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AN INNOVATIVE MULTI-CRITERIA SUPPLIER SELECTION BASED ON TWO-TUPLE MULTIMOORA AND HYBRID DATA

Abstract. *In this study, a multi-criteria decision making method MULTIMOORA (Multi-Objective Analysis by Ratio Analysis plus the Full Multiplicative Form) is extended to tackle fuzzy supplier selection problem, which is an important part of supply chain management model. More specifically, this study is aimed at extending MULTIMOORA with 2–tuple linguistic representation method. Hence, two–tuples are used to represent, convert and map into the basic linguistic term set various crisp and fuzzy numbers. Consequently, the fusion of crisp, fuzzy, and linguistic variables was performed when assessing the suppliers. The application of the new method was successful. Moreover, the indicator system can be customized according to the needs of certain decision maker, thus making MULTIMOORA–2T a powerful tool for such practices as e-commerce, e-procurement, and innovative procurements.*
Key Words: *MCDM, MULTIMOORA, MULTIMOORA–2T, 2–tuple linguistic representation, hybrid data, fuzzy number, uncertainty, supply chain management, supplier selection, innovative procurements.*

JEL Classification: C44, D81, M10.

1. INTRODUCTION

Supplier selection is an important part of supply chain management model (Kothari *et al.*, 2005). Moreover, the recent developments of information technologies pushed forward such practices as e-commerce, e-procurement, and innovative procurement, which, in turn, require sophisticated decision aiding tools. In this paper, a multi-criteria decision making (MCDM) method MULTIMOORA (Multi-Objective Analysis by Ratio Analysis plus the Full Multiplicative Form) is extended to handle these problems. Indeed, Degraeve and Roodhooft (2001) argue that purchasing products and services accounts for more than 60 per cent of the average company's total costs. Supplier selection, therefore, becomes an actual issue. Given supplier selection is a complex problem, there were many multi-criteria methods offered to deal with the issue (Amin *et al.*, 2011; Aissaoui *et al.*, 2007; Amin, Razmi, 2009;

Bevilacqua *et al.*, 2006; Chou, Chang, 2008; De Boer *et al.*, 2001; Demirtas, Ustun, 2008; Kahraman *et al.*, 2003; Liao, Rittscher, 2007; Xia, Wu, 2007). However, the robust decision of supplier selection requires the fusion of internal, external, objective, subjective, quantitative, and qualitative variables (Ulubeyli *et al.*, 2010; Zavadskas *et al.*, 2010a, 2010b; Plebankiewicz, 2010). For instance, price of goods can be represented in ordinary real numbers, whereas ratings for overseas partners can be obtained in linguistic form. Moreover, fuzzy number can store a wide variety of information (Behret, Kahraman, 2010; Zhang, Liu, 2010). Consequently, the two–tuple linguistic representation will be used when dealing with such granularity of uncertainty.

Herrera *et al.* (2000) contributed to the computing with words by introducing two–tuple linguistic representation approach. Two–tuples are used to represent, convert and map into the basic linguistic term set various crisp and fuzzy numbers. As 2–tuple linguistic is a powerful MCDM tool, many studies aimed at application and development of the method are present (Chen, Ben–Arieh, 2006; Martinez *et al.*, 2007; Tai, Chen, 2009; Halouani *et al.*, 2009; Wang, 2009; Liu, 2009; Wei, 2011; Li, 2009; Dursun, Karsak, 2010; Chang, Wen, 2010). This study is aimed at extending MULTIMOORA with 2–tuple linguistic representation method.

Brauers and Zavadskas (2006) introduced Multi-Objective Optimization by Ratio Analysis (MOORA). In 2010 these authors developed this method further under the name of MULTIMOORA. Numerous examples of application of these methods are present (Brauers *et al.*, 2007, 2008, 2010; Brauers and Ginevičius, 2009, 2010; Brauers and Zavadskas, 2009a, 2009b; Baležentis *et al.*, 2010; Chakraborty, 2010; Kracka *et al.*, 2010). This article presents MULTIMOORA–2T (2–tuple MULTIMOORA) and its application for supplier selection by performing fusion of hybrid data expressed in crisp, interval, and fuzzy number.

The article, hence, is organized as follows. Section 2 describes the basics of fuzzy number theory and 2–tuple linguistic representation method. The next Section 3 is focussed on the new MULTIMOORA–2T method as well as the crisp MULTIMOORA. Finally, a numerical simulation is offered in Section 4.

2. PRELIMINARIES

In this section we shall briefly describe the fuzzy number theory and 2–tuple linguistic representation model. The first method enables to express and evaluate uncertainty of the investigated phenomenon, whereas the second method is aimed at fusion of these data.

