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# VALUE AT RISK ESTIMATION USING GARCH-TYPE MODELS

***Abstract.*** *This paper examines five GARCH-type models, including RiskMetrics in the Value at Risk estimation. These models are applied to an optimized internally diversified portfolio, comprised of three benchmark indexes from three different countries (Romania, UK and USA) in order to asses the overall performance of the daily VaR estimates at various probability levels (1%, 2.5% and 5%). Study results indicate that all symmetric models outperform the asymmetric ones, both for normal and Student's t distributions. We also find that GARCH(1,1) underestimates 1% VaR in comparison to RiskMetrics and GARCH-t(1,1) that perform very well. Moreover, GARCH-t(1,1) gives better 2.5% VaR estimates and RiskMetrics outperforms GARCH(1,1) and GARCH-t(1,1) for 5% VaR estimates.*

**Key words:** *portfolio, volatility, Value at Risk, GARCH-type models.*

**JEL Classification: C53, G10.**

## 1. Introduction

Modern portfolio theory relies on the study developed by Markowitz (1952). Rubinstein (2002) appreciated that Markowitz's research represents the first mathematical formalization of the diversification concept of investments, emphasizing the fact that even though diversification reduces risk, it can not eliminate it completely. So, through diversification risk can be reduced without having any effects on the portfolio expected return. Thus, investing in different classes of financial assets and in different industrial sectors enables investors to improve the performance of their portfolios (Aloui, 2010).

Value at Risk ($VaR$) is considered to be one of the most important measures of market risk and it has been widely used for financial management by institutions including banks, regulators and portfolio managers. Since the risk management group J.P. Morgan developed the RiskMetrics model for $VaR$ measurement in 1994, this model has become a benchmark for measuring market risk (So,Yu, 2006). Other methods, such as those based on GARCH-type models have also been consistently used in the estimation of Value at Risk. A crucial factor for the accuracy of the $VaR$ estimates relies on the underlying measure of volatility (Moosa, Bollen, 2002). Another problem that arises in the estimated $VaR$ is finding a suitable performance measure that has the capacity to evaluate the performance of the estimates correctly.

Taking all these aspects into consideration, we aim to analyze in this paper the performance of the daily $VaR$ estimates of an optimized internationally diversified portfolio, estimates based on five volatility measures namely: RiskMetrics, GARCH(1,1), EGARCH(1,1), GARCH-t(1,1) and EGARCH-t(1,1). We chose precisely these measures due to the fact that the empirical distribution of financial assets exhibits some well-known stylized facts like volatility clusters, leptokurtosis and leverage effects (Tavares A.B., Curto, Tavares G.N., 2008), and these models are able to capture such characteristics.

The remainder of the paper is organized as follows: **Section 2** presents the theoretical background related to the estimation of the daily $VaR$, as well as aspects related to the volatility measurement of financial time series using GARCH-type models and also aspects regarding the evaluation of the performance of the $VaR$ estimates. In **Section 3** we report the empirical results of our study and in **Section 4** we provide a summary of our conclusions.

## 2. Value at Risk estimation using GARCH type models

### *2.1. Value at Risk* ($VaR$)

The Basel Committee on Banking Supervision at the Bank for International Settlements imposed banks and other authorized financial institutions to communicate at the beginning of each day the daily estimated risk to the closest monetary authority using one or more models of Value at Risk ($VaR$). These models have become a very popular tool for measuring the market risk of a portfolio of financial assets. $VaR$ represents the decline in the market value of an asset or a portfolio of financial assets that can be expected within a given time horizon with a given probability. In order to define the concept of $VaR$ of a portfolio of securities, we must first define the daily returns of the portfolio (Moosa, Bollen, 2002). In the empirical literature it is assumed that the behavior over time of the daily price of a portfolio of securities in a capital market can be represented as:

$$\ln(p_t) = \mu_t + \ln(p_{t-1}) + e_t \tag{1}$$

where $p_t$ represents the market price of the portfolio at time $t$, $\mu_t$ is the mean daily return, and $\text{var}(e_t) = \sigma_t^2$. In this study we concentrate only on daily data, and we shall therefore assume that $\mu_t = 0$.[1] We define the daily continuously compounded return of a portfolio at time $t$ as:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \tag{2}$$

$$e^{r_t} = \frac{p_t}{p_{t-1}} \tag{3}$$

Therefore

$$p_t = p_{t-1}e^{r_t} \tag{4}$$

Let $r_t^c$ be the critical portfolio return such that the observed return on day $t$ is less than or equal to the critical level with a given probability. Thus, for a probability of $\alpha\%$ we have:

$$P(r_t \le r_t^c) = \alpha\% \tag{5}$$

The critical portfolio value $p_t^c$ that corresponds to a probability of $\alpha\%$ implies the fact that the observed return on day $t$ will be less than or equal to $r_t^c$ and it can be obtained by combining equations (4) and (5). So

$$p_t^c = p_{t-1}e^{r_t^c} \tag{6}$$

For a portfolio whose market price is $p_t$, the $VaR$ represents the loss in the value of the portfolio with $\alpha\%$ probability. Thus

$$VaR_t = p_t - p_t^c \tag{7}$$

By combining equations (6) and (7) we obtain:

