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INSIDE THE BLACK BOX: INFORMATIONAL ENTROPY OF HIGH - FREQUENCY DATA ON FOREX MARKET

Abstract. *The current period of financial and real instability questions the usual approaches concerning financial markets' operating mechanisms. Thus, several alternatives have been proposed in order to provide a more realistic description of markets' mechanisms. Among these, the Fractal Market Hypothesis accounts for discontinuous and non-periodical evolutions in financial assets' prices. The proposed study analyzes the main properties of Rény's entropy estimated for 139,671 intra-day observations on USD/CAD exchange rate over various time scales. The paper argues that if the Fractal Market Hypothesis stands, than the respective properties are conserved despite the shifts from high to low frequency. Overall there are some empirical evidences supporting such time-scale invariance. However, these evidences ought to be interpreted with caution since the shifts from high to low frequency do not entirely preserve the entropic characteristics of data.*

Keywords: *FX market, USD/CAD exchange rate, Fractal Market Hypothesis, Rény's entropy, high frequency data*

JEL Classification: G14, G15

[1] ***Acknowledgement: This work was supported by the project "Post-Doctoral Studies in Economics: training program for elite researchers - SPODE" co-funded from the European Social Fund through the Development of Human Resources Operational Programme 2007-2013, contract no. POSDRU/89/1.5/S/61755.***

1. Introduction

The current global financial and real turmoil is undermining the efficiency of the international financial markets and, consequently, generates a global shock. Furthermore, it raises questions on how these markets really work. In particular, it reveals the necessity to take into consideration the heterogeneity of investors' beliefs, asymmetric and imperfect information, as well as bounded rationality portfolio decision. Thus, it is required a more realistic description of the interactions between market participants and of the fundamentals used in their decisions.

In the classic portfolio management theory build on the Markowitz's mean-variance model, selecting the structure of the investor's portfolio depends primarily on the first and second moments corresponding to the expected return and the variance-covariance matrix of the return. However, these moments are generally inadequate in explaining the *de facto* portfolio structure in the case of non-normal return distribution (see, for instance, Chunhachinda et al., 1997 for a detailed analysis). Hence, several extensions of the standard model were developed by including the skewness of return in portfolio selection. Still, some studies (e.g. Prakash et al., 2003) suggest that the portfolio weights obtained by using these versions often focus on a few assets or extreme positions and tend to offer limited possibilities for portfolio diversification. In this context, an increasing literature (Bera and Park, 2005, Jana et al., 2009, Usta and Kantar, 2011) attempts to use entropy as an objective function in multi-objective model portfolio selection. An underlying argument in such approach is that entropy is more able to capture self information - the information provided by a random process about itself (Gray, 2011) – compared to other measures, such as variance, by being a better descriptor of the intrinsic uncertainty associated with the evolutions of the financial assets' prices. Thus, the study of the entropic properties of financial assets' prices can provide relevant information for portfolio's structure and management, by improving the allocation decisions.

Starting with the *Fractal Market Hypothesis* (FMH), new approaches to the discontinuous and non-periodical evolutions of financial markets have been proposed. FMH postulates the significant impact of information and investment horizons on investors' behavior. It incorporates the following assumptions (see Peters, 1997):

- The market is formed by heterogeneous investors;
- Investors' decisions incorporate, in a non-uniform manner, information over different time horizons;
- Financial assets' prices reflect a combination of short-term technical trading and long-term fundamental valuation;
- Investors' preferences in regard to the trading time horizons for a specific financial asset largely depend upon the sensitivity of the respective asset's return to the macroeconomic cycles.

In this framework, Molgedey and Ebeling (2000) provide evidences on Dow Jones Index that in spite of high stochasticity in average there might be special local situations where local order exists and the predictability is considerably higher than average. Zunino et al. (2009) found robust evidences that the degree of market inefficiency is positively correlated with the number of forbidden patterns and negatively correlated with the permutation entropy.
Glattfelder et al (2011) have discovered 12 independent new empirical scaling laws in foreign exchange data series that hold for close to three orders of magnitude and across 13 currency exchange rates. They have shown that the scaling laws give an estimation of the length of the price-curve coastline, which in their study turns out to be significantly long. The main implication of such findings is perhaps the idea that the analysis of financial assets' prices should be linked more to their behavior as event-based processes, instead of focusing on their stochastic nature.
Mercik and Veron (2002) provide evidences that foreign exchange rate returns satisfy scaling with an exponent significantly different from that of a random walk. They also provide evidences that the conditionally exponential decay (CED) model can be used to identify the mathematical structure of the distributions of FX returns corresponding to the empirical scaling laws.