2. 1. Fuzzy number

Zadeh (1965) introduced the use of fuzzy set theory when dealing with problems involving fuzzy phenomena. Noteworthy, fuzzy sets and fuzzy logic are powerful mathematical tools for modeling uncertain systems (Badescu *et al.*, 2010; Turskis, Zavadskas, 2010a, 2010b). A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership, while fuzzy sets allow partial membership. The theoretical fundaments of fuzzy set theory are overviewed by Chen (2000).

In a universe of discourse *X*, a fuzzy subset $\tilde{A}$ of X is defined with a membership function $\mu_{\tilde{A}}(x)$ which maps each element $x \in X$ to a real number in the interval [0; 1]. The function value of $\mu_{\tilde{A}}(x)$ resembles the grade of membership of *x* in $\tilde{A}$. The higher the value of $\mu_{\tilde{A}}(x)$, the higher the degree of membership of *x* in $\tilde{A}$ (Keufmann and Gupta, 1991). Noteworthy, in this study any variable with tilde will denote a fuzzy number.

A fuzzy number $\tilde{A}$ is described as a subset of real number whose membership function $\mu_{\tilde{A}}(x)$ is a continuous mapping from the real line $\Re$ to a closed interval [0; 1], which has the following characteristics: 1) $\mu_{\tilde{A}}(x) = 0$, for all $x \in (-\infty; a] \cup [c; \infty)$; 2) $\mu_{\tilde{A}}(x)$ is strictly increasing in [a; b] and strictly decreasing in [d; c]; 3) $\mu_{\tilde{A}}(x) = 1$, for all $x \in [b; d]$, where *a*, *b*, *d*, and *c* are real numbers, and $-\infty < a \le b \le d \le c < \infty$. When *b* = *d* a fuzzy number $\tilde{A}$ is called a triangular fuzzy number represented by a triplet (a, b, c).

Triangular fuzzy numbers will therefore be used in this study to characterize the alternatives. The membership function $\mu_{\tilde{A}}(x)$ is thus defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{x-c}{b-c}, & b \le x \le c, \\ 0, & x > c. \end{cases} \tag{1}$$

In addition, the parameters *a*, *b*, and *c* in (1) can be considered as indicating respectively the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event (Torlak *et al.*, 2011: 3). Moreover, the robustness as well as precision of multi–criteria optimization can be improved by applying either intuitionistic fuzzy numbers (Zhang, Liu, 2010) or 2–tuple linguistic representation (Liu, 2009).

2. 2. Two–tuple linguistic representation

After introducing fuzzy set theory, Zadeh (1975) described the fuzzy linguistic variables. The linguistic variables are very useful when describing various vague phenomena, which cannot be reasonably expressed in ordinary quantitative terms (Wang, 2009). Indeed, linguistic terms are often peculiar with finite set, odd cardinality, semantic symmetric, ordinal level, and compensative operation (Herrera-Viedma *et al.*, 2003). Consequently, Herrera and Martinez (2000a, 2000b, 2001) developed the 2–tuple linguistic representation model with various aggregation operators (Herrera *et al.*, 2000). The main advantage of such representation is the continuity of its domain. Hence any counting of information might be expressed in the universe of discourse. Moreover appropriate techniques prevents from loss of information during computing with words.

The linguistic information is expressed in a pair of values—2–tuple—consisting linguistic term and a number. Let us take, for instance, a 2–tuple $L = (s, \alpha)$, where *s* stands for the linguistic label of the information and α represents the symbolic translation. Actually, one can define any ordered set of linguistic terms $S_{g+1} = (s_0, s_1, \ldots s_i, \ldots, s_g)$, containing $g+1$ labels. As it was mentioned before, there should be odd cardinality, namely $g+1$. Let the set S_{g+1} has the following characteristics (Martinez *et al.*, 2007): 1) a negation operator $Neg(s_i) = s_j$ such that $j = g - i$; 2) a *min* and *max* operators, i. e. $s_i \le s_j \Leftrightarrow i \le j$, where $i, j \in [0, g]$. It is considered, that seven or so linguistic terms can be effectively applied (Miller, 1956). Any label $s_i = (a_i, b_i, c_i)$ can be defined in the following way (Liu, 2009):

$$\begin{cases} a_o = 0; \\ a_i = \dfrac{i-1}{g}, 1 \le i \le g; \\ b_i = \dfrac{i}{g}, 0 \le i \le g; \\ c_i = \dfrac{i+1}{g}, 0 \le i \le g-1; \\ c_g = 0. \end{cases} \tag{2}$$

However, different decision makers can use different scales (so called granularity of uncertainty), which need to be mapped onto single basic linguistic term set (BLTS) S_T. The latter set should contain the maximum number of labels if compared to scales used by different decision makers.

