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**MCDM METHODS WASPAS AND MULTIMOORA:
VERIFICATION OF ROBUSTNESS OF METHODS WHEN
ASSESSING ALTERNATIVE SOLUTIONS**

***Abstract.** The paper employs a couple of Multiple Criteria Decision Making (MCDM) methods: innovative, newly developed WASPAS (Weighted Aggregated Sum Product ASsessment), reputed MOORA (Multiple Objective Optimisation on the basis of Ratio Analysis) method consisting of the Ratio System and the Reference Point approach, also MULTIMOORA (MOORA plus Full Multiplicative Form). Development of WASPAS as an aggregated method of two criteria of optimality, namely WSM (Weighted Sum Model) and WPM (Weighted Product Model) is shortly introduced with references to previous researches of the authors. Application of the method is presented for multiple criteria assessment of alternative building designs. Criteria representing economy of decisions, performance parameters, environmental impact, structural and physical properties of structures are involved for ranking of alternatives and selecting the optimal one. Next, to verify the decision and to validate robustness of the newly developed method, the rather known and reputed MOORA and MULTIMOORA are applied and ranking of alternatives is performed. Conclusions as concerns partial conformity of the methods depending on the weights of both criteria of optimality in the aggregated function are presented.*

***Key words:** MCDM, WASPAS, MOORA, MULTIMOORA, robustness, building design.*

JEL Classification: C02, C44, C52, C53, C61, C63.

1. Introduction

Multiple objective optimization and multiple criteria decision making problems are increasingly important in economics. There are a lot of methods employed and case studies available when complex decisions are needed. In a paper of Zavadskas and Turskis (2011) a complex overview of MCDM methods in economics is presented, starting from background approaches and the earliest

applications and ending by later developments and applications. Several important new concepts and trends for solving actual multiple criteria problems are considered by Liou and Tzeng (2012). Also new developments of MCDM methods as well as their applications in construction economics are presented by Kaplinski and Tupenaite (2011).

The further the more developments of decision making methodology appear and their applications become more advanced. The usual methods are modified, adapting them for vague or uncertain environment. Broad review of fuzzy multiple criteria decision making is presented in publications of El-Wahed (2008), Chu and Lin (2009).

The review study of Bragge *et al.* (2012) shows that the area of research keeps growing. Hybrid and modular crisp as well as fuzzified methods are numerous developed, when a couple of methods are integrated. Further a short review of recent papers on new theoretical developments of integrated methods and their applications to different problems in economy is provided.

The first group of papers represents integration of old, classic methods, usually AHP (Analytic Hierarchy Process) or ANP (Analytic Network Process) with TOPSIS (the Technique for the Order Preference by Similarity to Ideal Solution), VIKOR (in Serbian *Višekriterijumska optimizacija I Kompromisno Resenje*, which means Multicriteria Optimization and Compromise Solution), SAW (Simple Additive Weighting) and alike. Application of the latter developments for personal digital assistant selection is presented by Büyüközkan *et al.* (2012). Also Büyüközkan and Çifçi (2012) apply a novel hybrid approach to evaluate suppliers. Portfolio selection is explored by Tzeng and Tsai (2011), Ho *et al.* (2011). Important problems such as marketing or trade decisions are supported by research of Wang (2012), Wang and Tzeng (2012). Also the latter developments are applied for entrepreneurship policy evaluation (Tsai and Kuo, 2011), for evaluation of staff or equipment (Wu *et al.*, 2012; Lashgari *et al.*, 2011), for effective selecting the best team member (Hashemkhani Zolfani and Antucheviciene, 2012).

