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OPTIMAL CONTRACTS IN ASYMMETRIC INFORMATION WITH RANDOM RESERVATION UTILITY LEVEL

Abstract. *In the paper we consider an adverse selection model where the private information of the agent is represented by his marginal cost of production. While in the standard model the Agent's participation is taken as deterministic, depending on his outside opportunity utility level, we here relax this assumption allowing some randomness into Agent's decision to participate. We first present the standard model with two types of agent and then we generalize it, assuming that the agent's participation constraints are random and the type of agent belongs to a finite set of values. In the last part of the paper we derive and summarize the features of the optimal contracts in the situation of asymmetric information.*

Keywords: adverse selection, optimal contract, random participation constraints.

JEL Classification: **L21, D82**

1. Introduction

In the paper we present a problem of incentive theory in a static bilateral contracting framework. In this context, one of the partners (the Agent) has private information about his characteristics and this information affects the contract' results. The uninformed partner (the Principal) wants to delegate a production task to the Agent and is controlling the Agent's decisions by designing the contract and making a contractual offer to the Agent. This delegation problem corresponds to a classical adverse selection problem where the information gap between the Principal and the Agent has strong implications for the design of optimal contract to be signed: for an efficient allocation of economic resources, the Principal must give up some costly informational rent to the better informed Agent.

The literature on standard adverse selection models is huge. These models were developed in many theoretical and empirical settings such as: continuum of types (Salanie (1990), Laffont and Tirole (1993), Laffont and Rochet (1998), Cvitanic and Zhang (2007)); stochastic mechanisms (in insurance, nonlinear pricing, optimal taxation: Maskin and Riley (1984), Stiglitz (1987), Arnott and Stiglitz (1988)); relaxing the Spence-Mirrlees (single crossing) property (Guesnerie and Laffont (1984), Mathews and Moore (1987), Araujo and Moreira (2000)); multidimensional asymmetric information (two types case: Rochet and Chone(1998), continuum of types: McAfee and McMillan (1988), Wilson (1993), Sibley and Srinagesh (1997)), Armstrong and Rochet (1999)); type dependent participation constraints (Kahn and Moore (1985) (labor contracts), Lewis and Sappington (1989) (regulation), Feenstra and Lewis (1991), Brainard and Martimort (1996), Jeon and Laffont (1999) (downsizing the public sector); random participation constraints (Rochet and Stole (2002)); limited liability constraints (Sappington (1983)).

The pioneering papers on adverse selection considered some simplifying assumption regarding the Agent’s participation. Recent works show that this assumption can be extended; hence, Rochet and Stole (2002) presented and solved a class of two-dimensional type preferences in which the Agent’s participation is random. Saak (2009) proved, in a setting with random participation, that a seller achieves higher expected profits under intermediate private information when the heterogeneity in reservation utilities is not too small or too great. In the present paper we deal with a special case of an adverse selection model proposed by Laffont and Martimort (2002). In a previous paper, Marinescu and Marin (2011) extended it to the case where the adverse selection parameter can have three possible values asymmetric distributed. They also presented another extension, allowing for a generalized cost function of the Agent, dependent on two adverse selection parameters. While in the standard model the Agent’s participation is taken as deterministic, depending on his outside opportunity utility level, we here relax this assumption allowing some randomness into Agent’s decision to participate. We study a situation where the Agent (the firm) has private information about his marginal cost of production. The Principal’s objective is to maximize the profit and hence he is concerned in making a *take-it or leave-it offer* of a menu of contracts specifying the quantities to be produced and the transfers to be paid to the firm.

The paper is organized as follows. Section 2 presents the standard one dimensional adverse selection model of Laffont and Martimort (2002) and the main features of the optimal contract in this situation. In Section 3 we extend the standard model allowing for random participation constraints and three types of Agent. We also describe there the Principal’s optimization problem and a procedure of reducing the number of the problem’s constraints. In the next section the program is solved using as variables the informational rents. Next, we derive the optimal contracts; two situations are discussed here – separating and pooling contracts. The last section summarizes the features of the optimal contracts in both cases and concludes the paper.

2. The standard model

In this section we present the framework used as a benchmark in the rest of the paper. Our approach is based on the standard problem of adverse selection proposed by Laffont and Martimort (Laffont and Martimort (2002)). The main assumptions of the model are the following:

A1. The Principal (a firm or consumer) wants to consume a good and delegates to an Agent (a firm) the production of q units of this good. The value for the Principal of q units is $S(q)$, where the function $S(q)$ has the properties $S'(\cdot)>0$, $S''(\cdot)<0$ and $S(0)=0$. If the Agent accepts the Principal's offer, then he receives a transfer denoted by *t*. Therefore, the Principal's objective function is written as $S(q)-t$.