Definition 1. Let $\beta \in [0, g]$ be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S_{g+1}, i. e., the result of a symbolic aggregation operation. Let $i = round(\beta)$ and $\alpha = \beta - 1$ be two values, such that $i \in [0; g]$ and $\alpha \in [-0.5, 0.5)$, then α is called a symbolic translation (Herrera *et al.*, 2005; Wei, 2011).

Then linguistic representation model handles the linguistic evaluations by means of 2–tuples (s_i, α_i), where $s_i \in S_{g+1}$ and $\alpha_i \in [-0.5, 0.5)$.

Definition 2. Let $S_{g+1} = (s_0, s_1, ..., s_g)$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of symbolic aggregation operation. Then the following function returns the respective 2–tuple:

$$\Delta : [0, g] \to S_{g+1} \times [-0.5, 0.5)$$
$$\Delta(\beta) = \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases} \tag{3}$$

where *round* is the usual rounding operation, s_i has the closest index value to β, and α is called a symbolic translation (Herrera *et al.*, 2000).

Given Definitions 1 and 2, and if S_{g+1} is the BLTS, the linguistic term of BLTS may be represented by respective 2–tuple:

$$s_i \in S_{g+1} \overset{\Delta}{\Rightarrow} (s_i, 0) \tag{4}$$

Definition 3. Let $S_{g+1} = (s_0, s_1, ..., s_g)$ be a linguistic term set and (s_i, α_i) be a 2–tuple. There exists a function Δ^{-1} which, according to 2–tuple, returns its equivalent value $\beta \in [0, g] \subset \mathbb{R}$ (Herrera *et al.*, 2000; Wei, 2011):

$$\begin{aligned} &\Delta^{-1} : S_{g+1} \times [-0.5, 0.5) \to [0, g] \\ &\Delta^{-1}(s_i, \alpha_i) = i + \alpha = \beta \end{aligned} \tag{5}$$

Definition 4. Let (s_k, α_k) and (s_l, α_l) be two 2–tuples. Then (Herrera *et al.*, 2000):

If $k < l$, then $(s_k, \alpha_k) \prec (s_l, \alpha_l)$.

If $k = l$, then a) if $a_k = a_l$, then $(s_k, \alpha_k) \sim (s_l, \alpha_l)$;

b) if $a_k < a_l$, then $(s_k, \alpha_k) \prec (s_l, \alpha_l)$;

c) if $a_k > a_l$, then $(s_k, \alpha_k) \succ (s_l, \alpha_l)$.

Definition 5. A 2–tuple negation operator is the following (Herrera *et al.*, 2005):

$$Neg(s_i, \alpha_i) = \Delta\left(g - \left(\Delta^{-1}(s_i, \alpha_i)\right)\right) \tag{6}$$

Definition 6. Let $x = \{(r_1, \alpha_1), (r_2, \alpha_2), ..., (r_n, \alpha_n)\}$ be a set of 2–tuples from S_{g+1}. Then the 2–tuple arithmetic average is obtained as (Herrera, Martinez, 2000a, 2000b):

$$\bar{x} = (\bar{r}, \bar{a}) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(r_j, a_j)\right), \bar{r} \in S_{g+1}, \bar{a} \in [-0.5, 0.5) \tag{7}$$

Definition 7. Let $L_k = (s_k, \alpha_k)$ and $L_l = (s_l, \alpha_l)$ be two 2–tuples, then

$$d\left(L_k, L_l\right) = \Delta \,|\, \Delta^{-1}(s_k, \alpha_k) - \Delta^{-1}(s_l, \alpha_l) \,| \tag{8}$$

is called the distance between L_k and L_l (Herrera, Martinez, 2000a, 2000b).