The second group of papers is composed of researches concerning integrated classic MCDM methods with new ones, including COPRAS (COmplex PROportional Assessment), MOORA (Multiple Objective Optimisation on the basis of Ratio Analysis), MULTIMOORA and the other methods. Medineckiene and Björk (2011) demonstrate using AHP for determining criteria weights and several MCDM methods for calculations on the effects of a number of renovation measures. COPRAS and AHP under fuzzy environment are applied for measuring performance of institutions in India (Das *et al.*, 2012). AHP and COPRAS with grey numbers are used for selecting company supplier in Iran (Hashemkhani Zolfani *et al.*, 2012). Kosareva and Krylovas (2013) analyze application of interval and triangular intuitionistic fuzzy numbers for decision making in an uncertain environment together with COPRAS, TOPSIS and VIKOR methods.

Although a great variety of ranking methods are developed, the continual problem of proper choice remains under consideration. How to select the best one? Which method is the most robust and what assessment is the most accurate?

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Accordingly, comparative analysis of MCDM methods and evaluation of ranking results are presented in a number of publications. Opricovic and Tzeng (2004) provided a famous paper on comparative analysis of VIKOR and TOPSIS methods. Podvezko (2011) performed comparative analysis of classic SAW and more modern COPRAS. Kou *et al.* (2012) analyses disagreements among methods in rankings. The paper proposes an approach to resolve disagreements based on rank correlation and five MCDM methods are examined. The similar methodology is applied for fuzzified methods by Antucheviciene *et al.* (2011, 2012), analysis of ranking results is performed and their congruence (incongruence) is measured. Balezentis *et al.* (2012b) applied several fuzzy MCDM methods for assessment of economic sectors of a country. The paper of Chatterjee and Chakraborty (2012) focuses on the application of four MCDM methods for material selection and comparison of ranking results, while the paper of Karande and Chakraborty (2012) employs much more techniques for the same problem and evaluates the performance of the methods and their relations by calculating rank correlations.

Performance of methods can also be measured by evaluating robustness of the method. Meaning of robustness is discussed in a number of papers. Brauers and Zavadskas (2012) summarize definitions of robustness and conditions of robustness, including researches published several decades ago and recent works (Chakraborty, 2011). Robustness of MOORA and MULTIMOORA is also proved (Brauers and Zavadskas, 2009; Brauers and Zavadskas, 2012).

The problem of robustness of certain multi-criteria decision making method is still under consideration. According to conditions or robustness, the use of two different methods of multiple criteria optimization is more robust than the use of a single method; the use of three methods is more robust than the use of two, etc. (Brauers and Zavadskas, 2012). Accordingly, the authors of the current research propose a joint method as a combination of two criteria of optimality, namely WSM (Weighted Sum Model) and WPM (Weighted Product Model). Moreover, it is suggested to select the most appropriate multiple criteria decision making method based on its accuracy of measurement and a combination of two methods is proposed to increase the ranking accuracy. Optimization of aggregation is held and Weighted Aggregated Sum Product Assessment (WASPAS) method for ranking of alternatives is proposed (Zavadskas *et al.*, 2012). The next step is to verify performance of the approach and to validate robustness of the newly developed method. Therefore the well-known and reputed MOORA method consisting of the Ratio System and the Reference Point approach (Brauers and Zavadskas, 2006) as well as the Full Multiplicative Form and MULTIMOORA (Brauers and Zavadskas, 2010) are involved in the research. A case study of multiple criteria assessment of alternative building designs, considering economy of decisions, performance parameters, environmental impact, structural and physical properties of structures, is performed. Conclusions as concerns partial conformity of the methods depending on the weights of both criteria of optimality in the aggregated function are presented.

2. Weighted Aggregated Sum Product Assessment (WASPAS)

Two criteria of optimality were the initial source for development of the third joint criterion of optimality and the WASPAS method afterwards.

The first criterion of optimality, i.e. criterion of a mean-weighted success is similar to the well-known Weighted Sum Model (WSM). This is a method for multiple criteria decision making, i.e. it is applied for evaluating a number of alternatives in terms of a number of decision criteria.