[1] See Moosa, Bollen (2002).

$$VaR_t = p_t - p_{t-1}e^{r_t^c} \tag{8}$$

We know that $e^x \approx 1 + x$ when $x$ is very small, so the Value at Risk can be calculated as:

$$VaR_t = p_t - p_{t-1}(1 + r_t^c) \tag{9}$$

The critical return for $\alpha\%$ probability depends on the values chosen for $\alpha$. Therefore we chose in our study three common values: $1\%$, $2.5\%$ and $5\%$. An important hypothesis of the $VaR$ model is the fact that the portfolio returns are normally distributed, so the critical return for these probability levels will be given by:

**Table 1**

**The value of the critical return for different probability levels**

| Critical return | Probability level | | |
|---|---|---|---|
| | $\alpha = 1\%$ | $\alpha = 2.5\%$ | $\alpha = 5\%$ |
| $r_t^c$ | $-2.326\sigma_t$ | $-1.96\sigma_t$ | $-1.645\sigma_t$ |

The Value at Risk for all three probability levels can be calculated as:

**Table 2**

**Value at Risk for different probability levels**

| Value at Risk | Probability level | | |
|---|---|---|---|
| | $\alpha = 1\%$ | $\alpha = 2.5\%$ | $\alpha = 5\%$ |
| $VaR_t$ | $p_t - p_{t-1}(1 - 2.326\sigma_t)$ | $p_t - p_{t-1}(1 - 1.96\sigma_t)$ | $p_t - p_{t-1}(1 - 1.645\sigma_t)$ |

where $\sigma_t$ represents the volatility of the portfolio on trading day $t$. The expected value of $VaR_{t+1}$ at time $t$, respectively $E_t[VaR_{t+1}]$, is given by:

$$\begin{aligned} E_t[VaR_{t+1}] &= E_t[p_{t+1}] - E_t[p_t(1 + r_{t+1}^c)] \\ &= p_t - p_t - E_t[p_t \times r_{t+1}^c] \\ &= -p_t \times E_t[r_{t+1}^c] \end{aligned} \tag{10}$$

because $E_t\left[p_{t+1}\right]=p_t$ and $E_t\left[p_t\right]=p_t$.[2]

**Table 3**
**Expected value of the critical return at time $t$ for different probability levels**

| Expected value of the critical return at time $t$ | Probability level | | |
|---|---|---|---|
| | $\alpha = 1\%$ | $\alpha = 2.5\%$ | $\alpha = 5\%$ |
| $E_t[r_{t+1}^c]$ | $-2.326 \times E_t[\hat{\sigma}_{t+1}]$ | $-1.96 \times E_t[\hat{\sigma}_{t+1}]$ | $-1.645 \times E_t[\hat{\sigma}_{t+1}]$ |

An estimator of $VaR_{t+1}$ for all the chosen probability levels is defined as:

**Table 4**
**Estimator of the Value at Risk for different probability levels**

| Estimator of $VaR_{t+1}$ | Probability level | | |
|---|---|---|---|
| | $\alpha = 1\%$ | $\alpha = 2.5\%$ | $\alpha = 5\%$ |
| $\hat{VaR}_{t+1}$ | $p_t \times 2.326 \times \hat{\sigma}_{t+1}$ | $p_t \times 1.96 \times \hat{\sigma}_{t+1}$ | $p_t \times 1.645 \times \hat{\sigma}_{t+1}$ |

Taking all these aspects into consideration we can conclude that the estimation of *VaR* is dependent on the estimate of the volatility parameter $\sigma_t$. Providing accurate estimates is of crucial importance because poorly estimated risk may lead to a sub-optimal capital allocation with consequences on the profitability or even financial stability of institutions (Manganelli, Engle, 2001).

### *2.2. GARCH-type models*

In order to provide accurate estimates of the volatility parameter we use in our empirical study five volatility measures: (i) the variance given by the RiskMetrics model; (ii) the conditional variance derived from a GARCH(1,1) model; (iii) the conditional variance derived from an EGARCH(1,1) model; (iv) the conditional variance derived from a GARCH-t(1,1) model and (v) conditional variance derived from an EGARCH-t(1,1) model.
The basic GARCH model is based on the assumption of normal distribution for the asset returns and this model is able to capture several *stylized facts* of financial returns

[2] When financial prices are modeled as random walks, as represented in equation (1), it can be easily verified that $E_t[p_{t+1}] = p_t$ and $E_t[p_t] = p_t$.

series like *heteroscedasticity (time-dependent conditional variance)* and *volatility clustering*. One of the primary restrictions of this model is the fact that it enforces a symmetric response of volatility to positive and negative shocks. However, it has been argued that a negative shock to a financial time series is likely to cause volatility to rise more than a positive shock of the same magnitude (Brooks, 2010). Such asymmetries are typically attributed to *leverage effects* and one of the models that can explain these effects is the exponential GARCH (EGARCH) model proposed by Nelson (1991). Another extension of the standard GARCH-type models is to substitute the conditional normal density with a Student's $t$ density in order to allow for excess kurtosis in the conditional distribution (Bollerslev, 1986). As financial time series are generally skewed, the major drawback of the Student's $t$ density is the symmetry.

