

Figure 4. Sorted vector n

4.3.4. The first three (p) genes of sorted vector are selected to represent opened facilities as Fig. 5.

0.18 (2)	**0**.29 (5)	**0**.43 (1)	**0**.58 (3)	**0**.93 (4)

Opened facilities with regarding to $p = 3$

Figure 5. Opened facilities

4.3.5. In this step, the selected facilities are reported for assignment process. Besides, the position number of potential facilities before sorting process represents the number of selected facilities which is determined in Fig. 6.

0.18(2)	**0**.29 (5)	**0**.43 (1)	**0**.58 (3)	**0**.93(4)

Selected (active) facilities → *2, 5, 1*

Figure 6. Number of Selected facilities

4.3.6. The second part of chromosome (vector $\vec{m}$) with seven genes is generated which each gene indicates a random number between zero and one. (As Fig. 7)

0.65	**0**.32	**0**.19	**0**.09	**0**.73	**0**.97	**0**.66

Figure 7. Generated vector m

4.3.7. Compute vector $\vec{\mathrm{H}}$ where $\mathrm{H_i} = \lfloor \mathrm{v} \times \mathrm{m_i} \rfloor + 1$ which determined as Fig. 8.

2	**1**	**1**	**1**	**3**	**3**	**2**

Figure 8. Vector H

4.3.8. Last step respectively determine the active facilities and allocates each costumer to each number of vector H. Thus, the decision variables are obtained as $y_2 = 1$, $y_5 = 1$, $y_1 = 1$ and also $x_{15} = 1$, $x_{22} = 1$, $x_{32} = 1$, $x_{42} = 1$, $x_{51} = 1$, $x_{61} = 1$, $x_{75} = 1$. The other variables are all equal to zero.

4.4. Particles evaluation

In order to evaluate each individual in the swarm, the combined objective function value is considered. Each individual will be evaluated for the combined objective function of f_1, f_2 by L_P-metric technique. In the presence of constraints in the proposed model, some generated chromosomes may not be feasible. Moreover, in the meta-heuristic algorithms literature, the most best-developed approach for handling constraints is penalty functions policy. The policy of penalty function is that the penalty value will be considered zero when a chromosome is feasible and it will be chosen as a non-zero value, even though one of the constraints is not be satisfied. It requires to be mentioned that the average of violations has been considered for each sort of constraint as normalized one. Besides, when a chromosome is feasible, the penalty value will be zero and, otherwise, the penalty value will be multiplied to the cost function value as Eq. (14). As regards to the general form of constraints as$g(x) \leq b$, the penalty value of a chromosome is calculated as follows (Yeniay and Ankare (2005):

$$F(x) = \begin{cases} F(x) & ; x \in \text{feasible region} \\ F(x) \times P(x) & ; x \notin \text{feasible region} \end{cases} \quad and\ P(x) = U \times Max\left\{\left(\frac{g(x)}{b} - 1\right), 0\right\} \quad (14)$$

where$F(x)$, U,$g(x)$ and $P(x)$ indicate combined objective function value, a large number, constraint, and penalty value of chromosome *x*, respectively.

4.5. Adaptive scheme

One of the efficient methods of incorporating the PSO parameters is proposed by Clerc and Kennedy (2002). As regards Eq. (13), the main parameters of PSO are obtained by following formula as Eq. (15):

$$\gamma = \frac{2}{\emptyset_1 + \emptyset_2 - 2 + \sqrt{(\emptyset_1 + \emptyset_2)^2 - 4 \times (\emptyset_1 + \emptyset_2)}} \ , \ C_1 = \gamma \times \emptyset_1 \ , \ \ C_2 = \gamma \times \emptyset_2 \quad (15)$$

where $\emptyset_1 + \emptyset_2 > 4$. So, the optimal setting is acquired when $\emptyset_1 = \emptyset_2$ and$\emptyset_1 + \emptyset_2 = 4.1$. Implementing this process has been called PSO algorithm with no problem-specific parameters. Therefore, in order to increase efficiency of the proposed PSO, adaptive version of the metaheuristic is investigated.

4.6. Stopping criteria

Stopping criteria is a set of conditions such that when the method satisfies them, corresponding solution is obtained. The proposed APSO algorithm will be stopped after a fixed number of iterations. The computational performance of the algorithms on a set of problems is investigated in next section.

5. Results analysis

This section concentrates on two features which consisted how generated test problems and providing answer of the research questions which is stated in the Section 1. Actually, we have illustrated how the results support the answers and how the answers fit in with existing knowledge on the topic. In order to best performance of proposed metaheuristic algorithm, several test problems are generated. The proposed mathematical model is as a constrained Non-linear integer programming problem. Due to the nature of this problem is NP-hard, the exact methods are hard enough to select as a solving methodology (Pasandideh and Niaki (2010a); Pasandideh and Niaki (2010b); Pasandideh and Niaki (2011); Wang et al. (2002)). In this sort of problems with complicated calculation procedures and time consuming optimization models, the application of metaheuristic algorithms is very common and useful. To increase efficiency and intelligence of the proposed algorithm, adaptive version of PSO is investigated. Since applicability of the objective function value is goal of this paper, we investigate it as response variable. However several problems are solved by a solver-software such as Lingo 8.0 (Release 8.0) to demonstrate clarity of proposed model, it not to be required to report the Lingo outputs. The reason is that nature of FLPs is large so proposing efficient metaheuristic takes account an appropriate methodology.