Suppose that problem is defined on m alternatives and n decision criteria. The relative significance (weight) of the criterion is denoted by w_j . Variable x_{ij} stands for the performance value of alternative i when it is evaluated in terms of criterion j .

The relative importance of alternative i , denoted as $Q_i^{\bar{\cdot}}$, is defined as follows (MacCrimon, 1968; Triantaphyllou and Mann, 1989):

$$Q_i^{\bar{\cdot}} = \sum_{j=1}^n \bar{x}_{ij} w_j, \quad (1)$$

where linear normalization of initial criteria values is applied, i.e.

$$\bar{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}}, \quad (2)$$

if $\max_i x_{ij}$ value is preferable or

$$\bar{x}_{ij} = \frac{\min_i x_{ij}}{x_{ij}}, \quad (3)$$

if $\min_i x_{ij}$ value is preferable.

The second criterion of optimality, namely multiplicative exponential generalized criterion, in general coincides with Weighted Product Model (WPM).

The relative importance of alternative i , denoted as Q_i^{\bullet} , is defined as follows (Miller and Starr, 1969; Triantaphyllou and Mann, 1989):

$$Q_i^{\bullet} = \prod_{j=1}^n (x_{ij})^{w_j}. \quad (4)$$

The third joint generalized criterion of weighted aggregation of additive and multiplicative methods was proposed by Saparauskas *et al.* (2011):

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$$Q_i = 0.5Q_i^{\leftarrow} + 0.5Q_i^{\rightarrow} = 0.5 \sum_{j=1}^n \bar{x}_{ij} w_j + 0.5 \prod_{j=1}^n \left(\bar{c}_{ij}^{w_j} \right). \quad (5)$$

Supposing the increase of ranking accuracy and the effectiveness of decisions, methodology for optimization of weighted aggregated function was proposed and the Weighted Aggregated Sum Product Assessment (WASPAS) method for ranking of alternatives was presented (Zavadskas *et al.*, 2012):

$$Q_i = \lambda_i \sum_{j=1}^n \bar{x}_{ij} w_j + \left(1 - \lambda_i \right) \prod_{j=1}^n \left(\bar{c}_{ij}^{w_j} \right), \lambda_i = 0, \dots, 1. \quad (6)$$

Optimal values of λ_i can be found when searching the extreme of the function:

$$\lambda_i = \frac{\sigma^2 \left(Q_i^{\rightarrow} \right)}{\sigma^2 \left(Q_i^{\leftarrow} \right) + \sigma^2 \left(Q_i^{\rightarrow} \right)} \quad (7)$$

The variances $\sigma^2 \left(Q_i^{\leftarrow} \right)$ and $\sigma^2 \left(Q_i^{\rightarrow} \right)$ should be calculated as:

$$\sigma^2 \left(Q_i^{\leftarrow} \right) = \sum_{j=1}^n w_j^2 \sigma^2 \left(\bar{x}_{ij} \right) \quad (8)$$

$$\sigma^2 \left(Q_i^{\rightarrow} \right) = \sum_{j=1}^n \left(\frac{\prod_{j=1}^n \left(\bar{c}_{ij}^{w_j} w_j \right)}{\left(\bar{c}_{ij}^{w_j} \right) \left(\bar{c}_{ij}^{-w_j} \right)} \right)^2 \sigma^2 \left(\bar{c}_{ij}^{w_j} \right) \quad (9)$$

In the case of normal distribution of initial data with the credibility $q=0.05$, estimates of variances of normalized criteria values are calculated as follows:

$$\sigma^2 \left(\bar{c}_{ij}^{w_j} \right) = \left(0.05 \bar{x}_{ij} \right)^2. \quad (10)$$

Please, see Zavadskas *et al.* (2012) for detailed explanation of Eq. (7–10). The simplified application of WASPAS for ecological-economical assessment of houses modernization can be found in Staniunas *et al.* (2013).