Definition 8. Let $x = \{(r_1, \alpha_1), (r_2, \alpha_2), ..., (r_n, \alpha_n)\}$ be a set of 2–tuples from S_{g+1}. Then the 2–tuple geometric average is computed in the following way:

$$\prod_{j=1}^{n}(r_j, a_j) = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(r_j, a_j)\right)^{\frac{1}{n}}\right) \tag{9}$$

Definiton 9. Let *I* be real number, interval number, triangular fuzzy number, or trapezoidal fuzzy number etc., and $S_{g+1} = (s_0, s_1, ..., s_g)$ be linguistic term set (Gong, 2007; Gong, Liu, 2007; Liu, 2009). Thereafter *I* can be converted into 2–tuple linguistic set by the following mapping:

$$\begin{aligned} &\tau : [0,1] \to F(S_{g+1}) \\ &\tau(I) = \left\{(s_i, \alpha_i) \mid i \in [0, 1, ..., g]\right\} \\ &\alpha_i = \max_y \min\left\{\mu_I(y), \mu_{s_i}(y)\right\} \end{aligned} \tag{10}$$

where $\mu_I(y)$ and $\mu_{s_i}(y)$ are membership functions associated with *I* and s_i, respectively (Fig. 1).

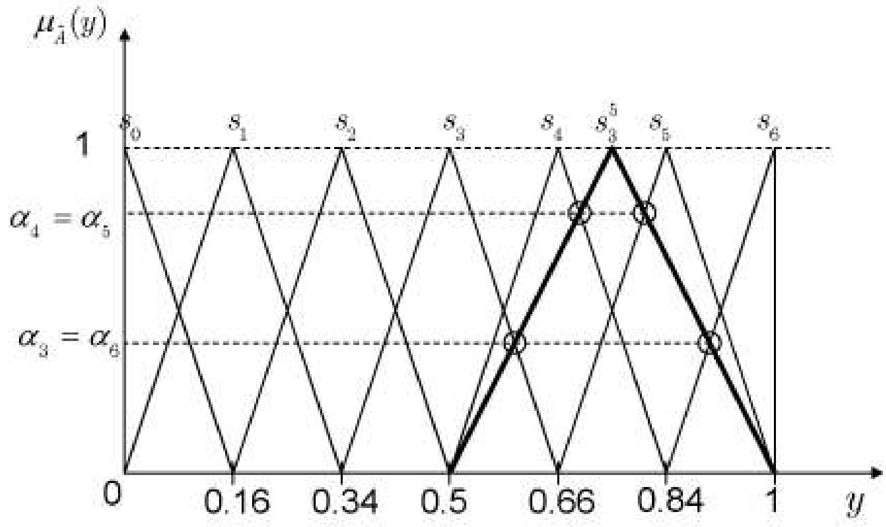

Figure 1 – Fuzzy linguistic term s_3^5 mapped into 7–label BLTS

Definition 10. Let $\tau(I)=\{(s_i,\alpha_i)\,|\,i\in[0,1,...,g]\}$ be 2–tuple linguistic value of the uncertain fuzzy number *I*, then 2–tuple linguistic set $\tau(I)$ can be converted into 2–tuple linguistic variable by mapping χ :

$$\chi : F(S_{g+1}) \rightarrow [0,g]$$

$$\chi\left(\tau(I)\right)=\chi\left(F(S_{g+1})\right)=\chi\left[(s_i,\alpha_i)\,|\,i\in[0,1,...,g]\right]=\sum_{i=0}^{g} i\alpha_i \Big/ \sum_{i=0}^{g}\alpha_i=\beta \tag{11}$$

3. THE MULTIMOORA METHOD

The following section defines the crisp MULTIMOORA method and the new 2–tuple based MULTIMOORA.

3. 1. The crisp MULTIMOORA

As already said earlier, Multi-Objective Optimization by Ratio Analysis (MOORA) method was introduced by Brauers and Zavadskas (2006). Brauers and Zavadskas (2010) extended the method into MULTIMOORA (MOORA plus the Full Multiplicative Form). These methods have been applied in numerous studies (Brauers *et al.*, 2007, 2010; Brauers, Ginevičius, 2009, 2010; Brauers, Zavadskas, 2009a, 2009b; Baležentis *et al.*, 2010) focused on regional studies, international comparisons and investment management.

MOORA method begins with matrix X where its elements x_{ij} denote i^{th} alternative of j^{th} objective ($i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$). MOORA method consists of two parts: the ratio system and the reference point approach. MacCrimmon (1968) defines two stages of *weighting*, namely *normalization* and voting on *significance* of objectives. The issue of weighting is discussed by Brauers, Zavadskas (2010: 10), Zavadskas *et al.* (2010b), while the problem of normalization is analyzed by Brauers (2007), Peldschus *et al.* (2010), and Turskis *et al.* (2009). The MULTIMOORA method includes internal normalization and treats originally all the objectives equally important. In principle all stakeholders interested in the issue only could give more importance to an objective. Therefore they could either multiply the dimensionless number representing the response on an objective with a significance coefficient or they could decide beforehand to split an objective into different sub-objectives (Brauers, Ginevičius, 2009: 124).