This study contributes by analyzing the properties of the entropy series estimated for a major currency pair in FX market, Unites States dollar / Canadian dollar, over various time scales ranging from 5 minutes to 5 trading days. The underlying argument is that if the exchange rate exhibits fractalic properties than the main characteristics of the associated entropy should be preserved with the shift from high to low frequency data. This should happen as a consequence of the self-similarity property of fractals which makes them scale-invariant. In other words, the impact of information shocks on the heterogeneous market participants should more or less be the same regardless of the decisional time frame.
The next section describes the methodology and research hypotheses. Section 3 presents the data while Section 4 reports the main results. Section 5 concludes.

2. Methodology and research hypotheses

Since entropy is a measure of the size of a data distribution contained within a bounded region, it appears naturally to use it for problems where the detailed scaling behavior of correlations over substantial scale intervals is of interest. Such a problem is represented by the study of financial assets' prices formation over different time frames when several types of effects may distort scale distributions, and where the correlation structure is not trivially expressible as a power law or other elementary function.
We are considering the intraday prices at the time τ_j, $x(\tau_j)$, defined as:

$$x\left(\tau_j\right)=\ln\left(\frac{p_{bid}\left(\tau_j\right)+p_{ask}\left(\tau_j\right)}{2}\right) \qquad (1)$$

Hence $\{\tau_j\}$ is the sequence of the tick recording times which is unequally spaced. This definition of tick-by-tick prices consider the average of the bid and ask price rather than either the bid or the ask series as a better approximation of the transaction price. Such approach seeks to account for the observation that market makers frequently skew the spread towards a more favorable price to offset their position. In the same, it should be noticed that since it works with the natural logarithm of the average of bid and ask prices, such definition implies a symmetrical behavior when prices are inverted.

Our purpose is to estimate the so-called *Rény's entropy of order α* (see Rény, 1961; Ullah, 1996; Jizba and Arimitsu, 2004; Karmeshu, 2003) which is an extension of the *Shannon's entropy* to an incomplete probability distribution of a probabilistic system with n states, $P_n=(p_1,p_2,...,p_n)\subset P, p_i\geq 0$, p_i reflecting the probability of the i-th state s_i and P being the set of all probability distributions on finite sets, with $i=1,2,...,n$ and $\sum_{i=1}^{n} p_i \leq 1$:

$$H^1{}_{R_n}\left(P_n;\alpha\right)=\frac{1}{1-\alpha}\log\left(\sum_{i=1}^{n} p_i^{\alpha}\right) \quad \left(\alpha>0;\alpha\neq 1\right) \qquad (2)$$

Here the entropy measure is a function $H^1{}_{R_n}:P\rightarrow[0,\infty)$ and *log* stands for log_2, with $\lim_{\alpha\to 1} H^1{}_{R_n}(P_n;\alpha)\rightarrow H^1{}_{S_n}(P_n)$ where $H^1{}_{S_n}(P_n)$ is the function of *Shannon's entropy* for a discrete probability distribution P_n.

We assimilate the probabilities with the *data frequency* at the observation t_i, $f(t_i)$, being defined as

$$f\left(t_i\right)\equiv f\left(t_i;S\right)\equiv\frac{1}{S}N\left(x\left(r_j\right)\mid r_j\in\left[t_i-S+1,t_i\right]\right) \qquad (3)$$

Here S is the number of successive prices on which the entropy is computed (for instance 100 or 1000 successive trade prices) and $N(x(r_j))$ is the counting function.

Our strategy is based on the estimation of the *Rény's entropy* on K non-overlapping "windows" which are splitting a database of N tick observations ($K=N/S$), each of them with the length S, length which is equally time spaced, for some arbitrary values of S (5, 15, 60, 1440 and 7200 successive observations) and of α (0.50 and 2.50). The purpose is to study the behavior of the entropy series depending on the length S of the computation sequence of prices. Since an increase in S is equivalent with an increase in the corresponding time span, this is as well an implicit study of the prices' behavior at a shift from "high" to "low" frequency in data.