*A2***.** The Agent's production cost is unobservable to the Principal. We consider that the Agent has private information about his marginal cost of production. He can have one of two types - low or high marginal cost:

$$\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}, \text{ with } \underline{\theta} < \overline{\theta}.$$

We will relax this assumption in the next section.

The fixed cost is neglected and this assumption doesn't affect the problem generality.

We also suppose that the Agent's outside opportunity utility level is normalized to zero.

Hence, the firm's objective function is $t-\theta q$.

A3. The economic variables of the problem (the contractual variables) are: q – production to be produced and *t* – the transfer received by the Agent.

A4. The timing of contracting in adverse selection situation (in asymmetric information):

$t=0$: the Agent discovers his type θ; but the Principal doesn't observe this type;

$t=1$: the Principal designs and offers the contract;

$t=2$: the Agent accepts or refuses the contract;

$t=3$: the contract is executed; payoffs for Principal and Agent.

The case of symmetric information (The first best solution)

First we suppose that there is no asymmetry of information between the Principal and the Agent: the Principal knows exactly the Agent's type. Therefore, he makes a contractual offer to the Agent and this offer corresponds to the solution of the following optimization problem:

$$\max_{t,q} \quad S(q)-t$$
$$s.t.$$
$$t \geq \theta q$$
$$q \geq 0, t \geq 0$$

The optimal solution for the above problem is given by the first order conditions:

$$\begin{cases} S'(q^*) = \theta \Rightarrow q^* = (S')^{-1}(\theta) \\ t^* = \theta q^* \end{cases}$$

We must note that the first best contract yields to a higher optimal production for the efficient type than the corresponding production for the inefficient type, i.e. $\underline{q}^* > \overline{q}^*$.

The case of asymmetric information

Suppose now that the Principal doesn't know the Agent's type, but he has some beliefs regarding the Agent's type. The probability that the Agent is efficient ($\underline{\theta}$) is denoted by ν; then, the probability that the Agent is inefficient ($\overline{\theta}$) is $1-\nu$.

In this situation, it is optimal for the Principal to design a menu of contracts – one for each type, hoping that each type of Agent chooses the contract designed for him. We denote by $\{(\underline{t},\underline{q}), (\overline{t},\overline{q})\}$ the menu of contracts being derived such that the Principal's expected profit is maximized:

$$\max_{\overline{t},\overline{q},\underline{t},\underline{q}} \quad \nu\left[S(\underline{q})-\underline{t}\right]+(1-\nu)\left[S(\overline{q})-\overline{t}\right]$$

The optimal contracts are chosen from the set of the incentive feasible contracts, meaning the contracts satisfying the participation constraints:

$$\underline{t}-\underline{\theta}\underline{q} \geq 0$$
$$\overline{t}-\overline{\theta}\overline{q} \geq 0$$

and the incentive compatibility constraints:

$$\underline{t}-\underline{\theta}\underline{q} \geq \overline{t}-\underline{\theta}\overline{q}$$
$$\overline{t}-\overline{\theta}\overline{q} \geq \underline{t}-\overline{\theta}\underline{q}$$

The following theorem (see for details Laffont and Martimort [9]) characterize the optimal contracts in adverse selection situation:

Theorem 1 (Laffont and Martimort, 2002) In the situation of asymmetric information, the features of the optimal contract *(the second best solution)* are:

i) The Agent with low marginal cost marginal (the efficient type of Agent) produces the first best quantity:

$$S'\left(\underline{q}\right)=\underline{\theta}$$

ii) The Agent with high marginal cost (the inefficient type of Agent) produces a distorted quantity with respect to the first best, and this quantity is given by the following relation:

$$S'\left(\overline{q}^{SB}\right)=\overline{\theta}+\Delta\theta\frac{\nu}{1-\nu}\text{, where } \Delta\theta=\overline{\theta}-\underline{\theta}\text{, and } \overline{q}^{SB}<\overline{q}^{*}.$$

iii) The Agent with type $\overline{\theta}$ gets no informational rent (hence he obtains exactly his outside opportunity level of utility). We have therefore:

$$\overline{U}^{SB}=0$$

or his optimal transfer is $\overline{t}^{SB}=\overline{\theta}\overline{q}^{SB}$.

iv) The efficient Agent gets a positive informational rent, given by:

$$\underline{U}^{SB}=\Delta\theta\overline{q}^{SB}$$

or his optimal transfer is

$$\underline{t}^{SB}=\underline{\theta}\underline{q}^{SB}+\Delta\theta\overline{q}^{SB}.$$

3. A generalization of the adverse selection model: random participation constraints and three type of Agent

In this section we use the standard model introduced above. We extend it, modifying the assumptions regarding the Agent's private information and the outside opportunity utility level.