3. Multiple Objective Optimisation on the basis of Ratio Analysis (MOORA and MULTIMOORA)

The MOORA method consists of the Ratio System and the Reference Point approach (Brauers and Zavadskas, 2006).

In the Ratio System each response of an alternative to the objective is normalized as follows:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, \quad (11)$$

where x_{ij} — response of alternative i to objective j ; $i = 1, 2, \dots, m$; m — the number of alternatives; $j = 1, 2, \dots, n$; n — the number of objectives (decision criteria); \bar{x}_{ij} — a dimensionless number representing the normalised response of alternative i to objective j .

Next, for optimisation, in a case of maximisation the responses (weighted normalized criteria) are added and, in a case of minimisation, weighted normalized criteria are subtracted, respectively:

$$\bar{y}_i = \sum_{j=1}^{j=g} \bar{x}_{ij} w_j - \sum_{j=g+1}^{j=n} \bar{x}_{ij} w_j, \quad (12)$$

where $j = 1, 2, \dots, g$ are maximised decision criteria; $j = g + 1, g + 2, \dots, g + n$ are minimised decision criteria; w_j is the relative significance (weight) of the criterion; \bar{y}_i stands for the calculated relative importance of alternative i with respect to all objectives according to the Ratio System approach. An ordinal ranking of \bar{y}_i shows the final preference of alternatives.

According to the second part of the MOORA, the maximal objective Reference Point approach is used. The desirable ideal alternative with coordinates r_j is formed selecting data from every decision alternative under consideration, considering optimization direction of every particular criterion.

Next, normalization according to Eq. (11) is performed. Having \bar{x}_{ij} the normalised response of alternative i to objective j and the relative significance (weight) of the criterion w_j , the Min–Max metric of Tchebycheff (Karlın and Studden, 1966) is applied for ranking of alternatives:

$$\text{Min}_i \left\{ \max_j |r_j - \bar{x}_{ij} w_j| \right\}. \quad (13)$$

Brauers and Zavadskas (2010) proposed MOORA to be updated by the Full Multiplicative Form. The Full Multiplicative Form is applied as follows:

$$U_i = \frac{A_i}{B_i}, \quad (14)$$

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where U_i denotes overall utility of alternative i .

$$A_i = \prod_{j=1}^g w_j \bar{x}_{ij}, \quad (15)$$

$$B_i = \prod_{j=g+1}^n w_j \bar{x}_{ij}, \quad (16)$$

where $j = 1, 2, \dots, g$ are maximised decision criteria; $j = g + 1, g + 2, \dots, g + n$ are minimised decision criteria.

The MULTIMOORA summarizes MOORA and the Full Multiplicative Form based on the theory of dominance (Brauers and Zavadskas, 2010).

The MOORA and MULTIMOORA are widely applied, and especially in economics. The economy of regions is tested and regional development considering multiple objectives is evaluated by Brauers *et al.* (2010). Economic ranking of European Union member states is performed (Brauers *et al.*, 2012a). Some more recent applications can be mentioned, as personnel selection (Balezentis *et al.*, 2012a) or calculations for energy savings in buildings (Brauers *et al.*, 2012b). Balezentis and Zeng (2013) propose an extension of technique for group decision making based upon interval-valued fuzzy numbers and provide an application for personnel selection.

4. Multiple criteria assessment of alternative building design solutions

4.1. Considered alternative solutions

Assessment of alternative building designs is far from being entirely a technological question. Harmony between economy, environment, quality of living environment, and required technological parameters should be supported and sustainable decision should be made.

To choose the best building's design alternative usually several different solutions are being considered. In the presented case study facade's alternatives for public or commercial building are assessed. Four building facades' alternatives are evaluated, namely cellular concrete masonry covered by Rockwool plates and decorative plaster surface, "sandwich" facade panels, gas silicate masonry covered by Rockwool and "Minerit" facade plates, and aluminium-glazing facade (Povilavicius, 2007). There are some advantages and disadvantages of structures mentioned (see Table 1).

