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QUALITY COMMITMENT UNDER MONOPOLY

Abstract: *If incomes of consumers are significantly grown, consuming structures correspondingly change. For a monopolization industry with a single type of products, a monopolist may introduce a new product and a quality commitment is simultaneously launched. By establishment a game theory model, the quality commitment under monopoly is characterized in this work. This paper argues that high prices of old products and high expenditures of new products are two major factors to deter the producer from releasing the new product under monopoly.*

Keywords: *Market structure, industrial organization, quality commitment, price, game theory.*

JEL Classification: C61, C72，D4, L1

I. Introduction

Schelling's pioneer work (1960) explored commitment in economics and subsequently some interesting papers developed mathematical model to characterize commitment. In the decision with multiple players, commitments move opponent's strategy such that the corresponding players significantly benefits from these commitments. There exist numerous researchers about commitments in economic field and in social science. Krueger and Uhlig (2006) developed commitment theory with one-side commitment. Caruana and Einav (2008) recently explored the theory of commitments with dynamic game theory. The conflicts and commitments were recently exploited in bilateral bargaining by Ellingsen and Miettinen (2008). Esteban and Shum (2007) discussed the car market with commitment and some interesting conclusions were achieved.

Commitments of firms have crucial effects on prices of corresponding goods. Based on the data of U.S.A storable goods, Krueger et al. (2008) confirmed that household consumption with commitments, yields low prices. Dudine, Hendel & Lizzeri (2006) compared commitments with non-commitment under monopoly for durable goods. For storable goods, prices with commitments are lower than those that without commitment under monopoly. Recently, Nie (Nie, 2009;Nie, 2011) derived the commitments with storable goods under vertical integration case. In the recent paper of Goering (2008), the level of commitments for socially concerned firms was addressed. Based on Stackelberg game, commitments were further explored by Kopel and Loffler (2008). According to commitment and non-commitment about the audit of the principal, Chen and Liu (2008) recently investigated the optimal contract. Gautier, Teulings and Van Vuuren (2010) showed that on-the-job search in combination with monopolistic wage setting without commitment created a "business-stealing" externality.

Quality commitment, in which the producer launches a commitment in the quality, is also exceedingly important in market. Firstly, quality commitment can efficiently improve the confidences of consumers. Secondly, by quality commitment, product differentiations are significantly promoted, Gupta, Grant and Melewar (2008) addressed this topic. Thirdly, quality commitment can change opponent’s strategies. Finally, there exists some laws in many countries regulated the quality commitment.

Quality commitment plays extremely important role in many industries. Especially, in some industries, producers own private information of products while consumers lack academic knowledge about products. In marble market, for example, it is very difficult for an average consumer to acknowledge the quality of marbles. On the other hand, the marble is an important type of adornment in China. In this situation, the quality guarantee seems extremely important in these industries. We also point out that the above assumptions hold for many industries in practice.

It is therefore important to highlight quality commitments in all aspects. This motivates the intensive research on the quality commitments under monopoly market structure in this work. Quality commitments are captured in the industrial organization theory field in this paper. We focus on the monopoly market structure because this type of market structure is quite tractable and very popular in economic communities. Monopoly is an extremely popular market structures all over the world. Some industries in some districts are actually monopoly in China. Many industries, such as railway, energy power and so on, are all in monopoly in China.

This paper is organized as follows: The model is outlined in Section 2. Some analysis and the main results are presented in Section 3. An example is presented in Section 4.

Some remarks are given in the final section.

II. The Model

Here we formally introduce the model about quality commitment and we focus on a unique producer. This unique producer releases two products or there exist two types of products in this industry. These two types of products are functionally identical but there exists difference about quality. When the two types of products are considered, the measure of hedonic prices is employed to handle this situation, see the interesting paper of Epple (1987). The market size is N for this industry. The zero cost incurred for the first product and the quantity of the first products is q_1 along with price p_1. Given the quantity of the products q_2 along with price p_2, the cost to the new products is $c(q_2)=c_0q_2$, where the marginal cost c_0 is a positive constant. (We note that the linear cost function is employed such that the problem is tractable. The model and the corresponding results can be easily extended to general situations.)