Due to the fact that the number of the states reached by the trajectory of prices tends to increase with a shift from "high" to "low" frequency data, a first plausible hypothesis is that the entropy will increase correspondingly to such shift.

Thus:
H1: *The entropy levels will be higher on lower frequency data as reflecting an increase in the uncertainty associated to prices' evolutions.*
However, such conclusion must be extended with an examination of the changes in the entropy levels at a given length of the estimation "window". If there is a "learning" mechanism for an individual data frequency, it should lead to adjustments in entropy as a consequence of changes in the degree of uncertainty concerning prices' evolutions in the reference time span.

H2: *If the investors are learning about current market dynamics - as more information about the observed values of prices is gathered - their portfolio corrective decisions should be reflected by low amplitude adjustments in information entropy. As a consequence, the entropy series must follow a random walk (eventually with drift) process.*

In order to test this hypothesis, a possible approach consists in the use of the so-called Lo and MacKinlay (1988; 1989) overlapping *Variance Ratio Test*. This test examines the predictability of a series by comparing variances of differences of data computed over different intervals (or successive observations). If the data are assumed to follow a random walk, the variance of a $-q$ period (or observation) difference should be q times the variance of the one-period (or observation) difference. Evaluating the empirical evidence for or against this restriction is the basis of the variance ratio test. More exactly, if the series $H^1{}_{R_n}$ satisfying

$$\Delta H^1{}_{R_n i} = \mu + \varepsilon_i \tag{4}$$

where μ is an arbitrary drift parameter, then the key properties of a random walk that are of interest for the test can be described as $E(\varepsilon_i) = 0, E(\varepsilon_i, \varepsilon_{i-j}) = 0$ for all i and any positive j.
The estimators for the mean of first difference and the scaled variance of the q-th difference are defined as:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\left(H^1{}_{R_n i} - H^1{}_{R_n i-1}\right)$$

$$\hat{\sigma}^2(q) = \frac{1}{nq}\sum_{i=1}^{n}\left(H^1{}_{R_n i} - H^1{}_{R_n i-q} - q\hat{\mu}\right)^2 \tag{5}$$

The corresponding variance ratio is $VR(q) = \frac{\hat{\sigma}^2(q)}{\hat{\sigma}^2(1)}$.

Lo and MacKinlay (1988) show that the variance ratio *z*-statistic:

$$z(q) = \left[VR(q) - 1\right] * \left[s^2(q)\right]^{-\frac{1}{2}} \qquad (6)$$

is asymptotically N(0,1) if the estimator s^2 is properly chosen.
The test is firstly performed for homoskedastic random walks using the wild bootstraps distribution to evaluate statistical significance. Such an approach is based on the strong assumption that ε_t is *iid* Gaussian but the normality assumption is not strictly required. Two different alternatives are considered: 1) the entropy series are random walks so that variances are computed for differences of data; or 2) the series contains the random walk innovations themselves.
Other specifications for tests' implementation includes the use of the *rank scores* (*van der Waerden scores*) instead of data level as has been proposed by Wright (2000), a *Rademacher* bootstrap error distribution, 10000 replications, the *Knuth* random number generator and an randomized assignation of ranks in the presence of tied data. The objective of such specification set is to provide the most general possible framework for the analysis of the two mentioned hypotheses and to account for possible local deviations of the entropy from its general pattern.
Furthermore, we estimate the *Hurst exponent* of the entropy series in order to evaluate their "persistent" / "anti-persistent" behavior. The term "persistent" ought to be carefully considered. In the context of entropy, it implies that there is a *dominant* state of prices for which there is maintained more or less the same amount of *ex-ante* information where roughly the same amount of *ex-ante* information is maintained.