Table 1. Advantages and disadvantages of different facades

Type of facade	Advantages	Disadvantages
Cellular concrete masonry covered by Rockwool plates and decorative plaster surface	The relatively low price	Vulnerable; Stubborn stains; Requires care; Breach threatens facade structures; It is difficult to find defects
“Sandwich” facade panels	Resistant to shock and corrosion; Easy to remove dirt	Violation difficult to sort out
Gas silicate masonry covered by Rockwool and “Minerit” facade plates	Resistant to water; Resistant to chemicals; Easily removed dirt, graffiti	The relatively high price
Aluminium-glazing facade	Can be easily observed structural damage; Easy to remove dirt	Threat of violation of construction and facade; High price

Potential alternatives are evaluated in terms of a number of quantitative and qualitative criteria. Twelve criteria are applied that represent economy of decisions (installation cost and labour intensity), performance parameters (user friendliness, durability and warranty), environmental impact of particular facades’ systems (environmental friendliness, recovery or utilization, aesthetics), structural properties (weight and thickness of structure) and physical properties of structures (sound isolation and fire resistance).

4.2. Ranking of alternatives applying WASPAS

A similar problem was analysed in previous researches of the authors. Three criteria of optimality were applied and alternative decisions were ranked by Saparauskas *et al.* (2011). Two criteria of optimality indicated the most preferable alternative; however the third criterion indicated the other alternative as the best one. Accordingly, the question remained unsolved.

Solution of the problem is continued in the current research and a joint method of the latter criteria of optimality called WASPAS is applied for ranking of facades.

The problem is defined on m alternatives ($m=4$ in the current case) and n decision criteria ($n=12$ in the current case). Variable x_{ij} stands for the performance value of alternative a_i ($i=1, \dots, 4$) when it is evaluated in terms of criterion x_j ($j=1, \dots, 12$).

Criteria under consideration are installation cost, Lt/m^2 (x_1); labour intensity by assembling, *days* (x_2); user friendliness, *points* (x_3); durability, *points* (x_4); warranty, *points* (x_5); environmental friendliness, *points* (x_6); recovery (utilization), *points* (x_7); aesthetics, *points* (x_8); weight of structure, kg/m^2 (x_9); thickness of

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structure, *mm* (x_{10}); sound isolation, *points* (x_{11}); fire resistance, *points* (x_{12}). Criteria x_1 , x_2 , x_9 and x_{10} are minimized, while the remaining $x_3 - x_8$, x_{11} and x_{12} are maximized in a process of optimization.

Four building facades' alternatives are evaluated considering the above criteria and ranked, namely cellular concrete masonry covered by Rockwool plates and decorative plaster surface (a_1), "sandwich" facade panels (a_2), gas silicate masonry, covered by Rockwool and "Minerit" facade plates (a_3) and aluminium-glazing facade (a_4).

Relative significances (weights) of the criteria are denoted by w_j ($j=1, \dots, 12$) and are determined by means of Entropy method (Shannon, 1948). Calculations of relative significances for the current case study were presented in Saparauskas *et al.* (2011).

Initial decision making matrix for description of alternatives in terms of particular criteria is presented in Table 2.

Table 2. Initial decision making matrix: initial criteria values and weights

Criteria x_j	Weights w_j	Alternatives a_i			
		a_1	a_2	a_3	a_4
x_1	0.0627	370.00	314.00	480.00	850.00
x_2	0.0508	11.00	7.00	10.00	16.00
x_3	0.1114	2.69	2.37	3.09	3.17
x_4	0.0874	2.75	3.27	3.67	4.10
x_5	0.0625	5.00	35.00	30.00	50.00
x_6	0.1183	1.63	1.72	1.87	1.91
x_7	0.0784	1.47	2.07	1.38	2.22
x_8	0.0984	7.11	5.60	7.82	8.25
x_9	0.0530	88.00	12.60	94.00	23.00
x_{10}	0.1417	410.00	100.00	410.00	65.00
x_{11}	0.0798	2.93	2.13	2.87	1.10
x_{12}	0.0557	1.98	3.21	2.94	4.37

Ranking of alternatives is performed applying Weighted Aggregated Sum Product Assessment, Eq. (1 – 4) and Eq. (6) when $\lambda=0, 0.1, \dots, 1$. Established relative significances of alternatives are presented in Table 3.