First, we suppose that the Agent can have one of following three types (marginal costs): $\theta\in\left\{\underline{\theta}<\tilde{\theta}<\overline{\theta}\right\}$, with $\overline{\theta}-\tilde{\theta}=\tilde{\theta}-\underline{\theta}=\Delta\theta>0$. Hence, $\underline{\theta}$ represents the most efficient type (the type with the lowest marginal cost), and $\overline{\theta}$ is the least efficient type (the type with the highest marginal cost).

The Agent has the type $\underline{\theta},\tilde{\theta}$ or $\overline{\theta}$ with the respective probabilities $\underline{\nu},\tilde{\nu}$ or $\overline{\nu}$, where $\underline{\nu}+\tilde{\nu}+\overline{\nu}=1$.

Second, let $\underline{U} = \underline{t} - \underline{\theta}\underline{q}, \tilde{U} = \tilde{t} - \tilde{\theta}\tilde{q}$ and $\bar{U} = \bar{t} - \bar{\theta}\bar{q}$ be the informational rent for each type of Agent. With the previous assumptions, the only constraint imposed on this type of variables is a nonnegative condition. We replace this condition with a new assumption.

We now assume that the Agent's decision to participate is random: instead of having a zero reservation utility level, this level is random and drawn from an interval $\varepsilon \in \ a,b \ $, with a cumulative distribution function $F \ \varepsilon \ $. We denote by $f' \ \varepsilon \ = F \ \varepsilon \ $ the density of the random variable.

3.1. The menu of incentive feasible contracts

Definition. A menu of contracts $\ \underline{t},\underline{q} \ , \ \tilde{t},\tilde{q} \ , \ \bar{t},\bar{q} \ $ is *incentive feasible* if it satisfies the following constraints:

- participation constraints (for each type of Agent):

$$\underline{U} = \underline{t} - \underline{\theta}\underline{q} \geq \varepsilon \quad (1)$$

$$\tilde{U} = \tilde{t} - \tilde{\theta}\tilde{q} \geq \varepsilon \quad (2)$$

$$\bar{U} = \bar{t} - \bar{\theta}\bar{q} \geq \varepsilon \quad (3)$$

- incentive compatibility constraints (for each type of Agent):

$$\underline{U} = \underline{t} - \underline{\theta}\underline{q} \geq \tilde{t} - \underline{\theta}\tilde{q} \text{ or } \underline{U} \geq \tilde{U} + \Delta\theta\tilde{q} \quad (4)$$

$$\underline{U} = \underline{t} - \underline{\theta}\underline{q} \geq \bar{t} - \underline{\theta}\bar{q} \text{ or } \underline{U} \geq \bar{U} + 2\Delta\theta\bar{q} \quad (5)$$

$$\tilde{U} = \tilde{t} - \tilde{\theta}\tilde{q} \geq \bar{t} - \tilde{\theta}\bar{q} \text{ or } \tilde{U} \geq \bar{U} + \Delta\theta\bar{q} \quad (6)$$

$$\tilde{U} = \tilde{t} - \tilde{\theta}\tilde{q} \geq \underline{t} - \tilde{\theta}\underline{q} \text{ or } \tilde{U} \geq \underline{U} - \Delta\theta\underline{q} \quad (7)$$

$$\bar{U} = \bar{t} - \bar{\theta}\bar{q} \geq \tilde{t} - \bar{\theta}\tilde{q} \text{ or } \bar{U} \geq \tilde{U} - \Delta\theta\tilde{q} \quad (8)$$

$$\bar{U} = \bar{t} - \bar{\theta}\bar{q} \geq \underline{t} - \bar{\theta}\underline{q} \text{ or } \bar{U} \geq \underline{U} - 2\Delta\theta\underline{q} \quad (9)$$

- sign constraints (for each variable):

$$\underline{q} \geq 0, \tilde{q} \geq 0, \bar{q} \geq 0, \underline{t} \geq 0, \tilde{t} \geq 0, \bar{t} \geq 0 \quad (10)$$

We note that the above incentive compatibility constraints can be grouped into two categories: local and global incentive constraints. Local incentive constraints are the upward incentive constraints (4) and (6) and the downward constraints (7) and (8), while the global incentive constraints are (5) and (9).

3.2. The objective function of the Principal and the optimization problem

The decisional problem of the Principal consists in designing a menu of optimal contracts he proposes to the Agent. This menu of contracts must be incentive feasible and maximizes the Principal's expected profit.