According to the theory of hedonic prices, the quasi-linear utility function of the consumers for the new products is u_2-p_2, while the utility function of the consumers for the old products is u_1-p_1. u_1 and u_2 are the utility with a unit of product for the consumers with the first product and the second one, respectively, and $u_1<u_2$. Furthermore, without loss of generality, we denote that $p_1<p_2$. (If $p_1\geq p_2$, no consumers buy product from the first producer.)The profits of the producer are therefore given as follows

$$\pi(p_1,q_1,p_2,q_2,c_q)=p_1q_1+p_2q_2-c(q_2)-c_q \tag{1}$$

where c_q represents the cost to launch the commitment about the quality. The third term of (1) describes the cost incurred in production for the second product. The fourth term is the expenditures to launce a commitment for the new product. Furthermore, u_2 is close related to the term c_q. In general, $\frac{\partial u_2}{\partial c_q}>0$. More cost to commit in the quality, higher utility for the corresponding consumers. For the second order differential, we have $\frac{\partial^2 u_2}{\partial (c_q)^2}<0$. In the other words, u_2 is concave in c_q.

About the terms q_1 and q_2, here we discuss them in detail. The density of the probability be $\varphi(y)$ along with the incomes y. The corresponding distribution function is given by $\Phi(y)$. If $u_1 - p_1 \le u_2 - p_2$ is satisfied, we further have the relations $q_2 = N\int_{p_2}^{\infty} F(y)dy = N[1-\Phi(p_2)]$ and $q_1 = N\int_{p_1}^{p_2} F(y)dy = N[\Phi(p_2)-\Phi(p_1)]$. If $u_1 - p_1 > u_2 - p_2$, there are no consumers for the second product. Therefore, it is very rational that $u_1 - p_1 \le u_2 - p_2$. We further note that the consumers are inclined to the second type of the products if $u_1 - p_1 = u_2 - p_2$. This is an ideal assumption to simplify the problem. The general situation is similar to this ideal case with moderate modification of the above model.

The problem of the producer is therefore rewritten as the following formulation.

$$\hat{\pi}(p_1, q_1, p_2, q_2, c_q) = p_1 N[\Phi(p_2)-\Phi(p_1)] + p_2 N[1-\Phi(p_2)] - c(N[1-\Phi(p_2)]) - c_q. \qquad (2)$$

This subjects to the constraint as follows.

$$u_1 - p_1 \le u_2 - p_2 \qquad (3)$$

We always assume that all consumers and the producer are rational in this work. Namely, they aim to maximize their utilities or the profits when they make decisions. Furthermore, the following assumption is given.

Assumption (A) u_2 is concave in c_q and $\frac{\partial u_2}{\partial c_q} > 0$.

(B) $\hat{\pi}$ is concave in p_2 and p_1.

(A) is consistent with the hypothesis in the advertisement theory, which is therefore considerably rational. (B) guarantees the existence of the solution to this system. (B) is easy to meet for some special distribution, which is discussed in the final section.

We point out that two types of the products may be not simultaneously produced. One type product may be firstly produced. The new products lately appear and we equally consider it in static situation if discounting factor is 1. The properties about this model are captured in the next section.

Ⅲ. The main results

Here we analyze the above model. The standard problem with the simplified version is restated as follows.

$$\underset{p_1,p_2,c_q}{Max} \quad \hat{\pi}(p_1,q_1,p_2,q_2,c_q) = p_1N[\Phi(p_2)-\Phi(p_1)] + p_2N[1-\Phi(p_2)] - c(N[1-\Phi(p_2)]) - c_q$$
$$S.t. \quad u_1 - p_1 \le u_2 - p_2. \tag{4}$$

Ⅲ.Ⅰ. The existence of the solution

Firstly, we define the feasible region $F = \{(p_2,c_q) \mid u_1 - p_1 \le u_2 - p_2\}$. The concavity of u_2 indicates the following result.