H3: *If the main corrective market mechanisms are preserved with/despite the shift from "high" to "low" frequency data, a certain dominant prices' level is supposed to be found at each frequency and the same degree of "persistence" is preserved even with the translation to different frequencies.*
With the purpose of testing this hypothesis, we firstly employ the so-called *Dispersional Analysis* (also known as the *Aggregated Variance* method). This procedure supposes that the series P_n are fractional Brownian motion processes. Their succesive increments ξ can be viewed as the equivalents of the fractional Gaussian noise (*fGn*):

$$\xi(i) = P(i + \delta) - P(i) \qquad (7)$$

The procedure averages the differenced *fGn* series over *bins* of width ℵ and calculates the variance of the averaged dataset. An implementation alghorithm for the procedure considers the following steps:

1) Set the *bin* size $\aleph = 1$;
2) Calculate the standard deviation of the *n* data points and record the point $(\aleph,\ \aleph\sigma_{\aleph})$;
3) Average neighbouring data points and store in the original dataset

$$\xi(i) \leftarrow \frac{\left[\xi(2i-1)+\xi(2i)\right]}{2} \tag{8}$$

In the same time, *n* and □ are rescaled as:

$$n \leftarrow \frac{n}{2}$$
$$\aleph \leftarrow 2\aleph \tag{9}$$

4) As long as more than a certain number of data points remain (n > predefined number of *bins*) return to Step 2;
5) Perform a linear regression on the log-log graph as:

$$\log\left(\aleph\sigma_{\aleph}\right) = H\log\left(\aleph\right)+C \tag{10}$$

The calculated slope of this regression can be seen as an estimator of the *Hurst exponent*.

Secondly, we use an alternative evaluation method based on the so-called *Performance Persistence Analysis* (Amenc and Le Sourd, 2007). This method implies the recalculation of the series ξ by subtracting the mean value of the sample, $\bar{\xi}$:

$$Z(i) = \xi(i) - \bar{\xi} \tag{11}$$

Defining:

$$Y(s) = \sum_{s=1}^{i} Z(i)$$
$$Y_1 = \max_{1<i<n} Y(i)$$
$$Y_2 = \min_{1<i<n} Y(i)$$

The estimation of the *Hurst exponent* is obtained as:

$$H = \frac{1}{\ln(n)}\ln\left(\frac{Y_1 - Y_2}{\sigma}\right)$$
$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} Z(i)^2} \tag{12}$$

3. Data

Prices data represents 1 minute bar close prices of the USD/CAD exchange rate on FOREX market for a time span between 8/1/2011 (15:30:00) and 12/16/2011 (23:59:00) (139671 observations). These data are provided by *TeleTrader* (http://www.teletrader.com).
The bootstraps *BDS test* values reported in Table 1 shows that the null of identical and independent distributed values of exchange rates can clearly be rejected for almost all areas of data for $m = 2,\ldots,10$ embedded dimensions.
However, it should be noticed that the test does not clearly separate the independent and, respectively, identically distributed patterns. However, by taking into account both the changes in distribution over time and the autoregressive dynamics of exchange rates, the test leads to the conclusion that there are important deviations from the independence in data. The nature of such deviations requires further investigation. Still, it can be presumed the existence of some structural changes in data distribution as well as some hysteresis effects leading to low order autocorrelations behaviors.

Table 1. BDS portmanteau test for identical and independent distributed values of USD/CAD exchange rate

Dimension	BDS Statistic	Std. Error	z-Statistic	Normal Prob.	Bootstrap Prob.
2	0.00006	0.00000	25081.53000	0.00000	0.00000
3	0.00012	0.00000	22556.69000	0.00000	0.00000
4	0.00018	0.00000	20297.92000	0.00000	0.00000
5	0.00024	0.00000	18537.79000	0.00000	0.00000
6	0.00030	0.00000	17144.73000	0.00000	0.00000
7	0.00036	0.00000	16022.47000	0.00000	0.00000
8	0.00043	0.00000	15091.88000	0.00000	0.00000
9	0.00049	0.00000	14302.45000	0.00000	0.00000
10	0.00055	0.00000	13629.93000	0.00000	0.00000

Raw epsilon: 0.10305

4. Results

Table 2 reports the main statistics of entropy estimations while the computation "windows" varies from 5 to 7200 successive observations (ranging from 5 minutes frequencies to weekly ones) and α parameter is modified from 0.5 to 2.50.
These values show that the entropy highlights a clear tendency to increase with the shift from "high" to "low" frequency data and also to slowly decrease for higher values of α parameter. Thus, the results can be seen as providing some support for *H1*.