In what follows, we use as optimization variables the informational rents and the quantities corresponding to each type of Agent.

If the length of the interval $[a,b]$ is small, then, simplifying the analysis, we can admit that if the Agent with type $\bar{\theta}$ accepts the contract with the probability $F(\bar{U})$ (alternatively he will not participate with the probability $1-F(\bar{U})$), the other types of Agent accept the contract. Hence, we have $F(\underline{U})=F(\tilde{U})=1$.

The Principal's optimization problem can now be written as:

$$\max_{\underline{U},\tilde{U},\bar{U},\underline{q},\tilde{q},\bar{q}} G(\cdot)=\underline{\nu}\left[S(\underline{q})-\underline{\theta}\underline{q}-\underline{U}\right]+\tilde{\nu}\left[S(\tilde{q})-\tilde{\theta}\tilde{q}-\tilde{U}\right]+\bar{\nu}F(\bar{U})\left[S(\bar{q})-\bar{\theta}\bar{q}-\bar{U}\right]$$

$s.t.$

$(4)-(10)$

We will solve this problem in the next section. Before, we need to reduce the number of the relevant constraints involved in the optimization problem.

3.3. Reducing the dimension of the Principal's problem

The main difficulty in solving the Principal's problem is the huge number of the constraints: we have three participation constraints and six incentive compatibility constraints. In order to reduce the number of the relevant constraints we follow some steps (given bellow):

Proposition 1. If the set of incentive feasible contracts is nonempty, then the *implementability condition* holds, i.e.:

$$\underline{q}\geq\tilde{q}\geq\bar{q} \qquad \text{(IC)}$$

Proof

Indeed, this condition is immediately. Summing the constraints (4) and (7), we get:

$$\Delta\theta\underline{q}\geq\Delta\theta\tilde{q} \text{ or } \underline{q}\geq\tilde{q}$$

Similarly, using the constraints (6) and (8) we get:

$$\Delta\theta\tilde{q}\geq\Delta\theta\bar{q} \text{ or } \tilde{q}\geq\bar{q}$$

Proposition 2. Suppose that the local upward incentive constraints (4) and (6) are satisfied. Then, the global upward constraint (5) is also satisfied. Similarly, the global downward constraint (9) is implied by the local incentive constraints (7) and (8).

Proof

This conclusion is easy to derive. Hence, using the constraints (4), (6) and the implementability condition, we successively have:

$$\underline{U} \geq \tilde{U} + \Delta\theta\tilde{q} \geq \bar{U} + \Delta\theta\bar{q} + \Delta\theta\tilde{q} \geq \bar{U} + 2\Delta\theta\bar{q}$$

and this is exactly the global constraint (5).

In the same way, using (7) and (8) and the condition (IC) we have:

$$\bar{U} \geq \tilde{U} - \Delta\theta\tilde{q} \geq \underline{U} - \Delta\theta\underline{q} - \Delta\theta\tilde{q} \geq \underline{U} - 2\Delta\theta\underline{q}$$

and this yields to the global constraint (9).

We can conclude that the set of the six incentive constraints can be replaced by the set of local incentive constraints together with the implementability condition and the participation constraints.

The next question (step) is whether the set of remaining constraints can be reduced further. We ignore for a while the downward local constraints (8) and (9). Later, we will show that these constraints are indeed satisfied at the optimum. We are interested now in finding which of the remaining constraints is binding at the optimum. The answer is given in the following proposition.

Proposition 3. At the optimum, the local upward constraints are binding.

Proof

We denote $(\underline{U}, \tilde{U}, \bar{U}, \underline{q}, \tilde{q}, \bar{q})$ the optimal solution and let $\alpha > 0$ be a small arbitrarily chosen value.

Suppose that the constraint (6) is not binding, that is:

$$\tilde{U} > \bar{U} + \Delta\theta\bar{q}$$

Then we can reduce the informational rents $\tilde{U}$ and $\underline{U}$ with $\alpha > 0$ such that:

$$\tilde{U} - \alpha \geq \bar{U} + \Delta\theta\bar{q}$$

and

$$\underline{U} - \alpha \geq \tilde{U} - \alpha + \Delta\theta\tilde{q}$$

The new solution $(\underline{U} - \alpha, \tilde{U} - \alpha, \bar{U}, \underline{q}, \tilde{q}, \bar{q})$ derived from the optimal solution is feasible and yields to a higher value for the objective function, i.e.:

$$G(\underline{U}, \tilde{U}, \bar{U}, \underline{q}, \tilde{q}, \bar{q}) = G(\underline{U} - \alpha, \tilde{U} - \alpha, \bar{U}, \underline{q}, \tilde{q}, \bar{q}) + \alpha(\underline{\nu} + \tilde{\nu})$$

and this contradicts the optimality of the solution.