Lemma 1. If $u_2(c_q)$ is concave in c_q, the set $F = \{(p_2,c_q) \mid u_1 - p_1 \le u_2 - p_2\}$ is convex.
Proof. According to the concavity of $u_2(c_q)$, for any $\lambda \in [0,1]$, $(p_2,c_q) \in F$ and $(\tilde{p}_2,\tilde{c}_q) \in F$, we have $\lambda u_2(c_q) + (1-\lambda)u_2(\tilde{c}_q) \le u_2(\lambda c_q + (1-\lambda)\tilde{c}_q)$.
$(p_2,c_q) \in F$ suggests $u_1 - p_1 \le u_2(c_q) - p_2$. From $(\tilde{p}_2,\tilde{c}_q) \in F$, we obtain the relation $u_1 - p_1 \le u_2(\tilde{c}_q) - \tilde{p}_2$. Combined $u_1 - p_1 \le u_2(c_q) - p_2$ and $u_1 - p_1 \le u_2(\tilde{c}_q) - \tilde{p}_2$, we have the following relation $u_1 - p_1 \le \lambda u_2(c_q) - \lambda p_2 + (1-\lambda)u_2(\tilde{c}_q) - (1-\lambda)\tilde{p}_2$
$\le u_2(\lambda c_q + (1-\lambda)\tilde{c}_q) - \lambda p_2 - (1-\lambda)\tilde{p}_2$ for any $\lambda \in [0,1]$. If $(p_2,c_q) \in F$ and $(\tilde{p}_2,\tilde{c}_q) \in F$, we obtain ($\lambda c_q + (1-\lambda)\tilde{c}_q$, $\lambda p_2 + (1-\lambda)\tilde{p}_2$) $\in F$ for any $\lambda \in [0,1]$. The set $F = \{(p_2,c_q) \mid u_1 - p_1 \le u_2 - p_2\}$ is convex if $u_2(c_q)$ is concave in c_q.

The result is therefore obtained and the proof completes.

Here we show the existence of the above system. There exists the solution for the maximization problem of a concave function in a convex region, which is an existed conclusion of Rockafellar (1970). According to the above Lemma 1, the result is given as follows and the proof is omitted since it is a direct result in the monograph of Rockafellar (1970).

Proposition 1. Under Assumption, there exist solutions for (4). Furthermore, the constraints of (4) is binding.
Proof. The first part of the above proposition is a direct result. Here we show the second part by contradiction. If the constraint were not binding, we should have $u_1 - p_1 < u_2 - p_2$. Given the expenditures to commitment c_q, the producer can increase the profits by improving the prices. This contradicts the optimal prices for the producer. Thus, the constraint of (4) is binding and the second part of the result is achieved. This completes the proof.

Then, we consider the solution of (4). The results are based on Lagrangian function of (4), which is outlined as follows.

$$L(p_1, p_2, c_q, \mu) = p_1 N[\Phi(p_2) - \Phi(p_1)] + p_2 N[1 - \Phi(p_2)] - c(N[1 - \Phi(p_2)]) - c_q + \mu[(u_2(c_q) - p_2) - (u_1 - p_1)] \quad (5)$$

where $\mu \geq 0$ is the corresponding Lagrangian multiplier. The first order optimal conditions of (5) are obtained.

$$\frac{\partial L}{\partial p_2} = f_1 = p_1 N\varphi(p_2) + N[1 - \Phi(p_2)] - p_2 N\varphi(p_2) + c_0 N\varphi(p_2) - \mu = 0 \quad (6)$$

$$\frac{\partial L}{\partial c_q} = f_2 = -1 + \mu u_2^{'}(c_q) = 0, \quad (7)$$

$$\frac{\partial L}{\partial \mu} = f_3 = (u_2(c_q) - p_2) - (u_1 - p_1) = 0 \quad (8)$$

$$\frac{\partial L}{\partial p_1} = f_4 = N[\Phi(p_2) - \Phi(p_1)] - p_1 N\varphi(p_1) + \mu = \quad (9)$$

The solution of (4) is determined by (6)-(9). Combined (6)-(7), we have the following relation between $p_2^{(2)}$ and c_q.

$$f_5 = \{p_1 N\varphi(p_2) + N[1 - \Phi(p_2)] - p_2 N\varphi(p_2) + c_0 N\varphi(p_2)\}u_2^{'}(c_q) - 1 = 0\,. \quad (10)$$

The solution of (4) is also given by (8), (9) and (10) equivalently. The further discussion of the solution is presented in next subsection.

III.II. The properties of the solution

Here the solution based on (8)-(10) is further investigated. When the price of the first product changes, p_2 and c_q are all correspondingly varied. The following results hold.