Table 2. **Main statistics of entropy series**

	Mean	Median	Minimum	Maximum	Standard deviation	Skewness	Kurtosis	Observations
Length of window:5								
α= 0.50	2.05	2.16	0.26	2.85	0.54	-0.87	3.79	27934
α= 2.50	1.15	1.15	-0.44	2.15	0.59	-0.46	2.87	27934
Length of window: 15								
α= 0.50	2.79	2.86	0.09	4.09	0.60	-0.74	3.54	9311
α= 2.50	2.19	2.26	-0.15	3.84	0.65	-0.52	3.10	9311
Length of window: 60								
α= 0.50	3.88	3.97	1.45	5.42	0.67	-0.56	3.08	2327
α= 2.50	3.34	3.42	0.89	5.09	0.71	-0.50	2.96	2327
Length of window: 1440								
α= 0.50	6.40	6.40	5.36	7.54	0.47	0.04	2.48	96
α= 2.50	5.71	5.70	4.44	6.89	0.54	-0.10	2.40	96
Length of window: 7200								
α= 0.50	7.67	7.62	6.98	8.57	0.49	0.28	1.87	19
α= 2.50	7.08	7.05	6.03	8.22	0.52	0.17	2.84	19

The *Variance Ratio* tests reported in Table 3 clearly reject the *random walk* null in each tested form for the low and intermediary levels estimation "windows". However, for windows equals to 7200 the null of entropy series being *random walk* processes cannot be any longer rejected. So, the *H2* hypothesis can have certain viability but is more probable to happen at "low" frequencies of prices data.

Table 3. **Lo-MacKinlay *Variance Ratio Test* for the entropy series (computed using *rank scores*)**

	***Null**: Random walk*		***Null**:Cumulated data are random walk processes*	
	Max |z| (at "period" 2)	Wald (Chi-Square)	Max |z| (at "period" 16)	Wald (Chi-Square)
Length of window: 5				
α= 0.50	97.18 (0.00)	9508.528 (0.00)	231.70 (0.00)	63425.00 (0.00)
α= 2.50	97.87 (0.00)	9624.42 (0.00)	225.64 (0.00)	60131.20 (0.00)

Length of window: 15				
α= 0.50	42.90 (0.00)	1857.94 (0.00)	137.77 (0.00)	19960.79 (0.00)
α= 2.50	43.66 (0.00)	1927.88 (0.00)	129.42 (0.00)	17570.49 (0.00)
Length of window: 60				
α= 0.50	16.32 (0.00)	323.33 (0.00)	46.97 (0.00)	2465.62 (0.00)
α= 2.50	15.22 (0.00)	300.11 (0.00)	45.59 (0.00)	2368.15 (0.00)
Length of window: 1440				
α= 0.50	5.63 (0.00)	57.63 (0.00)	6.77 (0.00)	56.05 (0.00)
α= 2.50	4.16 (0.00)	29.81 (0.03)	7.06 (0.00)	55.60 (0.00)
Length of window: 7200				
α= 0.50	1.34 (0.40)	8.07 (0.84)	2.15 (0.04)	29.91 (0.04)
α= 2.50	1.13 (0.63)	6.47 (0.94)	2.41 (0.02)	32.02 (0.03)

Probability in (); *Equal spaced* grid: - Minimum: 2; - Maximum: 16; -Step:1;
Bootstrap replications: 10000; Random generator: *Knuth*; Tie-handling: Average

Table 4 suggests that the pattern of the entropy series is "persistent" (there is a certain conservation of the total degree of uncertainty in the market) but such "persistence" tends somehow to decline with the shift to "low" frequency data. Thus, it appears that the behavior of the entropy series is close to a fractional Brownian motion process (or to other processes that generate such "persistence" like for instance, *fractional ARIMA (0,d,0)* processes which also exhibit scaling with an *exponent Hurst* greater than 0.5) especially for "high" frequency data. This conclusion is robust to the change in the estimation method and is especially valid for "extreme" values of estimation "windows".