The Ratio System of MOORA. Ratio system defines data normalization by comparing alternative of an objective to all values of the objective:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{12}$$

where x_{ij}^* denotes i^{th} alternative of j^{th} objective. Usually these numbers belong to the interval [-1; 1]. These indicators are added (if desirable value of indicator is a maximum) or subtracted (if desirable value is a minimum) and summary index of an alternative is derived in this way:

$$y_i^* = \sum_{j=1}^{g} x_{ij}^* - \sum_{j=g+1}^{n} x_{ij}^* \tag{13}$$

where $g = 1, \cdots, n$ denotes number of objectives to be maximized. Then every ratio is given the rank: the higher the index, the higher the rank.

The Reference Point of MOORA. Reference point approach is based on the Ratio System. The Maximal Objective Reference Point (vector) is found according to ratios found in formula (9). The j^{th} coordinate of the reference point can be described as $r_j = \max_i x_{ij}^*$ in case of maximization. Every coordinate of this vector represents maxima or minima of certain objective (indicator). Then every element of normalized responses matrix is recalculated and final rank is given according to deviation from the reference point and the Min-Max Metric of Tchebycheff:

$$\min_i \left(\max_j \left| r_j - x_{ij}^{*} \right| \right). \tag{14}$$

The Full Multiplicative Form and MULTIMOORA. Brauers and Zavadskas (2010: 13–14) proposed MOORA to be updated by the Full Multiplicative Form method embodying maximization as well as minimization of purely multiplicative utility function. Overall utility of the i^{th} alternative can be expressed as dimensionless number:

$$U_i^{'} = \frac{A_i}{B_i}, \tag{15}$$

where $A_i = \prod_{j=1}^{g} x_{ij}$, $i = 1, 2, \cdots, m$ denotes the product of objectives of the i^{th} alternative to be maximized with $g = 1, \cdots, n$ being the number of objectives (indicators) to be maximized and

where $B_i = \prod_{j=g+1}^{n} x_{ij}$ denotes the product of objectives of the i^{th} alternative to be minimized with $n - g$ being the number of objectives (indicators) to be minimized. Thus MULTIMOORA summarizes MOORA (i. e. Ratio System and Reference point) and the Full Multiplicative Form. Brauers and Zavadskas (2011) developed the dominance theory to tackle the latter issue. Meanwhile, Ameliorated Nominal Group and Delphi techniques can also be used to reduce remaining subjectivity (Brauers and Zavadskas, 2010: 17–19).

As one can note, the Reference Point prevents the MULTIMOORA from becoming a fully compensatory technique. Whereas the Ratio System and the Full Multiplicative Form are fully compensatory methods, the Reference Point is not one. For the latter method is based on Min–Max metric of Tchebycheff, which identifies certain alternatives peculiar with relative backwardness in either of criteria. Hence, the MULTIMOORA is quite an effective tool for assessing sustainability of various phenomena resulting in unbiased ranking of alternatives.

3. 2. The proposed two–tuple MULTIMOORA method

This subsection describes the MULIMOORA extended with 2–tuple linguistic representation (MULTIMOORA–2T). The new method is aimed at fusion of hybrid data, namely 1) real number; 2) interval number; 3) fuzzy number and linguistic variables. Hence, the MULTIMOORA–2T will be able to handle both objective and subjective criteria.

Data fusion. Let $A=(a_1,a_2,...,a_m)$ be the set of alternatives considered with respect to criteria $C=(c_1,c_2,...,c_n)$. Additionally, let $J_1 \subset C$ and $J_2 \subset C$ be subsets of benefit and cost criteria, respectively. The initial data are pooled in the decision matrix $X=x_{ij}$, where $i=1,2,...,m$ and $j=1,2,...,n$. Suppose $x_{ij} \mid j=1,2,...,r_1$ is real number; $x_{ij}=[x_{ij}^a,x_{ij}^d] \mid j=r_1+1,r_1+2,...,r_2$ is interval number; $x_{ij}=\left(x_{ij}^a,x_{ij}^b,x_{ij}^d\right) \mid j=r_2+1,r_2+2,...,r_3$ is triangular fuzzy number; $x_{ij}=s_{ij} \in S_{g+1} \mid j=r_3+1,r_3+2,...,r_4$ is linguistic variable; $x_{ij}=(x_{ij}^a,x_{ij}^b,x_{ij}^c,x_{ij}^d) \mid j=r_4+1,r_4+2,...,n$ is trapezoidal fuzzy number.