We have therefore, $\tilde{U} = \bar{U} + \Delta\theta\bar{q}$.

Similarly, we can prove that $\underline{U} = \bar{U} + \Delta\theta(\bar{q} + \tilde{q})$.

Using the above proposition's results we can now show that the local downward incentive constraints are satisfied at the optimum.

Proposition 4. If all the local upward constraints are binding at the optimum, then all the downward incentive constraints are satisfied.

Proof

We use the expressions of the informational rents derived in the previous proposition, $\tilde{U} = \bar{U} + \Delta\theta\bar{q}$ and $\underline{U} = \bar{U} + \Delta\theta\left(\bar{q} + \tilde{q}\right)$.

The constraint (7) becomes:

$$\bar{U} + \Delta\theta\bar{q} \geq \bar{U} + \Delta\theta\left(\bar{q} + \tilde{q}\right) - \Delta\theta\underline{q}$$

or $\underline{q} \geq \tilde{q}$, which is true from the implementability condition.

The constraint (8) can be written as:

$$\bar{U} \geq \bar{U} + \Delta\theta\bar{q} - \Delta\theta\tilde{q}$$

or $\tilde{q} \geq \bar{q}$, which is also true, from the implementability condition.

4. Solving the reduced problem of the Principal

With the results obtained in the previous section, the problem is significantly reduced and we can now solve this final form of the Principal's problem. It becomes an unconstrained optimization problem, with only for optimization variables.

The Principal's problem reduces to:

(P reduced)

$$\max_{\bar{U},\underline{q},\tilde{q},\bar{q}} H(\cdot) = \underline{\nu}\left[S(\underline{q}) - \underline{\theta}\underline{q}\right] + \tilde{\nu}\left[S(\tilde{q}) - \tilde{\theta}\tilde{q}\right] + \bar{\nu}F(\bar{U})\left[S(\bar{q}) - \bar{\theta}\bar{q}\right] - \underline{\nu}\left(\bar{U} + \Delta\theta\bar{q} + \Delta\theta\tilde{q}\right)$$
$$-\tilde{\nu}\left(\bar{U} + \Delta\theta\bar{q}\right) - \bar{\nu}\bar{U}F(\bar{U})$$

The first order conditions, assuming an interior solution, are:

$$\frac{\partial H}{\partial \underline{q}} = 0 \text{ or } \quad \underline{\nu}\left[S'(\underline{q}) - \underline{\theta}\right] = 0 \qquad (11)$$

$$\frac{\partial H}{\partial \tilde{q}} = 0 \text{ or } \quad \tilde{\nu}\left[S'(\tilde{q}) - \tilde{\theta}\right] - \underline{\nu}\Delta\theta = 0 \qquad (12)$$

$$\frac{\partial H}{\partial \bar{q}} = 0 \text{ or } \quad \bar{\nu}F(\bar{U})\left[S'(\bar{q}) - \bar{\theta}\right] - \Delta\theta\left(\underline{\nu} + \tilde{\nu}\right) = 0 \qquad (13)$$

$$\frac{\partial H}{\partial \bar{U}} = 0 \text{ or } \quad \bar{\nu} F'(\bar{U})\left[S(\bar{q}) - \bar{\theta}\bar{q} - \bar{U}\right] - (\underline{\nu} + \tilde{\nu} + \bar{\nu} F(\bar{U})) = 0 \qquad (14)$$

Using these first order conditions we derive the features of the optimal contracts (the second best solution). The second order conditions are derived in the Appendix.

From (11) we can derive the second best quantity $\underline{q}^{SB}$ produced by the type $\underline{\theta}$. We have:

$$S'(\underline{q}^{SB}) = \underline{\theta} \qquad (15)$$

or

$$\underline{q}^{SB} = (S')^{-1}(\underline{\theta}) = \underline{q}^{*}$$

Note that for this type of Agent, the optimal solution is not distorted with respect to the first best solution.

Using (12) we get the relation defining the second best quantity $\tilde{q}^{SB}$ produced by the type $\tilde{\theta}$:

$$S'(\tilde{q}^{SB}) = \tilde{\theta} + \frac{\underline{\nu}}{\tilde{\nu}}\Delta\theta > \tilde{\theta} \qquad (16)$$

The above relation yields to:

$$\tilde{q}^{SB} = (S')^{-1}\left(\tilde{\theta} + \frac{\underline{\nu}}{\tilde{\nu}}\Delta\theta\right) < (S')^{-1}(\tilde{\theta}) = \tilde{q}^{*}$$

such that, the optimal quantity produced by the type $\tilde{\theta}$ of Agent is distorted downward.