*2.2.1. The RiskMetrics Model*

In JP Morgan's RiskMetrics system for market risk management, the portfolio returns are generated as follows:

$$r_t = e_t \ , \ e_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \tag{11}$$

where $0 \le \lambda \le 1$ is the smoothing parameter (So, Yu, 2006). The formulation in the mean equation implies that the conditional distribution of returns is normal with zero mean and $\sigma_t^2$ variance. One main characteristic of the RiskMetrics model is that the conditional variance can be written as an exponentially moving average (EWMA) of the past squared returns (innovations):

$$\sigma_t^2 = (1-\lambda)(r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + ...) \tag{12}$$

The smaller the smoothing parameter, the greater is the weight given to recent return data. RiskMetrics found that the estimates were quite similar across assets, and therefore advised that $\lambda = 0.94$ can be used for daily data and $\lambda = 0.97$ for monthly data. One of the most important advantages of the RiskMetrics model is the fact that it tracks variance changes in a way that is broadly consistent with observed returns (Christofersen, 2004).

*2.2.2. GARCH(1,1)*

The GARCH model has been developed independently by Bollerslev (1986) and Taylor (1986). This model allows the conditional variance to be dependent upon previous own lags. The GARCH(1,1) model is defined as:

$$r_t = e_t\,,\ e_t \sim D(0, \sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{13}$$

where $D(0, \sigma_t^2)$ represents a conditional distribution with zero mean and variance $\sigma_t^2$, and $\alpha_0$, $\alpha_1$, $\beta$ are non-negative parameters with the restriction of $\alpha_1 + \beta < 1$ in order to ensure that the conditional variance is positive. In this model the conditional variance can be interpreted as a weighted function of a long term average value (dependent on $\alpha_0$), of the information related to the volatility during the previous period ($\alpha_1 r_{t-1}^2$) and of the variance during the previous period ($\beta\sigma_{t-1}^2$). In academic literature a GARCH(1,1) model is considered to be sufficient in capturing the evolution of the volatility.

### *2.2.3. EGARCH(1,1)*

The exponential GARCH model was proposed by Nelson (1991). There are various ways to express the conditional variance equation, but one possible specification is given by:

$$\ln(\sigma_t^2) = \omega + \beta\ln(\sigma_{t-1}^2) + \gamma\frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha\left[\frac{|r_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \tag{14}$$

This model has several advantages in comparison to the GARCH model because there is no need to artificially impose non-negativity constraints on the model parameters due to the fact that the log form allows the parameters to be negative without conditional variance becoming negative. Moreover, asymmetries are allowed under the EGARCH formulation, because the coefficient $\gamma$ captures the asymmetric impact of news. Thus if the relationship between volatility and returns is negative, then $\gamma$ will be negative, negative shocks having therefore a greater impact than positive shocks of the same magnitude. Moreover, a significant $\alpha$ captures the *volatility clustering* effect and $\omega$ represents the non-conditional variance coefficient. Even though, in the original relationship, Nelson assumed a Generalized Error Distribution (GED) structure for the errors, almost all applications of EGARCH employ conditionally normal errors (Brooks, 2010).

*2.2.4. GARCH-t(1,1) and EGARCH-t(1,1)*

GARCH models are estimated on the assumption that the conditional errors follow a normal distribution, but in the context of stock market returns, the distribution of the errors is often characterized by fat tails or excess kurtosis. One of the extensions of the GARCH models is to substitute the conditional normal density by the Student's $t$ density in order to allow for excess kurtosis in the conditional distribution. Thus, the errors follow a Student's $t$ distribution ($e_t \sim T_v(0, \sigma_t^2)$) instead of a normal distribution ($e_t \sim N(0, \sigma_t^2)$). $T_v(0, \sigma_t^2)$ denotes the Student's $t$ distribution with zero mean, $\sigma_t^2$ variance and $v$ degrees of freedom. The new parameter $v$ determines among other characteristics the kurtosis of the conditional distribution.

*2.3. The performance of the $VaR$ estimates*

In order to asses the overall performance of the Value at Risk estimates we use a performance measure:

$$Performance = |\alpha - \hat{\alpha}| \tag{15}$$

where $\alpha$ represents the probability used in the $VaR$ estimation and $\hat{\alpha}$ represents the sample coverage. The sample coverage $\hat{\alpha}$ represents the proportion of profits/losses of the portfolio ($p_t - p_{t-1}$) greater than the $VaR$ estimates. When $\hat{\alpha}$ is close to $\alpha$ we consider that the $VaR$ estimation method is a good one. Therefore, the smaller discrepancy between $\hat{\alpha}$ and $\alpha$, the better performance is the estimation method (So, Yu, 2006).