Table 4. "Long-range" memory in the entropy series (Hurst exponent)

	Dispersional analysis	*Performance persistence analysis*
Length of window: 5		
α= 0.50	0.80	0.75
α= 2.50	0.79	0.75
Length of window: 15		
α= 0.50	0.76	0.75
α= 2.50	0.75	0.75
Length of window: 60		
α= 0.50	0.75	0.73
α= 2.50	0.74	0.72
Length of window: 1440		
α= 0.50	0.75	0.69
α= 2.50	0.75	0.69
Length of window: 7200		
α= 0.50	0.57	0.63

α= 2.50	0.66	0.65

In other words, there seems to be a certain degree of "persistence" for individual frequencies, as *H3* predicts, but such degree is not necessarily the same for "high" and "low" frequency data.
Another approach of the entropy "persistence" issue may consist in the examination of its *long-run variance*. A *heteroskedasticity and autocorrelation consistent* (HAC) estimator of this - $\hat{\Omega}$ - can be obtain, for instance, based on nonparametric kernel approach (Andrews, 1991; Newey-West, 1987):

$$\hat{\Omega} = \frac{T}{T-K}\sum_{j=-\infty}^{\infty} k\left(\frac{j}{b_T}\right)\hat{\Gamma}(j) \tag{13}$$

For a sequence of mean-zero random p-vectors $\{V_t(\theta)\}$ that may depend on a *K*-vector of parameters θ, the considered sample autocovariances $\hat{\Gamma}(j)$ are given by:

$$\begin{cases} \hat{\Gamma}(j) = \dfrac{1}{T}\sum\limits_{t=j+1}^{T} \hat{V}\hat{V}'_{t-j} & j \geq 0 \\ \hat{\Gamma}(j) = \hat{\Gamma}(-j)' & j < 0 \end{cases} \tag{14}$$

K is a symmetric kernel (or lag window) function that, among other conditions, is continuous at the origin and satisfies $|k(x)| \leq 1$ for all x and $k(0) = 1$. b_T is a bandwidth parameter while the term $\frac{T}{T-K}$ represents a correction for degrees-of-freedom associated with the estimation of the *K* parameters in θ.
We are choosing the *Quadratic Spectral* kernel function as:

$$k(x) = \frac{\alpha}{\beta\pi^2 x^2}\left(\frac{\sin\left(\frac{\chi\pi x}{\delta}\right)}{\frac{\chi\pi x}{\delta}} - \cos\left(\frac{\chi\pi x}{\delta}\right)\right) \tag{15}$$

The values of the parameters are set to $\alpha = 25$, $\beta = 12$, $\chi = 6$, $\delta = 5$. The results are reported in Table 5. It appears that the long-run variance of entropy series slowly decline with the shift from high to low frequencies with the corresponding adjustments in the "persistence".

Table 5. Long-run variance of the entropy series

	Long-run variance
Length of window: 5	

α= 0.50	8.76
α= 2.50	9.71
Length of window:15	
α= 0.50	6.28
α= 2.50	6.78
Length of window: 60	
α= 0.50	1.54
α= 2.50	1.55
Length of window: 1440	
α= 0.50	0.77
α= 2.50	0.93
Length of window: 7200	
α= 0.50	0.81
α= 2.50	1.38

Bandwidth method: Newey-West; Lag selection: Hannan-Quinn Info criterion; Window: Symmetric

Finally, a global evaluation of the shifts in the values of the entropy estimated over different time horizons can be obtained by computing the time-scale differences. This can be done for two different estimations of the entropy $H^1{}_{R_m}$ and $H^1{}_{R_n}$ computed on two time scales, τ_1 and τ_2, with $r = \frac{\tau_2}{\tau_1} > 1$ and $n < m$, for instance in a simple way like:

$$\Delta^{n,m}{}_{\tau_1,\tau_2} = \frac{\left(H^1{}_{R_n} - H^1{}_{R_m}\right)}{r} \qquad (16)$$

If the changes in the main properties of the entropy are not substantially different over different time scales, it can be expected that the $\Delta^{n,m}{}_{\tau_1,\tau_2}$ series fulfill the stationarity requirements, at least around a linear trend (or, alternatively, around a non-linear one). In other words, the $H^1{}_{R_n}$ values should act as attractors for the $H^1{}_{R_m}$ values.

Table 6 reports on several unit root versus trend stationarity tests on the considered frequencies. Overall, these tests shows that the $\Delta^{n,m}{}_{\tau_1,\tau_2}$ series can be viewed as being stationary around a (possible linear) trend.