In addition, calculating the difference:

$$S'(\underline{q}^{SB}) - S'(\tilde{q}^{SB}) = -\Delta\theta - \frac{\underline{\nu}}{\tilde{\nu}}\Delta\theta < 0$$

we get $S'(\underline{q}^{SB}) < S'(\tilde{q}^{SB})$. Given the assumption that the revenue function is concave, it follows that $\underline{q}^{SB} > \tilde{q}^{SB}$, and so the first inequality from the implementability condition holds strictly.

The second best quantity $\bar{q}^{SB}$ is determined from (13):

$$S'(\bar{q}^{SB}) = \bar{\theta} + \frac{\underline{\nu} + \tilde{\nu}}{\bar{\nu} F(\bar{U})}\Delta\theta > \bar{\theta} \qquad (17)$$

It follows immediately that $\bar{q}^{SB} < \bar{q}^{*}$.

It remains to derive the optimal informational rent for the type $\bar{\theta}$. This is obtained from (14):

$$S(\bar{q}^{SB}) - \bar{\theta}\bar{q}^{SB} = \bar{U}^{SB} + \frac{\underline{\nu} + \tilde{\nu} + \bar{\nu} F(\bar{U}^{SB})}{\bar{\nu} F'(\bar{U}^{SB})} \quad (18)$$

In fact, the conditions (17) and (18) form a system of equations; its solution is exactly $(\bar{q}^{SB}, \bar{U}^{SB})$.

4.1. Verifying the implementability condition

While we have already proved that the first inequality from the implementability condition is always satisfied, the second inequality is not so easy to derive. In order to verify it, we compute and analyze the sign of the following difference:

$$S'(\bar{q}^{SB}) - S'(\tilde{q}^{SB}) = \left(1 - \frac{\underline{\nu}}{\tilde{\nu}} + \frac{\underline{\nu} + \tilde{\nu}}{\bar{\nu} F(\bar{U})}\right)\Delta\theta > \left(1 - \frac{\underline{\nu}}{\tilde{\nu}} + \frac{\underline{\nu} + \tilde{\nu}}{\bar{\nu}}\right)\Delta\theta$$

If $1 - \frac{\underline{\nu}}{\tilde{\nu}} + \frac{\underline{\nu} + \tilde{\nu}}{\bar{\nu}} > 0$ or equivalently $\tilde{\nu} > \underline{\nu}\bar{\nu}$, then $\tilde{q}^{SB} > \bar{q}^{SB}$ and the second inequality holds. In this case, the second best solution is fully characterized by the conditions (15)-(18).

We now discuss the form of the optimal solution if this condition is not true. The difference is negative if:

$$1 - \frac{\underline{\nu}}{\tilde{\nu}} + \frac{\underline{\nu} + \tilde{\nu}}{\bar{\nu} F(\bar{U})} \le 0 \text{ or } F(\bar{U})(\bar{\nu}\tilde{\nu} - \underline{\nu}\bar{\nu}) + \underline{\nu}\tilde{\nu} + \tilde{\nu}^2 \le 0$$

The necessary condition for the above relation to be true is $\tilde{\nu} < \underline{\nu}$. Moreover, if probabilities are such that $\tilde{\nu} \le \underline{\nu}\bar{\nu}$, then we have a bunching of types $\tilde{\theta}$ and $\bar{\theta}$, i.e. $\tilde{q}^{SB} = \bar{q}^{SB}$. In this case, the optimal solution has a different form, derived below.

4.2. Bunching contracts

As noted above, when $\tilde{\nu} \le \underline{\nu}\bar{\nu}$, there is a pooling of types. We now discuss the optimal solution. We first replace the variables $\tilde{q}$ and $\bar{q}$ with q^G in the Principal's objective function. It becomes:

(P bunching)

$$\max_{\bar{U},\underline{q},\tilde{q},\bar{q}} G(\cdot) = \underline{\nu}\left[S(\underline{q}) - \underline{\theta}\underline{q}\right] + \tilde{\nu}\left[S(q^G) - \tilde{\theta}q^G\right] + \bar{\nu}F(\bar{U})\left[S(q^G) - \bar{\theta}q^G\right] - \underline{\nu}\left(\bar{U} + 2\Delta\theta q^G\right)$$
$$-\tilde{\nu}\left(\bar{U} + \Delta\theta q^G\right) - \bar{\nu}\bar{U}F(\bar{U})$$