Proposition 2 For the model in the above section, we have the following conclusion: If the prices of the first type product increase, the prices of the new product and the expenditures to commit in the quality correspondingly increase.

$$\frac{\partial p_2}{\partial p_1} > 0 \quad (11)$$

and

$$\frac{\partial c_q}{\partial p_1} \geq 0 \quad (12)$$

Proof. Considering (10), we have $\frac{\partial f_5}{\partial p_1} = N\varphi(p_2)u_2^{'}(c_q) > 0$ and $\frac{\partial f_5}{\partial p_2} < 0$ by virtue of the concavity of the function $\hat{\pi}$. We therefore obtain $\frac{\partial p_2}{\partial p_1} = -\frac{\frac{\partial f_5}{\partial p_1}}{\frac{\partial f_5}{\partial p_2}} \geq 0$.

The concavity of $u_2(c_q)$ indicates $u_2^{''}(c_q) < 0$ and (6) suggests $p_1 N\varphi(p_2)$ $+N[1-\Phi(p_2)] - p_2 N\varphi(p_2) + c_0 N\varphi(p_2) \geq 0$. According to the partial derivative of (10), we have $\frac{\partial f_5}{\partial c_q} = \{p_1 N\varphi(p_2) + N[1-\Phi(p_2)] - p_2 N\varphi(p_2) + c_0 N\varphi(p_2)\}u_2^{''}(c_q) < 0$. We therefore obtain $\frac{\partial c_q}{\partial p_1} = -\frac{\frac{\partial f_5}{\partial p_1}}{\frac{\partial f_5}{\partial c_q}} \geq 0$.

The result is therefore obtained and the proof completes.

Remarks: The above conclusion illustrates that higher price of the first product yields both higher expenditures of the commitment and higher price of the new products. When the price of the first product is high enough, the firm in general is not going to launch a new product. These conclusions are highly consistent with the economic phenomena in reality.

According to the above result, when the incomes of the consumers improve (Wang, 2009), the producer may release a new type of product to earn more. In the cigarette industry in China, for example, the prices of the old products remain while the new products are released. This highly fits the above model.

Here we consider the relation between the new product and the cost of the new products. Similarly considering (10), we have the following results.

Proposition 3 For the model in the above section, if the cost of new product increases, the prices of new products and the expenditures to commitment correspondingly increase.

$$\frac{\partial p_2}{\partial c_0} > 0 \tag{13}$$

and

$$\frac{\partial c_q}{\partial c_0} \geq 0 \tag{14}$$

Proof (10) implies $\frac{\partial f_5}{\partial c_0} = N\varphi(p_2)u_2^{'}(c_q) > 0$. With the similar way, we therefore obtain $\frac{\partial p_2}{\partial c_0} = -\frac{\frac{\partial f_5}{\partial c_0}}{\frac{\partial f_5}{\partial p_2}} \geq 0$ and $\frac{\partial c_q}{\partial p_1} = -\frac{\frac{\partial f_5}{\partial p_1}}{\frac{\partial f_5}{\partial c_q}} \geq 0$, where $\frac{\partial f_5}{\partial p_2} < 0$ and $\frac{\partial f_5}{\partial c_q} < 0$ are all outlined in Proposition 2.

The result is therefore obtained and the proof completes.

Remarks: This result manifests that high cost is an important factor to deter the introduction of new products. Actually, in many industries all over the world, the high costs hinder the release of new products. This result is therefore rational.

According to results in Proposition 2 and 3, the costs and the prices of old products are two crucial factors to hinder the release of new products for the monopolist. In some industries, such as milk industry and cigarette industry, the cost of new products is not much higher than old products. Many new products are thus launched for these industries in recent years in China.

We here consider the profits of the producer when the cost of the second product changes with envelop theorem. The following result holds.

Proposition 4. When the parameter c_0 increases, the profits of the producer decrease.

$$\frac{\partial \hat{\pi}}{\partial c_0} \leq 0 \tag{15}$$

Proof. We show (15) by the envelop theorem to the objective function. According to the Lagrangian function (5), we have $\frac{\partial \hat{\pi}}{\partial c_0} = -N[1-\Phi(p_2)] \leq 0$. (15) therefore holds. When the parameter c_0 increases, the profits of the producer decrease.
The result is therefore achieved and the proof completes.

Remarks: It is rational that higher cost issues in lower profits of the producer in the above system. When the price of the first product improves, the producer's profits are increased in some situations. The producer's profits are decreased in other situations. The detail discussion abut this is omitted in this work.