**3. Empirical results**

*3.1. Data and descriptive statistics*

We selected three benchmark indexes from three different countries, namely Romania (BET), UK (FTSE100) and USA (S&P500) in order to construct the internationally diversified portfolio. We chose these indexes because FTSE100 and S&P500 represent the most liquid and efficient financial assets in the world (Tavares A.B., Curto, Tavares G.N., 2008). We computed a database containing daily returns over the period January 4, 2005 to June 1, 2010, being registered 1297 observations using the following formula:

$$r_{it} = \ln\left(\frac{p_{it}}{p_{it-1}}\right) \times 100\% \qquad (16)$$

where $r_{it}$ represents the continuously compounded daily return on the index $i$ at time $t$, $p_{it}$ represents the price on the index $i$ at time $t$, with $i = \overline{1,3}$ and $t = \overline{1,1297}$. In the first part of our analysis we used the Markowitz (1952) model of portfolio selection in order to find the optimal weights that minimize the variance of the internationally diversified portfolio. Thus, we obtained the resulting portfolio weights: $0.137$ for the BET index, $0.466$ for the FTSE100 index and $0.397$ for the S&P500 index. In **Table 5** we summarize the descriptive statistics for the indexes and the optimized portfolio.

**Table 5**

**Descriptive statistics**

| | BET | FTSE100 | S&P500 | Portfolio |
|---|---|---|---|---|
| Min | -0.135461 | -0.092646 | -0.094695 | -0.081198 |
| Max | 0.105645 | 0.093842 | 0.109572 | 0.093732 |
| Mean | 0.000011 | 0.000049 | -0.000080 | -0.000017 |
| Standard Deviation | 0.021893 | 0.014304 | 0.015155 | 0.012772 |
| Skewness | -0.665793 | 0.062484 | -0.029192 | -0.101215 |
| Kurtosis | 8.364870 | 11.03812 | 12.24093 | 11.36731 |
| Jarque-Bera | 1651.241 Prob. 0.000000 | 3492.551 Prob. 0.000000 | 4615.061 Prob. 0.000000 | 3785.775 Prob. 0.000000 |

The results show that the riskiest market is the national capital market, the less risky being the chosen portfolio (this result can be attributed to the fact that by using the model developed by Markowitz the variance of the portfolio is minimized). The lowest return is obtained in the case of the S&P500 index, the biggest one being obtained by the FTSE100 index. According to the skewness and kurtosis indexes, all data series are asymmetrical and exhibit excess kurtosis. The Jarque-Bera statistics are highly significant for all return series for a significance level of 1%, being confirmed the assumption that the series are not normally distributed. **Figure 1** below depicts both the daily closing prices of the optimized portfolio and the daily returns.

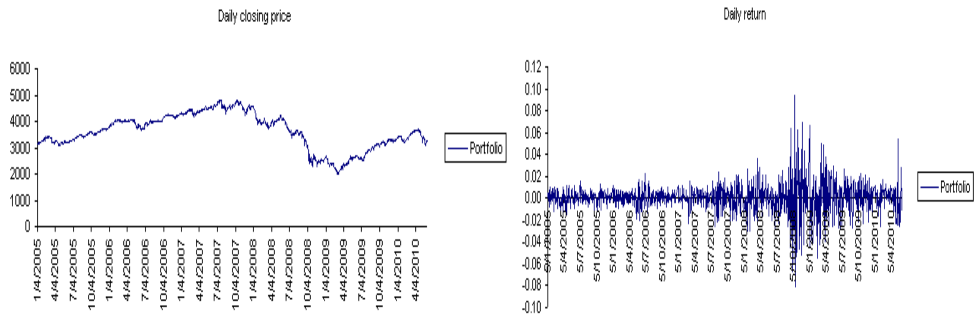


**Figure 1. Plots of the Portfolio-Daily closing prices and returns**

We can observe from the graphic that the returns were fairly stable over the period January 2005 to September 2008. After this date all return series manifested instability especially due to the effects of the global financial crisis. Moreover, we can observe that the series presents a feature specific to nonlinear models, namely *volatility clustering*. Thus, the nonlinear dependencies can be explained by the presence of conditional heteroscedasticity. In order to confirm this conclusion we performed the Engle (1982) test aiming to detect any ARCH effects in the portfolio returns series. According to the test results there are ARCH effects in the series (see **Table 6**). Such behavior can be captured using the GARCH-type models described in **Section 2**.