The first order conditions for the above problem are similar to those derived for the problem (P reduced), except the condition written for the variable q^G. Hence, we have:

i) $\dfrac{\partial G}{\partial \underline{q}} = 0$ or $\underline{\nu}\left[S'(\underline{q}) - \underline{\theta}\right] = 0$

which is exactly the condition (11) and yields again to

$$\underline{q}^{SBG} = \underline{q}^* \qquad (19)$$

ii) $\dfrac{\partial G}{\partial q^G} = 0$ or

$$\tilde{\nu}\left[S'(q^G) - \tilde{\theta}\right] + \bar{\nu}F(\bar{U})\left[S'(q^G) - \bar{\theta}\right] - 2\Delta\theta\underline{\nu} - \tilde{\nu}\Delta\theta = 0$$

We get:

$$S'(q^G) = \tilde{\theta} + \frac{\bar{\nu}F(\bar{U})}{\tilde{\nu} + \bar{\nu}F(\bar{U})}\Delta\theta + \frac{2\underline{\nu} + \tilde{\nu}}{\tilde{\nu} + \bar{\nu}F(\bar{U})}\Delta\theta > \underline{\theta} \qquad (20)$$

Thus, it follows that $S'(\underline{q}^*) < S'(q^G)$ and $\underline{q}^* > q^G$.

iii) $\dfrac{\partial G}{\partial \bar{U}} = 0$ or $S(q^G) - \bar{\theta}q^G = \bar{U}^{SBG} + \dfrac{\underline{\nu} + \tilde{\nu} + \bar{\nu}F(\bar{U}^{SBG})}{\bar{\nu}F'(\bar{U}^{SBG})} \qquad (21)$

The expressions (19)-(21) characterize the features of the optimal solution in the case of pooling contracts.

5. Conclusions

We determined above the solution of the Principal's optimization problem. We can state now the main features of the optimal contracts in the situation of asymmetric information. The following theorem summarizes these features.

Theorem (separating contracts). In the case of asymmetric information with random participation constraints and three types of Agent, if $\tilde{\nu} > \underline{\nu}\bar{\nu}$, the optimal contracts (*separating contracts*) are characterized by the following:

A. Optimal productions:

$\rightarrow$ The Agent with the lowest marginal cost (the type $\underline{\theta}$) produces efficient, such that the optimal second best quantity is not distorted with respect to the first best solution, $\underline{q}^{SB} = \underline{q}^{*}$. We have therefore:

$$S'(\underline{q}^{SB}) = \underline{\theta}.$$

$\rightarrow$ All other types of Agent (the types $\tilde{\theta}$ and $\bar{\theta}$) produce optimal quantities distorted downward with respect to the first best solution. The second best productions are given by the equations:

$$S'(\tilde{q}^{SB}) = \tilde{\theta} + \frac{\underline{\nu}}{\tilde{\nu}}\Delta\theta, \text{ with } \tilde{q}^{SB} < \tilde{q}^{*}$$

and

$$S'(\bar{q}^{SB}) = \bar{\theta} + \frac{\underline{\nu} + \tilde{\nu}}{\bar{\nu}F(\bar{U})}\Delta\theta, \text{ with } \bar{q}^{SB} < \bar{q}^{*}.$$

B. Optimal informational rents

$\rightarrow$ The Agent with the highest marginal cost (the type $\bar{\theta}$) gets an informational rent $\bar{U}^{SB}$ given by the expression:

$$S(\bar{q}^{SB}) - \bar{\theta}\bar{q}^{SB} = \bar{U}^{SB} + \frac{\underline{\nu} + \tilde{\nu} + \bar{\nu}F(\bar{U}^{SB})}{\bar{\nu}F'(\bar{U}^{SB})}$$

$\rightarrow$ The Agent with medium marginal cost (the type $\tilde{\theta}$) gets an informational rent increasing in the quantity produced by the type $\bar{\theta}$ and given by:

$$\tilde{U}^{SB} = \bar{U}^{SB} + \Delta\theta\bar{q}^{SB}.$$

$\rightarrow$ The Agent with the lowest marginal cost (the type $\underline{\theta}$) gets the second best informational rent written as:

$$\underline{U}^{SB} = \bar{U}^{SB} + \Delta\theta(\bar{q}^{SB} + \tilde{q}^{SB}).$$

C. Optimal transfers

$\rightarrow$ The Agent with the highest marginal cost of production gets a second best transfer expressed as:

$$\bar{t}^{SB} = \bar{U}^{SB} + \bar{\theta}\bar{q}^{SB}$$

$\rightarrow$ The Agent with the type $\tilde{\theta}$ gets an optimal second best transfer given by:

$$\tilde{t}^{SB} = \bar{U}^{SB} + \Delta\theta\bar{q}^{SB} + \tilde{\theta}\tilde{q}^{SB}$$

→ The Agent with the lowest marginal cost gets an optimal second best transfer equal to:

$$\underline{t}^{SB} = \bar{U}^{SB} + \Delta\theta \;\; \bar{q}^{SB} + \tilde{q}^{SB} \;\; + \underline{\theta}\underline{q}^{*}$$

Note that if the condition $\tilde{\nu} > \underline{\nu}\bar{\nu}$ is not satisfied (i.e., if $\tilde{\nu} \le \underline{\nu}\bar{\nu}$), we are in the presence of *pooling contracts*. The features of these contracts were derived in the previous section.