Table 3. Ranking of alternatives applying WASPAS

λ	Relative significances Q_i and ranks of alternatives							
	Alternative a_1		Alternative a_2		Alternative a_3		Alternative a_4	
	Q_1	Rank	Q_2	Rank	Q_3	Rank	Q_4	Rank
$\lambda = 0$	0.4912	4	0.8173	1	0.5873	3	0.8015	2
$\lambda = 0.1$	0.5033	4	0.8185	1	0.5983	3	0.8066	2
$\lambda = 0.2$	0.5154	4	0.8797	1	0.6093	3	0.8116	2
$\lambda = 0.3$	0.5274	4	0.8209	1	0.6203	3	0.8167	2
$\lambda = 0.4$	0.5394	4	0.8221	1	0.6313	3	0.8217	2
$\lambda = 0.5$	0.5516	4	0.8233	2	0.6423	3	0.8268	1
$\lambda = 0.6$	0.5637	4	0.8244	2	0.6523	3	0.8318	1
$\lambda = 0.7$	0.5758	4	0.8256	2	0.6642	3	0.8369	1
$\lambda = 0.8$	0.5878	4	0.8268	2	0.6752	3	0.8419	1
$\lambda = 0.9$	0.5999	4	0.8282	2	0.6862	3	0.8470	1
$\lambda = 1$	0.6120	4	0.8292	2	0.6972	3	0.8520	1

One can observe from calculation results that the most preferable alternative depends on λ value when applying WASPAS. Alternative a_2 ("sandwich" facade panels) is ranked as the best and alternative a_4 (aluminium-glazing facade) remains in the second place in several cases among analyzed eleven variants with different λ values. While ranking order of the particular alternatives changes in several other cases and a_4 is preferred.

Accordingly, the question of selection of the best alternative designs remains not completely solved. Therefore, MOORA method and the Full Multiplicative Form are applied to validate the decision as well as robustness of the newly developed method.

4.3. Ranking of alternatives applying MOORA, the Full Multiplicative Form and MULTIMOORA methods

Four alternatives considering twelve criteria (Table 2) are ranked applying the Ratio System, Eq. (11–12); the Reference Point approach, Eq. (11), Eq. (13); the Full Multiplicative Form, Eq. (14–16) and MULTIMOORA. Established relative significances of alternatives, also ranking order of alternatives are presented in Table 4.

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Table 4. Ranking of alternatives applying the Ratio System, the Reference Point approach, the Full Multiplicative Form and MULTIMOORA

a_i	MOORA				Full Multiplicative Form		MULTIMOORA
	Ratio System		Reference Point		U_i	Rank	Rank based on dominance
	\bar{y}_i	Rank	$\max r_j - \bar{x}_{ij} w_j $	Rank			
a_1	0.1082	4	0.0825	3, 4	2.92E-07	4	4
a_2	0.2740	1	0.0179	1	2.23E-04	1	1
a_3	0.1614	3	0.0825	3, 4	3.68E-06	3	3
a_4	0.2681	2	0.0308	2	6.31E-05	2	2

“Sandwich” facade panels are selected as the most preferable alternative when applying the Ratio System, the Reference Point approach, the Full Multiplicative Form and MULTIMOORA.

4.4. Verification of ranking results

When applying MOORA method consisting of the Ratio System and the Reference Point approaches as well as the Full Multiplicative Form and MULTIMOORA, the best ranked alternative decisions coincide in all cases and “sandwich” facade panels (a_2) are preferred in the analysed case study.