III.III. Compare with the benchmarks

Here we compare the above system with other cases. No new product is introduced and the price of the first type of products is considered. The market size is N, which is always fixed. The producer aims to attack the following problem.

$$\underset{p_1}{Max} \qquad \pi = p_1 N[1-\Phi(p_1)]. \tag{16}$$

Denote the optimal solution of (16) to be p_1^* and the corresponding optimal value to be π^*. To simplify the problem with the quality commitment, we also assume that $c_0 = 0$ and the corresponding problem is given (4).

Denote the optimal solution of (4) to be $\overline{p}_1, \overline{p}_2$ and $\overline{c}_q$, and the corresponding optimal value to be $\hat{\pi}^*$. Comparing (4) with (16), the following result holds.

Proposition 5. Under the hypothesis in Assumption, if $\dfrac{1}{u_2^{'}(\overline{c}_q)} - p_1 N\varphi(\overline{p}_2) > 0$, we have $\overline{p}_2 > p_1^*$. Otherwise, $\overline{p}_2 \leq p_1^*$.

Proof. Since the optimal solution to (16) is p_1^*, we have $\left.\dfrac{\partial \pi}{\partial p_1}\right|_{p_1^*} = 0$. Since the optimal solution to (4) is $\overline{p}_1, \overline{p}_2$, we have (6) or,

$$p_1 N\varphi(p_2) + N[1-\Phi(p_2)] - p_2 N\varphi(p_2) - \mu = p_1 N\varphi(p_2) + \left.\frac{\partial \pi}{\partial p_1}\right|_{\overline{p}_2} - \mu = 0 \tag{17}$$

(7) is rewritten as $\mu = \dfrac{1}{u_2^{'}(\overline{c}_q)}$. By virtue of (17), we have

$$\left.\frac{\partial \pi}{\partial p_1}\right|_{\overline{p}_2} = \frac{1}{u_2^{'}(\overline{c}_q)} - p_1 N\varphi(\overline{p}_2) \tag{18}$$

When $\dfrac{1}{u_2^{'}(\overline{c}_q)} - p_1 N\varphi(\overline{p}_2) \geq 0$, from the concavity of π, we obtain $\overline{p}_2 \leq p_1^*$.

When $\frac{1}{u_2^{'}(\bar{c}_q)} - p_1 N\varphi(\bar{p}_2) < 0$, we have $\bar{p}_2 > p_1^*$.

The result is therefore obtained and the proof completes.

Remarks: When new product appears, the prices of old products are in general low. Therefore, $\frac{1}{u_2^{'}(\bar{c}_q)} - p_1 N\varphi(\bar{p}_2) \geq 0$ holds and we have $\bar{p}_2 \leq p_1^*$. For p_1^*, by virtue of the first order optimal conditions to (16), we have $\left.\frac{\partial \pi}{\partial p_1}\right|_{p_1^*} = 0$ and

$$1 - \Phi(p_1^*) - p_1^* \varphi(p_1^*) = 0 \tag{19}$$

We further compare the profits of the procedure under two cases. Since the constraint of (4) is binding, we have $u_1 - p_1 = u_2(c_q) - p_2$. Or,

$$\mathrm{c_q} = u_2^{-1}(u_1 - p_1 + p_2) \tag{20}$$

where $u_2^{-1}(u_1 - p_1 + p_2)$ is the inverse function of the utility function u_2. It is apparent that u_2^{-1} should satisfy $\frac{\partial u_2^{-1}}{\partial c_q} = \frac{1}{\frac{\partial u_2}{\partial c_q}} > 0$ and $\frac{\partial^2 u_2^{-1}}{\partial (c_q)^2} = -\frac{\frac{\partial^2 u_2}{\partial (c_q)^2}}{[\frac{\partial u_2}{\partial c_q}]^2} > 0$.

Namely, u_2^{-1} is convex. (4) is equivalent to the following problem.

$$\underset{p_2^{(2)}}{Max} \quad \hat{\pi} = p_1 N[\Phi(p_2) - \Phi(p_1)] + p_2 N[1 - \Phi(p_2)] - u_2^{-1}(u_1 - p_1 + p_2)\,. \tag{21}$$

The corresponding solution to (4) is therefore given by the following first order conditions to (21), which is stated as follows.

$$p_1 N\varphi(p_2) + N[1 - \Phi(p_2)] - p_2 N\varphi(p_2) - [u_2^{-1}(u_1 - p_1 + p_2)]^{'} = 0 \tag{22}$$

Considering (22) and (21), the following result hold

Proposition 6. If $p_1 N[\Phi(p_1^*) - \Phi(p_1)] - u_2^{-1}(u_1 - p_1 + p_1^*) > 0$, we have $\hat{\pi}^* > \pi^*$.