Table 1
General data

Problem Number	Number of customer (*M*)	Number of potential facilities (*N*)	Number of on-duty servers (*P*)
1	4	5	3
2	9	6	3
3	16	7	5
4	20	9	6
5	35	12	9
6	42	15	11
7	57	17	12
8	62	21	14
9	77	25	18
10	81	30	20
11	90	38	22
12	105	42	25

$\lambda_i \sim Uniform[2,16]\, \mu_i \sim Uniform[50,100]\, t_{ij} \sim Uniform[65,95]$

$S \sim Geometric(0.5)\, C_j \sim Uniform[100,500]\, \alpha = 0.5\, \beta = 0.95\, \varpi = [0.4 \quad 0.3 \quad 0.3]$

More to the point, the experiments are implemented on 12 problems. Actually, various problems with different size are provided to demonstrate the performance of the proposed metaheuristic. Moreover, to achieve better solution and eliminate the uncertainty in the random generations, each problem has been run five times. Following this, averages of these five runs are calculated as ultimate solution. General data of the proposed model has been provided in Table 1. To code the proposed metaheuristic algorithm, MATLAB (Version 7.10.0.499, R2010a) on a laptop with a Pentium 1860 processor and one GB RAM has been utilized.

It requires to be mentioned that we solve the problem number 1 and 2 with Lingo 8 software to demonstrate the proposed model acts correctly. Besides, some points have caused to withdraw Lingo outputs reports. At first, since response variable of our research is cost function value (not computational time), the lingo outputs not to be considered to be reported. Actually, the winning of metaheuristic algorithms in large size problem is strictly obvious. Thus, since the nature of FLPs is large, we concentrate on metaheuristics solutions. To do so, implementing the proposed APSO algorithm with the obtained values of problem number 10, after one hundred generations, the algorithm has been converged with a combined cost function of 0.44536 as Fig. 9.

To wrap up the discussion, when a batch of primary customers arrives into the QFLP system, pervious models would be encountered with a difficulty. Therefore, a new bi-objective QFLP within batch arrival queuing framework has proposed to make a model more realistic. Additionally, proposing new type of objective function which causes to be place equilibrium in facilities system; providing weight for each part of the first objective function; considering the coefficient to increase service quality level in capacity constraints; proposing adaptive version of PSO algorithms with specific coding process in QFLP area, are the contributions of this article. Thus, as answer of research question, the new model in QFLP area associate with an efficient APSO algorithm are proposed in this article. Although dimension of QFLPs in the realistic world is usually large, the proposed metaheuristic are efficient and economical enough for solving proposed model.

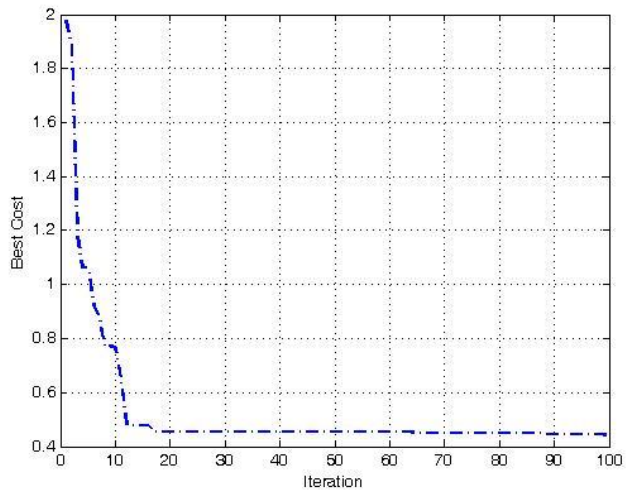

Figure 9. The convergence path of proposed APSO

6. Conclusion and suggestion for future research

In this article, a new bi-objective facility location problem within $M^{[x]}/M/1$ queuing framework has proposed to obtain the number of required facilities along with the relevant allocation process. Two objective functions are considered which are consisted: (I) minimizing summation of the travel time and waiting and (II) minimizing maximum of ideal time pertinent to each facility. As a contribution, second objective function is proposed to determine the best combination of the facilities. Moreover, two features are executed in modeling area including the weight for each part of the first objective function and the coefficient to increase service quality level. Due to the problem is NP-hard, an efficient APSO algorithm is proposed. The adaptive version of PSO causes to tune the parameters of the algorithm. Finally, applicability of the proposed metaheuristic has demonstrated by computational results. The future research directions include in the following:

(I) Other queue rules to model the QFLPs can be investigated

(II) When a customer faced with multi-echelon queuing network, the costumer arrival rate is changed. Thus, this feature can be modeled:

(III) A new all-feasible chromosome representation can be developed

(IV) The demand rate of service requests and service rate can be considered as fuzzy inputs (as $FM/FI1$ queue)

(V) Other multi-objective solving methodologies and compare them with each other able to be provided.

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