**Table 6**

**ARCH Test Results**

| Heteroskedasticity Test: ARCH | | | |
|---|---|---|---|
| F-statistic | 76.19271 | Prob. F(5,1285) | 0.0000 |
| Obs*R-squared | 295.2189 | Prob. Chi-Square(5) | 0.0000 |

*3.2. Value at Risk estimation results using GARCH-type models*

We estimated the daily portfolio volatility as an one-step forecast of the GARCH-type models, setting $\sigma_0^2$ (the variance of the first observation) equal to the variance of the entire section of portfolio squared returns and $r_0^2$ (the squared return of the first observation) equal to average return of the entire section of portfolio squared returns.

*3.2.1. Value at Risk estimates based upon RiskMetrics volatility*

The daily estimated volatility of the chosen portfolio was calculated using a value of $0.94$ for the smoothing parameter and as we mentioned in **Section 2.1**, the $VaR$ daily estimates were calculated for three different probability levels, namely: $1\%$, $2.5\%$ and $5\%$ (see **Table 4**). The estimated model conducted to the following result:

$$\hat{\sigma}_t = \sqrt{0.94\sigma_{t-1}^2 + 0.06r_{t-1}^2} \tag{17}$$

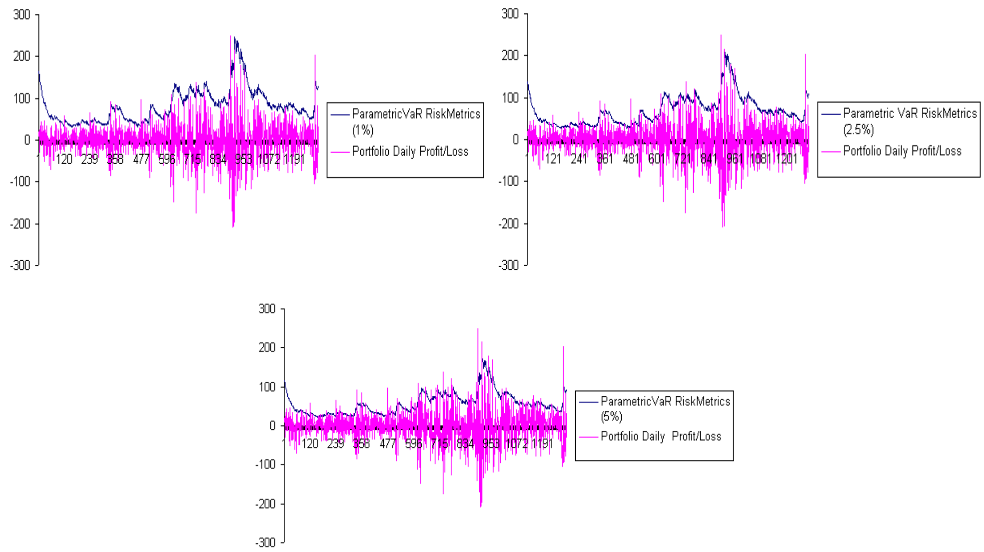


**Figure 2**
**Value at Risk estimates based upon RiskMetrics volatility for different probability levels ($1\%$, $2.5\%$ and $5\%$) and the Portfolio daily Profit/Loss**

In order to asses the overall performance of the Value at Risk estimates we used the performance measure described in equation (15).

**Table 7**
**Performance results of the Value at Risk estimates based upon the RiskMetrics model**

| Model | Probability level $\alpha = 1\%$ | | Probability level $\alpha = 2.5\%$ | | Probability level $\alpha = 5\%$ | |
|---|---|---|---|---|---|---|
| | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* |
| RiskMetrics | 1.5% | 0.5% | 3.4% | 0.9% | 6.6% | 1.6% |

The results show that the sample coverages are very close to their corresponding probability levels, leading to the conclusion that the estimation method leads to a good *Performance* in this case. The smallest discrepancy between the sample coverage and the corresponding probability level is obtained for the $1\%$ level. **Figure 2** depicts the $VaR$ estimated each day using this methodology and the profits/losses that occurred the next day. There are about $1\%$ of times that the Value at Risk is exceeded, a result consistent with the values obtained for the *Performance* measure.

*3.2.2. Value at Risk estimates based upon GARCH(1,1) volatility*

The estimation of a GARCH(1,1) model conducted to the following result:

$$\sigma_t^2 = 0.910640\sigma_{t-1}^2 + 0.086914 r_{t-1}^2 \; {}^{3} \tag{18}$$

The estimated coefficients of this model are highly statistically significant for all relevant significance levels. The coefficient of the conditional variance is 0.91 and this implies that the shocks to the conditional variance are persistent and that large changes in the conditional variance are followed by other large changes and small changes are followed by other small changes. Due to the fact that the variance intercept coefficient $\alpha_0$ is very small we did not take it into consideration in the calculation of the conditional variance. **Figure 3** depicts the daily $VaR$ estimates based upon GARCH(1,1) volatility and the profits/losses that occurred the next day.

[3] Where $\hat{\sigma}_t = \sqrt{\sigma_t^2}$ .