It can be seen from these results that the presence of asymmetric information has an important impact on the form of optimal contracts. The fact that the informational rent of the efficient Agent is increasing in the quantity produced by the most inefficient type has an important consequence for the Principal. He is constrained to allow lower quantities in order to reduce the informational rents received by the Agents.

We proposed in the paper a generalization of a standard model of adverse selection, assuming that the adverse selection parameter (here, the marginal cost of production) can have one of three possible values and the agent's participation is random. We presented a detailed procedure of solving this extended model and at the end we derived the main features of the optimal contracts in the situation of asymmetric information.

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Appendix. Second order conditions for the Principal's optimization problem

Using the first order conditions, the Hessian matrix of the objective function $H\ \cdot$ of the problem (P reduced) is:

$$H_H\ \ \underline{q},\tilde{q},\bar{q},\bar{U}\ \ = \begin{pmatrix} \underline{v}S''\ \underline{q} & 0 & 0 & 0 \\ 0 & \tilde{v}S''\ \tilde{q} & 0 & 0 \\ 0 & 0 & H_{33} & H_{34} \\ 0 & 0 & H_{43} & H_{44} \end{pmatrix}$$

where:

$$H_{33} = \bar{v}F'\ \bar{U}\ \ S''\ \bar{q}\ \ ,\ H_{34} = H_{43} = \bar{v}F'\ \bar{U}\ \left[S'\ \bar{q}\ -\bar{\theta}\right],$$

$$H_{44} = \bar{v}F''\ \bar{U}\ \left[S\ \ \bar{q}\ -\bar{\theta}\bar{q} - \bar{U}\right] - 2\bar{v}F'\ \bar{U}$$

To analyze if the matrix is negative definite we compute all leading principal minors:

$$\Delta_0 = 1$$

$$\Delta_1 = \underline{v}S''\ \underline{q}\ < 0$$

$$\Delta_2 = \underline{v}\tilde{v}S''\ \underline{q}\ \ S''\ \tilde{q}\ > 0$$

$$\Delta_3 = \underline{v}\tilde{v}\bar{v}F\ \ \bar{U}\ \ S''\ \underline{q}\ \ S''\ \tilde{q}\ \ S''\ \bar{q}\ < 0$$

$$\Delta_4 = \Delta_2\left[H_{33}H_{44} - \ H_{34}\ ^2\right]$$

It can be seen that the first for leading minors have well defined signs. The only problem is created by the sign of last minor.

At the optimum, from the first order conditions, we have:

$$S'\left(\bar{q}^{SB}\right)-\bar{\theta}=\frac{\underline{\nu}+\tilde{\nu}}{\bar{\nu}F\left(\bar{U}\right)}\Delta\theta$$ and

$$S\left(\bar{q}^{SB}\right)-\bar{\theta}\bar{q}^{SB}-\bar{U}^{SB}=\frac{\underline{\nu}+\tilde{\nu}+\bar{\nu}F\left(\bar{U}^{SB}\right)}{\bar{\nu}F'\left(\bar{U}^{SB}\right)}$$

such that Δ_4 becomes:

$$\Delta_4=\Delta_2\bar{\nu}^2\left\{F\left(\bar{U}\right)S''\left(\bar{q}\right)\left[\frac{F''\left(\bar{U}\right)}{F'\left(\bar{U}\right)}\frac{\underline{\nu}+\tilde{\nu}+\bar{\nu}F\left(\bar{U}\right)}{\bar{\nu}}-2F'\left(\bar{U}\right)\right]-\left[\frac{F'\left(\bar{U}\right)}{F\left(\bar{U}\right)}\frac{\underline{\nu}+\tilde{\nu}}{\bar{\nu}}\Delta\theta\right]^2\right\}$$

A necessary condition for $\Delta_4>0$ is that the following relation to be true at the optimum:

$$F''\left(\bar{U}\right)\frac{\underline{\nu}+\tilde{\nu}+\bar{\nu}F\left(\bar{U}\right)}{\bar{\nu}}<2\left(F'\left(\bar{U}\right)\right)^2$$