First of all, the initial data need to be normalized and summarized in the normalized decision matrix $B=b_{ij}$ (Liu, Liu, 2010):

$$b_{ij}=x_{ij}\Big/\sqrt{\sum_{i=1}^{m}x_{ij}^2},\ \forall j\in[1,2,...,r_1]; \tag{16}$$

$$b_{ij}=[b_{ij}^a,b_{ij}^d]=\begin{cases} b_{ij}^a=x_{ij}^a\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^a)^2+(x_{ij}^d)^2\right]} \\ b_{ij}^d=x_{ij}^d\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^a)^2+(x_{ij}^d)^2\right]} \end{cases},\ \forall j\in[r_1+1,r_1+2,...,r_2]; \tag{17}$$

$$b_{ij}=b_{ij}=(b_{ij}^a,b_{ij}^b,b_{ij}^d)=\begin{cases} b_{ij}^a=x_{ij}^a\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^a)^2+(x_{ij}^b)^2+(x_{ij}^d)^2\right]} \\ b_{ij}^d=x_{ij}^b\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^a)^2+(x_{ij}^b)^2+(x_{ij}^d)^2\right]} \\ b_{ij}^d=x_{ij}^d\Big/\sqrt{\sum_{i=1}^{m}\left[(x_{ij}^a)^2+(x_{ij}^b)^2+(x_{ij}^d)^2\right]} \end{cases},\ \forall j\in[r_2+1,r_2+2,...,r_3]. \tag{18}$$

The normalization of trapezoidal fuzzy number can be carried out by extending Eq. (19) with additional variable (Liu, Liu, 2010). Linguistic variables computed according to Eq. (2) do not need to be normalized.

Secondly, we have to choose the BLTS, namely $S_T = (s_0, s_1, ..., s_g)$. Usually, set with maximum granularity is chosen from the applied linguistic sets (Herrera *et al.*, 2005). Then each response b_{ij} is converted into 2–tuple $t_{ij} = (s_k, \alpha)_{ij}, s_k \in S_T, \alpha \in [-0.5, 0.5)$ by employing Eq. (11), (10), and (3):

$$t_{ij} = \Delta\left(\chi\left(\tau_{S_{g+1}S_T}\left(b_{ij}\right)\right)\right) = (s_k, \alpha)_{ij}, \forall i, j. \tag{19}$$

Subsequently, a negation operator is used according with Eq. (6) to transform cost criteria into benefit ones:

$$u_{ij} = \begin{cases} t_{ij}, \forall j \in J_1 \\ Neg(t_{ij}), \forall j \in J_2 \end{cases} \tag{20}$$

As a result, the transformed normalized decision matrix $U = u_{ij}$ is formed. Now we may proceed with aggregation of responses.

The Ratio System of MULTIMOORA–2T. The arithmetic mean will be calculated instead of simple sum of responses, since the sum could not be expressed in 2–tuples. The Eq. (7), hence, is employed:

$$y_i = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(s_k, \alpha)_{ij}\right), i = 1, 2, ..., m \tag{21}$$

where y_i stands for summarizing ratio of the i^{th} alternative. The alternatives with higher values of y_i are given higher ranks.

The Reference Point of MULTIMOORA–2T. The maxima of every criteria is found according to Definition 4. However, application of Δ^{-1} function would return the same results. The alternatives, therefore, are ranked by applying Min–Max metric and Eq. (8):

$$\min_i\left(\max_j d(u_{ij}, \max_i u_{ij})\right), \forall i, j. \tag{22}$$

The Full Multiplicative Form of MULTIMOORA–2T. Again, the geometric mean will be calculated instead of simple product, since the latter could not be successfully expressed in the 2–tuple form. As a result, the Eq. (9) is applied:

$$U_i = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(u_{ij})\right)^{\frac{1}{n}}\right), i = 1, 2, ..., m. \tag{23}$$

Alternatives with higher values of U_i are attributed with higher ranks. The final ranks for each alternative are provided according to the dominance theory (Brauers, Zavadskas, 2011).

4. A NUMERICAL EXAMPLE: SUPPLIER SELECTION CASE

Supplier selection is a complex problem due to its uncertainty involving subjective and objective, crisp and fuzzy information. Hence, many authors tried to offer MCDM tools for costs reduction and risk management (Amin *et al.,* 2011; De Boer *et al.*, 2001).