**Figure 3**
**Value at Risk estimates based upon GARCH(1,1) volatility for different probability levels ($1\%$, $2.5\%$ and $5\%$) and the Portfolio daily Profit/Loss**

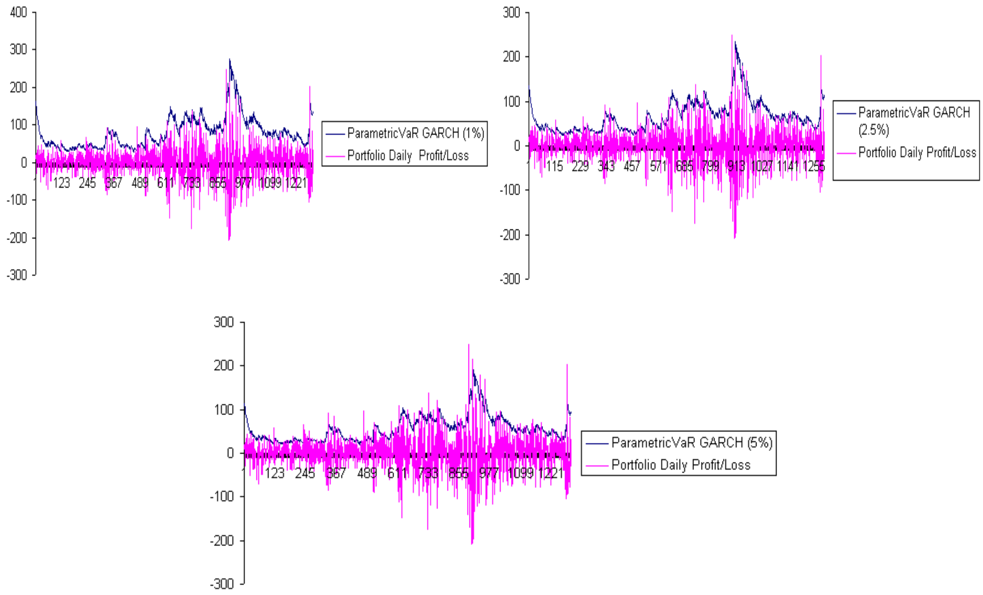


**Table 8**
**Performance results of the Value at Risk estimates based upon the GARCH(1,1) model**

| Model | Probability level $\alpha = 1\%$ | | Probability level $\alpha = 2.5\%$ | | Probability level $\alpha = 5\%$ | |
|---|---|---|---|---|---|---|
| | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* |
| GARCH(1,1) | 2% | 1% | 3.6% | 1.1% | 7% | 2% |

According to the results from **Table 8** the smallest discrepancy between the sample coverage and the corresponding probability level is obtained once again for the 1% $VaR$. As it can be observed also from **Figure 3** the $VaR$ estimated each day using the GARCH(1,1) model represents a good estimation method.

*3.2.3. Value at Risk estimates based upon EGARCH(1,1) volatility*

The estimation of this model conducted to the following result:

$$\ln(\sigma_t^2) = -0.214915 + 0.987445\ln(\sigma_0^2) - 0.099907\frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.126508\left[\frac{|r_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \tag{19}$$

The estimated coefficients of this model are highly statistically significant for all relevant significance levels. The coefficient estimate on $\frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}}$ is negative, suggesting that negative shocks imply a higher next period conditional variance than positive shocks of the same magnitude (the *leverage effect* specific to asymmetric models). This coefficient reflects the asymmetric impact of news on contemporaneous volatility. Moreover, the significance of the $\alpha$ coefficient captures the *volatility clustering* effect.

**Figure 4**
**Value at Risk estimates based upon EGARCH(1,1) volatility for different probability levels ($1\%$, $2.5\%$ and $5\%$) and the Portfolio daily Profit/Loss**