Proof. For p_1^* along with the term $u_2^{-1}(u_1 - p_1 + p_1^*)$,it is also a feasible point to (4).

That is, $\hat{\pi}_2^* \geq \pi^* + p_1 N[\Phi(p_1^*) - \Phi(p_1)] - u_2^{-1}(u_1 - p_1 + p_1^*)$. When $p_1 N[\Phi(p_1^*) - \Phi(p_1)] - u_2^{-1}(u_1 - p_1 + p_1^*) > 0$ holds, apparently we have $\hat{\pi}^* > \pi^*$. The result therefore holds and the proof completes.

Remarks: The hypothesis $p_1 N[\Phi(p_1^*) - \Phi(p_1)] - u_2^{-1}(u_1 - p_1 + p_1^*) > 0$ is rational for some industries, in which the quality commitment has dramatic effects on the consumers. In this industry, if the producer spends a little money to commit, the utility of the consumers is improved to a great degree. In milk industry in China, for example, the quality commitment seems to have such effects.

Ⅳ. A linear example

A linear example is outlined to illustrate the above results. Let $u_1 = 1, u_2 = 1 + c_q^{\frac{2}{3}}, p_1 = 1$, $c_0 = 0$ and $\varphi(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-2)^2}{2}}$. The profits of the unique producer are given as follows. The corresponding parameters are $Ey = 2$ and $Dy = 1$.

$$\underset{p_2^{(2)}, c_q}{Max} \qquad \hat{\pi} = N[\Phi(p_2) - \Phi(1)] + p_2 N[1 - \Phi(p_2)] - c_q$$

$$S.t. \qquad 0 \leq 1 + c_q^{\frac{2}{3}} - p_2.$$

The solution to the above problem is therefore given by the following system of the equations.

$$1 + c_q^{\frac{2}{3}} - p_2 = 0,$$

$$N\varphi(p_2) + N[1 - \Phi(p_2)] - Np_2\varphi(p_2) - \frac{3}{2}(p_2 - 1)^{\frac{1}{2}} = 0 .$$

The solutions are achieved with the software Matlab 6.5. The solution is $\overline{p}_2 = 2.13$ and $\overline{c}_q = 1.20$. The number N has not major effect on the price of the new products but has decisive effects on the profits of the producer. The profits of the producers are $\hat{\pi}^* = 1.34788N - 1.20$.

When the second product is not introduced, the corresponding prices are determined by (21). And the solution u is $p_1^* = 1.66$. The profits of the producer in this situation are $\pi^* = 1.051N$.

For the above result, the quality commitment has major effects on the utility of the consumers. The producer is inclined to employing quality commitment because of $\pi^* < \hat{\pi}^*$. Furthermore, $p_1^* < \overline{p}_2$, which is substantially consistent with Proposition 5.

V. Concluding Remarks

In this paper, quality commitment is captured by game theory model. The existence theory is correspondingly established. We find that it is a good strategy for the firm to release the new product for the monopolist under certain conditions.

Here we further consider (B) of Assumption. According to the above analysis, calculating the first-order differential, we evidently have $\frac{\partial \hat{\pi}}{\partial p_2} = p_1 N\varphi(p_2) + N[1-\Phi(p_2)] - p_2 N\varphi(p_2) + c_0 N\varphi(p_2)$. Comparing this formulation with (6), we immediately have $\frac{\partial \hat{\pi}}{\partial p_2} \geq 0$. For the second order differential of the function $\hat{\pi}$, we have $\frac{\partial^2 \hat{\pi}}{\partial (p_2)^2} = p_1 N\varphi^{'}(p_2) - 2N\varphi(p_2) - p_2 N\varphi^{'}(p_2) + c_0 N\varphi^{'}(p_2)$. Since there are many consumers, it is rational to assume that the distribution for the incomes of the consumers satisfies normal distribution. It is apparent that (B) of Assumption should be met under moderate restriction.