In this particular case, an enterprise decides on choosing the best supplier from four candidates. The following criteria are taken into consideration: product price, product quality, time of delivery, percentage of on–time deliveries, required payment in advance, remoteness of the facilities (location), and credibility of supplier. The price is expressed in Euro per unit (crisp number) and should be minimized. The product quality is identified by percentage of non–damaged goods in delivery (interval number). The time of delivery (TOD) is measured in working days (interval number). Percentage of on–time deliveries is expressed as triangular number (to be maximized). The location of supplier is evaluated by linguistic terms expressed in five–point linguistic scale (to be minimized). The rating of certain supplier provided by corresponding banks or trade insurance companies is expressed in seven–point linguistic scale and should be maximized. The payment in advance is measured in per cent of account payable to be settled before delivery and expressed by means of trapezoidal fuzzy number (to be minimized). Table 1 summarizes linguistic variables, whereas Table 2 exhibits initial decision matrix.

Table 1. The two linguistic term sets applied in the decision making.

<table>
<tr><td>Linguistic term set</td><td colspan="7">None … Perfect</td></tr>
<tr><td>S_5</td><td>(0, 0, .25)</td><td colspan="2">(0, .25, .5)</td><td>(.25, .5, .75)</td><td>(.5, .75, 1)</td><td colspan="2">(.75, 1, 1)</td></tr>
<tr><td>S_7</td><td>(0, 0, .16)</td><td>(0, .16, .34)</td><td>(.16, .34, .5)</td><td>(.34, .5, .66)</td><td>(.5, .66, .84)</td><td>(.66, .84, 1)</td><td>(.84, 1, 1)</td></tr>
</table>

Table 2. Initial decision matrix *X*.

Supplier	Price (EUR)	Quality (per cent)	TOD (days)	On–time (per cent)	Payment (per cent)	Location (S_5)	Rating (S_7)
	min	max	min	max	min	min	max
A	0.49	[95, 99]	[5, 14]	(90, 95, 99)	(50, 60, 70, 80)	s_4^5	s_6

B	0.45	[89, 93]	[5, 7]	(80, 90, 95)	(60, 70, 72, 75)	s_2^5	s_3
C	0.40	[92, 96]	[3, 10]	(90, 92, 94)	(40, 60, 65, 70)	s_3^5	s_5
D	0.37	[95, 98]	[10, 15]	(85, 89, 95)	(65, 70, 75, 78)	s_3^5	s_4

The initial data were normalized according to Eqs. (16) – (18). Thereafter, all the variables lie in the interval of [0, 1] (Table 2).

Table 3. Normalized decision matrix *B*.

Supplier	Price (EUR)	Quality (per cent)	TOD (days)	On–time (per cent)	Payment (per cent)	Location (S_5)	Rating (S_7)
	min	max	min	max	min	min	max
A	0.66	[.35, .37]	[.19, .52]	(.28, .3, .31)	(.19, .22, .26, .3)	s_4^5	s_6
B	0.61	[.33, .35]	[.19, .26]	(.25, .28, .3)	(.22, .26, .27, .28)	s_2^5	s_3
C	0.54	[.34, .36]	[.11, .37]	(.28, .29, .3)	(.15, .22, .24, .26)	s_3^5	s_5
D	0.50	[.35, .37]	[.37, .56]	(.27, .28, .3)	(.24, .26, .28, .29)	s_3^5	s_4

Now the data fusion began with application of Eq. (10). As a result, a series of 2–tuples were obtained. The series were aggregated into single value β by employing Eq. (11). Finally, Eq. (3) enabled to retrieve the final linguistic 2–tuples belonging to the target BLTS, namely S_7 (Table 4). The ratings of supplier credibility were already expressed in terms of S_7, hence Eq. (4) was valid for them. For instance, the normalized response b_{36}—a linguistic variable from S_5 represented by triangular fuzzy number (0.5, 0.75, 1)—was translated into BLTS 2–tuple in the following way (Figure 1):

$$\tau(b_{36}) = \tau(s_3^5) = \{(s_0,0),(s_1,0),(s_2,0),(s_3,0.39),(s_4,79),(s_5,79),(s_6,0.39)\};$$

$$\chi\left(\tau(s_3^5)\right) = \frac{3\times 0.39 + 4\times 0.79 + 5\times 0.79 + 6\times 0.39}{0.39+0.79+0.79+0.39} = 4.5;$$

$$\Delta\left(\chi\left(\tau(s_3^5)\right)\right) = \Delta\left(4.5\right) = (s_5,-0.5).$$

Table 4. Normalized values expressed in 2–tuples t_{ij}.