While applied aggregated criteria of success (Eq. 1 – 6) produce partly different ranking results. The case study proves that the most preferable alternative depends on λ value when applying a joint weighted method WASPAS. Alternative a_2 (“sandwich” facade panels) is ranked as the best and alternative a_4 (aluminium-glazing facade) remains in the second place when $\lambda=0, 0.1, \dots, 0.4$. While ranking order of the particular alternatives changes their places and aluminium-glazing facade (a_4) is preferred when $\lambda=0.5, 0.6, \dots, 1$.

The robustness of MOORA and MULTIMOORA methods is proved by Brauers and Zavadskas (2009, 2012). Accordingly, the calculation results applying the latter methods are reliable and applicable.

Ranking order applying WASPAS method coincides with the well-known, reputed and robust approach when λ is less than 0.5.

The above identified interval of λ less than 0.5 is consistent with the proposed approach to calculate optimal values of λ (Zavadskas *et al.* 2012). Calculation results when searching optimal λ_i in conformity with Eq. (7–10) are presented in Table 5. Calculated relative significances of alternatives Q_i^* (Eq. 6) using the established optimal λ_i values are also presented in Table 5.

One can observe from the Table 5 that calculated optimal λ_i values really are less than 0.5. Also, when calculating standard deviations in the case of normal distribution of initial data with the credibility $q=0.05$, the higher ranking accuracy

is reached when applying weighted aggregated function as compared to accuracy of WSM or WPM individually. It means that one of the conditions of robustness as concerns the use of two different methods of multiple criteria optimization instead of a single one is not only a presumption. The condition was proved by calculations. Therefore, the conclusion that newly developed WASPAS method appears to be robust can be approved and validation of the innovative method for real life applications can be made.

Table 5. Optimal λ and relative significances of alternatives

Alternatives a_i	Optimal λ_i	Relative significances of alternatives Q_i
a_1	0.32	0.5303
a_2	0.49	0.8232
a_3	0.37	0.6284
a_4	0.43	0.8233

5. Conclusions

The paper employs innovative, newly developed WASPAS (Weighted Aggregated Sum Product ASsessment) method and reputed MOORA (Multiple Objective Optimisation on the basis of Ratio Analysis) method consisting of the Ratio System and the Reference Point approach as well as the Full Multiplicative Form and MULTIMOORA.

A case study of multiple criteria assessment of four alternative building design solutions considering economy of decisions, performance parameters, environmental impact and physical properties of structures is performed applying the above methods.

It is assessed that the most preferable alternative depends on λ value when applying a joint weighted method WASPAS. Alternative a_2 (“sandwich” facade panels) is ranked as the best and alternative a_4 (aluminium-glazing facade) remains in the second place when $\lambda=0, 0.1, \dots, 0.4$ in a current case. While ranking order of the particular alternatives changes and aluminium-glazing facade is preferred when $\lambda=0.5, 0.6, \dots, 1$.

Whereas the best ranked alternative coincides when applying all approaches of MOORA and MULTIMOORA and “sandwich” panels (a_2) are preferred.

Robustness of the MOORA and MULTIMOORA methods were tested in previous researches. Accordingly, calculation results of the latter methods can be considered as reliable.

It is estimated that priority order of alternatives by applying WASPAS method coincides with the above reputed approaches in a case when λ is less than 0.5.

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It is proved that the higher ranking accuracy is reached when applying weighted aggregated function as compared to accuracy of WSM or WPM individually, i.e. a particular condition of robustness is fulfilled.

Supposing the increase of ranking accuracy and applying the proposed methodology for optimization of weighted aggregated function, optimal λ_i values are calculated. Optimal λ_i values vary from 0.32 to 0.49 and are also less than 0.5 in the current case.

Based on the results of the research, the conclusion that newly developed WASPAS method appears to be robust can be approved. Validation of the method for real life applications can be made.

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