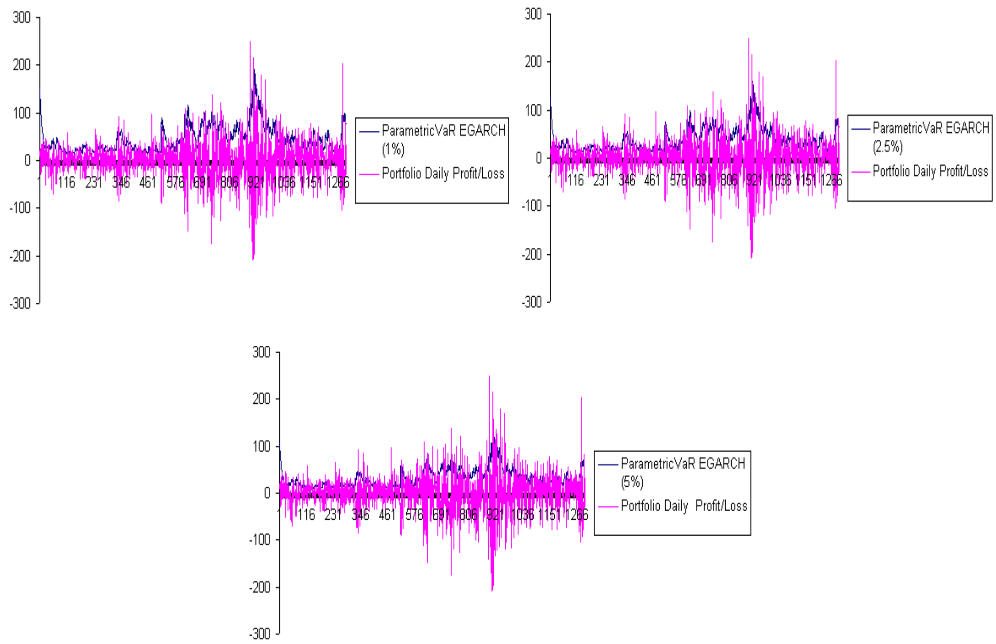

**Table 9**
**Performance results of the Value at Risk estimates based upon the EGARCH(1,1) model**

| Model | Probability level $\alpha = 1\%$ | | Probability level $\alpha = 2.5\%$ | | Probability level $\alpha = 5\%$ | |
|---|---|---|---|---|---|---|
| | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* |
| EGARCH(1,1) | 12.6% | 11.6% | 16.3% | 13.8% | 20.7% | 15.7% |

According to the results from **Table 9** and as it can be observed from **Figure 4** this method does not represent a good estimation method, because the discrepancies between the sample coverages and their corresponding probability levels are very large.

*3.2.4. Value at Risk estimates based upon GARCH-t(1,1) volatility*

The estimation of a GARCH-t(1,1) model conducted to the following result:

$$\sigma_t^2 = 0.908674\sigma_{t-1}^2 + 0.092773r_{t-1}^2 \tag{20}$$

The estimated coefficients of this model are highly statistically significant for the significance levels of 1%, 5% and 10%, with the exception of the variance intercept coefficient $\alpha_0$ that is significant only for $5\%$ and $10\%$ significance levels.

**Figure 5**
**Value at Risk estimates based upon GARCH-t(1,1) volatility for different probability levels ($1\%$, $2.5\%$ and $5\%$) and the Portfolio daily Profit/Loss**

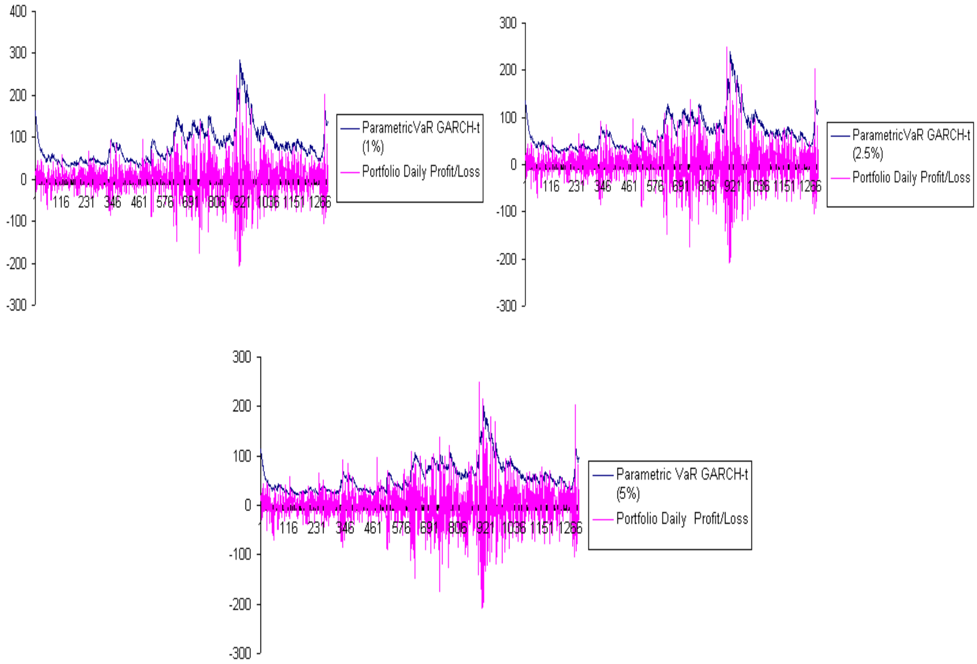


**Table 10**
**Performance results of the Value at Risk estimates based upon the GARCH-t(1,1) model**

| Model | Probability level $\alpha = 1\%$ | | Probability level $\alpha = 2.5\%$ | | Probability level $\alpha = 5\%$ | |
|---|---|---|---|---|---|---|
| | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* |
| GARCH-t(1,1) | 1.5% | 0.5% | 3.1% | 0.6% | 7% | 2% |

The results from **Table 10** show that GARCH-t(1,1) gives better $1\%$ and $2.5\%$ $VaR$ estimates in comparison to GARCH(1,1), but underestimates $5\%$ $VaR$ in comparison to RiskMetrics. The *Performance* results for the GARCH-t(1,1) model are consistent with the graphics depicted in **Figure 5**.