Supplier	Price (EUR)	Quality (per cent)	TOD (days)	On–time (per cent)	Payment (per cent)	Location (S_5)	Rating (S_7)
	min	max	min	max	min	min	max
A	$(s_4, 0.01)$	$(s_2, 0.17)$	$(s_2, 0.12)$	$(s_2, -0.26)$	$(s_2, -0.45)$	$(s_5, 0.43)$	$(s_6, 0)$
B	$(s_4, -0.32)$	$(s_2, 0.01)$	$(s_1, 0.39)$	$(s_2, -0.36)$	$(s_2, -0.46)$	$(s_3, 0)$	$(s_3, 0)$
C	$(s_3, 0.26)$	$(s_2, 0.11)$	$(s_1, 0.43)$	$(s_2, -0.29)$	$(s_1, 0.35)$	$(s_5, -0.5)$	$(s_5, 0)$
D	$(s_3, 0)$	$(s_2, 0.15)$	$(s_3, -0.22)$	$(s_2, -0.34)$	$(s_2, -0.42)$	$(s_5, -0.5)$	$(s_4, 0)$

Since certain indicators need to be minimized, they are converted into benefit criteria according – a transformed normalized decision matrix *U* (Table 5) is formed according to Eq. (20). Coordinates of the maximal objective reference point were found according to Definition 4.

Table 5. Transformed normalized decision matrix *U* and maximal objective reference point.

Supplier	Price (EUR)	Quality (per cent)	TOD (days)	On–time (per cent)	Payment (per cent)	Location (S_5)	Rating (S_7)
	max	max	max	max	max	max	max
A	$(s_2, -0.01)$	$(s_2, 0.17)$	$(s_4, -0.12)$	$(s_2, -0.26)$	$(s_4, 0.45)$	$(s_1, -0.43)$	$(s_6, 0)$
B	$(s_2, 0.32)$	$(s_2, 0.01)$	$(s_5, -0.39)$	$(s_2, -0.36)$	$(s_4, 0.46)$	$(s_3, 0)$	$(s_3, 0)$
C	$(s_3, -0.26)$	$(s_2, 0.11)$	$(s_5, -0.43)$	$(s_2, -0.29)$	$(s_5, -0.35)$	$(s_2, -0.5)$	$(s_5, 0)$
D	$(s_3, 0)$	$(s_2, 0.15)$	$(s_3, 0.22)$	$(s_2, -0.34)$	$(s_4, 0.42)$	$(s_2, -0.5)$	$(s_4, 0)$
$\max_i u_{ij}$	$(s_3, 0)$	$(s_2, 0.17)$	$(s_5, -0.39)$	$(s_2, -0.26)$	$(s_5, -0.35)$	$(s_3, 0)$	$(s_6, 0)$

The considered suppliers were ranked by the Ratio System, the Reference Point, and the Full Multiplicative Form according to Eqs. (21), (22), and (23), respectively (Table 6).

Table 6. Results of the MULTIMOORA–2T for supplier selection.

Supplier	Ratio System		Reference Point		Full Multiplicative Form		Final Rank
	y_i	Rank	$\max_j d_{ij}$	Rank	U_i	Rank	
A	$(s_3,-0.03)$	3	$(s_2,0.43)$	3	$(s_2,0.39)$	4	4
B	$(s_3,0.01)$	2	$(s_3,0)$	4	$(s_3,-0.18)$	2	2
C	$(s_3,0.18)$	1	$(s_2,-0.5)$	1	$(s_3,-0.13)$	1	1
D	$(s_3,-0.15)$	4	$(s_2,0)$	2	$(s_3,-0.35)$	3	3

The final ranks were provided according to the Dominance theory (Brauers, Zavadskas, 2011: 181–182). The absolute dominance of supplier C over the remaining suppliers is observed. Hence, the order of preference for the considered suppliers is the following: $C \succ B \succ D \succ A$.

5. Conclusion

A new MCDM method has been developed, by extending the MULIMOORA with 2–tuple linguistic representation (MULTIMOORA–2T). The new method is aimed at fusion of hybrid data, namely 1) real number; 2) interval number; 3) fuzzy number and linguistic variables. Hence, the MULTIMOORA–2T will be able to handle both objective and subjective criteria.

The MULTIMOORA–2T was applied for solving supplier selection problem. Supplier selection is a complex problem due to its uncertainty involving subjective and objective, crisp and fuzzy information. In this particular case, an enterprise decides on choosing the best supplier from four candidates. The following criteria are taken into consideration: product price, product quality, time of delivery, percentage of on–time deliveries, required payment in advance, remoteness of the facilities (location), and credibility of supplier.

The application of the new method was successful. Moreover, the indicator system can be customized according to the needs of certain decision maker, thus making MULTIMOORA–2T a powerful tool for such practices as e-commerce, e-procurement, and innovative procurements.

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