Table 6. Various stationarity tests for time-scale differences of entropy series

	Augmented Dickey-Fuller (ADF) *H0: unit root with drift process* against *H1: Linear trend stationarity*	Bierens DHOAC tests *H0: unit root with drift process* against *H1: Linear trend stationarity*	Phillips-Perron *H0: unit root with drift process* against *H1: Linear trend stationarity*	Bierens-Guo (type 5 and 6) *H0: Trend stationarity* against *H1: unit root with drift process*
$\tau_1 = 5$; $\tau_2 = 15$				
α= 0.50	-13.79 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-808046.44 DHOAC(2,2)=-802640.31 H0 is rejected at the 5% significance level	-67229.52 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 0.06 (p-value=0.96) Type 6: 0.78 (p-value=0.58) H0 is accepted at the 5% significance level
α= 2.50	-14.28 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-994751.69 DHOAC(2,2)=-993476.63 H0 is rejected at the 5% significance level	-60721.49 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 0.11 (p-value=0.93) Type 6: 0.67 (p-value=0.63) H0 is accepted at the 5% significance level
$\tau_1 = 5$; $\tau_2 = 60$				
α= 0.50	-14.80 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-702681.63 DHOAC(2,2)=-703553.25 H0 is rejected at the 5% significance level	-55170.92 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 0.15 (p-value=0.90) Type 6: 0.93 (p-value=0.52) H0 is accepted at the 5% significance level
α= 2.50	-15.21 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-147591.61 DHOAC(2,2)=-147815.11 H0 is rejected at the 5% significance level	-55742.19 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 0.16 (p-value=0.90) Type 6: 1.11 (p-value=0.47) H0 is accepted at the 5% significance level
$\tau_1 = 5$; $\tau_2 = 1440$				
α= 0.50	-9.79 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-26393.78 DHOAC(2,2)=-26352.37 H0 is rejected at the 5% significance level	-75886.28 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 0.62 (p-value=0.65) Type 6: 2.39 (p-value=0.25) H0 is accepted at the 5% significance level
α= 2.50	-9.14 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-18938.07 DHOAC(2,2)=-18958.66 H0 is rejected at the 5% significance level	-77651.37 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 1.10 (p-value=0.47) Type 6: 2.56 (p-value=0.24) H0 is accepted at the 5% significance level
$\tau_1 = 5$; $\tau_2 = 7200$				

α= 0.50	-7.62 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-18782.38 DHOAC(2,2)=-18811.63 H0 is rejected at the 5% significance level	-74373.11 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 2.32 (p-value=0.26) Type 6: 9.51 (p-value=0.06) H0 is accepted at the 5% significance level by Type 5 but rejected by Type 6 tests
α= 2.50	-7.70 (p-value=0.00) H0 is rejected at the 5% significance level	DHOAC(1,1)=-28509.44 DHOAC(2,2)=-28530.91 H0 is rejected at the 5% significance level	-81654.86 (p-value=0.00) H0 is rejected at the 5% significance level	Type 5: 2.71 (p-value=0.23) Type 6: 4.03 (p-value=0.15) H0 is accepted at the 5% significance level

5. Conclusions

The market mechanisms are far more complex than described by standard models which are based on the assumptions of rational decisions and perfect information. If a zoom-in approach on high frequency data is considered, then a picture of alternating local stability and chaotic evolutions can be revealed. Such complex dynamics is governed by some power laws that are relatively, but not perfectly, stable over various decisional time frames.
The adjustments in the corresponding parameters of such laws reveals the changes in the degree of uncertainty associated to the shifts from high to low frequency with impact on prices' mechanisms. Thus, a more detailed analysis of these adjustments can contribute not only by improving the understanding of such mechanisms, but also in the construction and implementation of trading strategies with better management of various risks faced by investors.

By focusing on a large volume of intra-day data related to USD/CAD exchange rate, this study provides some empirical evidences that in FX market the levels of Rény's entropy tends to increase on low frequency data as an expression of an increased degree of uncertainty. Moreover, it is revealed the fact that the entropy series estimated over various time scales cannot be described as random walk processes. In addition, it is observed that the pattern of the entropy series is "persistent", although such "persistence" tends to decline with the shift to low frequency data.
Finally, the conducted research shows that the time-scale differences between different entropy estimations are trend stationary as the high frequency values tend to be convergent to low frequency data. We believe that such findings can provide some useful insights and can support a more complex analysis of the market in an evolutionary perspective.

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