*3.2.5. Value at Risk estimates based on EGARCH-t(1,1) volatility*

The estimation of the model conducted to the following result:

$$\ln(\sigma_t^2) = -0.209312 + 0.989335\ln(\sigma_0^2) - 0.069329\frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.198067\left[\frac{|r_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \tag{21}$$

The estimated coefficients of the model are highly statistically significant for all levels and the coefficient estimate on $\frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}}$ is negative, suggesting that there are asymmetric information effects on the daily portfolio volatility.

**Figure 6**
**Value at Risk estimates based upon EGARCH-t(1,1) volatility for different probability levels ($1\%$, $2.5\%$ and $5\%$) and the Portfolio daily Profit/Loss**

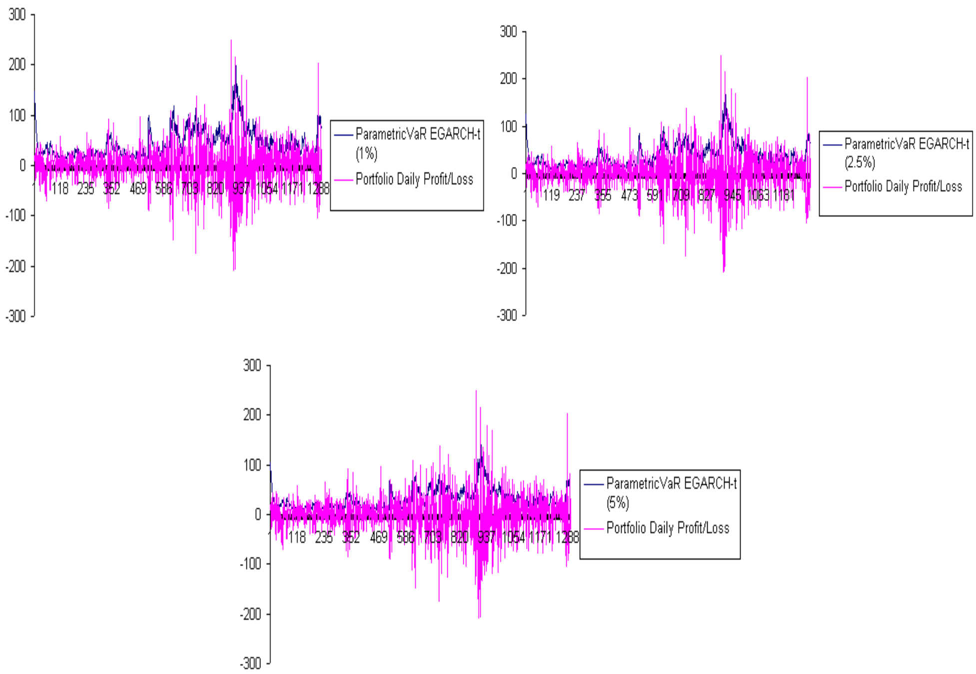

**Table 11**

**Performance results of the Value at Risk estimates based upon the EGARCH-t(1,1) model**

| Model | Probability level $\alpha = 1\%$ | | Probability level $\alpha = 2.5\%$ | | Probability level $\alpha = 5\%$ | |
|---|---|---|---|---|---|---|
| | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* | Sample coverage ($\hat{\alpha}$) | *Performance* |
| EGARCH-t(1,1) | 13.1% | 12.1% | 17.2% | 14.7% | 21.1% | 16.1% |

Although the estimated coefficients of the model are highly statistically significant, as it can be observed from **Figure 6** and also according to the results from **Table 11** this method does not represent a good estimation method, because this model underestimates the $VaR$ for all probability levels.

## 4. Conclusions

We explore in this paper the performance of the daily $VaR$ estimates of an optimized internationally diversified portfolio, estimates based on five volatility measures namely: RiskMetrics, GARCH(1,1), EGARCH(1,1), GARCH-t(1,1) and EGARCH-t(1,1). In order to asses the overall performance of the estimates we use a *Performance* measure, focusing on three probability levels ($1\%$, $2.5\%$ and $5\%$) and we analyze the graphics of the $VaR$ estimated each day and the profits/losses that occurred the next day for all three probability levels. We find that the $VaR$ daily estimates are sensitive to the methodology employed to estimate the daily volatility. Moreover, all symmetric models outperform the asymmetric ones, both for the normal and Student's $t$ distributions, because the discrepancies between the probability levels and the sample coverages are very large. We also find that GARCH(1,1) underestimates 1% $VaR$ in comparison to RiskMetrics and GARCH-t(1,1), that perform very well. Moreover, GARCH-t(1,1) gives better 2.5% $VaR$ estimates and RiskMetrics outperforms GARCH(1,1) and GARCH-t(1,1) for 5% $VaR$ estimates.

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