We also point out that we just consider the goods that the producer owned the private information about the quality of the products. When the information is complete, the quality commitment seems out of work. In the private computer industry, for example, the information is complete and no quality commitment appears.

We assume that the consumers are inclined to the second type of the products if $u_1 - p_1 = u_2 - p_2$. If this hypothesis does not hold, we can moderately modify the model to obtain the similar results. The detail modification and the corresponding analysis are omitted.

There exist many types of quality commitment in reality. In some industry, the quality commitment is guaranteed by the government or some organizations. The other types of the quality commitment are given by the producer. For the model in Section 2, both types of the quality commitment meet this assumption.

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REFERENCES

[1]**Caruana, G. and Einav, L.(2008),** ***A Theory of Endogenous Commitment***. *Review of Economic Studies*, 75 (1), 99-116;
[2]**Chen, H.C. and Liu, S.M.(2008),** ***Incentive Contracts under Imperfect Auditing***. *Manchester School*, 76 (2), 131-159;
[3]**Dudine, P., Hendel, I. & Lizzeri, A. (2006),** ***Storable Good Monopoly:The Role of Commitment***. *American Economic Review*, 96 (5), 1706-1719;
[4]**Ellingsen, T. and Miettenen,T. (2008),** ***Commitment and Conflict in Bilateral Bargaining***. *American Economic Review*, 98 (4), 1629-1635;
[5]**Epple, D. (1987),** ***Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products***. *Journal of Politics Economics*, 95(1), 59-80;
[6]**Esteban, S. and Shum, M. (2007),** ***Durable-goods Oligopoly with Secondary Markets: The Case of Automobiles.*** *Rand Journal of Economics*, 38(2),332-354;
[7]**Gaffard, J.L. & Quere, M. (2006),** ***What's the Aim for Competition Policy: Optimizing Market Structure or Encouraging Innovative Behaviors?*** *Journal of Evolutionary Economics*, 16 (1-2), 175-187;
[8]**Gautier, P.A., Teulings, C.N. and Van Vuuren, A.(2010),** ***On-the-job Search, Mismatch and Efficiency*** . Review of Economic Studies, 77 (1), 245-272;
[9]**Goering, G.E. (2008),** ***Socially Concerned Firms and the Provision of Durable Goods***. Economic Modelling, 25(3), 575-583;
[10]**Gupta, S., Grant, S. and Melewar, T.C. (2008),** ***The Expanding Role of Intangible Assets of the Brand***. *Management Decision*, 46(5-6), 948-960;
[11]**Hays, J. M. and Hill, A. V. (2006),** ***Service Guarantee Strength: The Key to Service Quality.*** *Journal of Operations Management*, 24(6), 753–764;
[12]**Kopel, M. and Loffler, C. (2008),** ***Commitment, First-mover and Second-mover Advantage***. *Journal of Economics*, 94(2), 143-166;
[13]**Krueger, D. Lustig, H. & Perri, F. (2008),** ***Evaluating Asset Pricing Models with Limited Commitment Using Household Consumption Data.*** *Journal of the European Economic Association*, 6(2-3), 715-726;
[14]**Krueger, D. & Uhlig, H. (2006),** ***Competitive Risk Sharing Contracts with***

One-sided Commitment. *Journal of Monetary Economics*. 53 (7), 1661-1691;

[15]**Kultti, K. (2003),** ***About Market Structure***. *Review of Economic Dynamic*,6 (1) , 240-251;

[16]**Nie, P.Y. (2009),** ***Commitment for Storable Goods under Vertical Integration***. *Economic Modelling*, 26(2), 414-417;

[17]**Nie, P.Y.(2011),*****Spatial Maintenance Goods under Monopoly***, *Ekonomska Istrazivanja - Economic Research*, 24(4), 2011,16-26;

[18]**Rockafellar, R.T. (1970)** ***Convex Analysis*****;** *Princeton University Press*, Princeton;

[19]**Schelling, T.C. (1960),** ***The Strategy of Conflict*****.** *Cambridge, MA, Harvard University Press;*

[20]**Tirole, J. (1988),** ***The Theory of Industrial Organization***. *Massachusetts Institute of Technology Press;*

[21]**Wang ,N. (2009)** ***Optimal Consumption and Asset Allocation with Unknown Income Growth*****.** *Journal of Monetary Economics*, 56(4), 524-534.