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INTEGRATED OPTIMAL POLICY FOR DETERIORATING INVENTORY SYSTEM OF VENDOR- BUYER WITH TRAPEZOIDAL DEMAND

Abstract. *An inventory model for the joint vendor-buyer venture is proposed for deteriorating items when demand is trapezoidal. It is illustrated by numerical example that the joint decision lowers the total joint cost than the independent decision of both the players. To attract the buyer to place a larger order, a permissible credit period is offered by the vendor to the buyer. A negotiation factor is incorporated to share the cost savings.*

Keywords: *Vendor-Buyer Joint Decision, Constant Deterioration, Credit Period, Trapezoidal Demand*

JEL classification: C61, L11

1. Introduction

The inventory models for linearly trended demand and exponentially time-varying demand are widely studied by the researchers. Silver and Meal (1969), Silver (1979), Xu and Wang (1991), Chung and Ting (1993,1994), Bose et al. (1995), Hariga (1995), Giri and Chaudhari (1997), Lin et al. (2000) etc. discussed optimal ordering policy when demand is linearly changing with respect to time but in practice, demand for items can not go on to raise over time. We can see such inventories in the market of fashion good, smart phones etc. Mehta and Shah (2003, 2004) assumed the demand to be exponential time varying which is again unrealistic for a newly launched product. Shah et al. (2008) introduced the quadratic demand which is again not observed in the market for an indefinite period. In order to have an alternative demand pattern, we can take into account the trapezoidal demand. This type of demand increases up to a certain time, then becomes stable for some time and afterwards decreases exponentially with time.

Most of the inventory models are derived with the assumption that the buyer is the dominant player to make the decision for placing an order. This strategy may not be economical for the vendor. An integrated vendor-buyer policy should be analyzed which is beneficial to all the players of the supply chain. Clark and Scarf (1960) and thereafter, Goyal (1977) proposed a mathematical model for vendor-buyer integration. Banerjee (1986) discussed an economic lot-size model when production is finite. Goyal (1988) extended Banerjee's model by relaxing the assump tion of the lot-for-lot production. Shah et al. (2008) analyzed joint decision policy when demand is quadratic.

Deterioration is defined as the decay, change, spoilage, evaporation and loss of utility of a product from the original one. Fruit and vegetables, cosmetics and medicines, electronic items, blood components, radioactive chemicals, agriculture products are some of the examples of deteriorating commodities. Refer to reviews on deteriorating inventory models by, Nahmias (1982), Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001) and Shah et al (2009). Yang and Wee (2000) defined a heuristic approach to formulate a joint vendor-buyer inventory model for deteriorating items. Yang and Wee (2005) derived a win – win strategy for an integrated system of vendor-buyer with deterioration. Shah et al. (2008) extended above model by incorporating salvage value to the deteriorated units.

In this article, we consider a joint vendor – buyer inventory system for deteriorating items when demand is trapezoidal. Inventories in the model deteriorate at the same rate for vendor as well as buyer. A negotiation factor is incorporated to share the savings. The credit period is offered to attract the buyer for placing a larger order. A numerical example is illustrated to support the proposed model. Sensitivity analysis is carried out to study the changes in cost savings.

2. Notations and Assumptions

The proposed study uses following notations and assumptions.

2.1 Notations

A_b Buyer's ordering cost per order

A_v	Vendor's ordering cost per order
C_b	Buyer's purchase cost per unit
C_v	Vendor's purchase cost per unit
I_b	Inventory carrying charge fraction per unit per annum for the buyer
I_v	Inventory carrying charge fraction per unit per annum for the vendor
θ	Deterioration rate of items in inventory system; $0<\theta<1$
$I_b(t)$	Buyer's inventory level at any instant of time t
$I_v(t)$	Vendor's inventory level at any instant of time t
n	Number of orders during cycle time for the buyer (a decision variable)
K_b	Buyer's total cost per unit time
K_v	Vendor's total cost per unit time
K_{NJ}	Total cost for vendor-buyer inventory System when they take independent decision
K_J	Total cost for vendor-buyer inventory System when they take joint decision
T	Vendor's cycle time (a decision variable)
T_b	$(=T/n)$, Buyer's cycle time (a decision variable)
M	Credit period offered by the vendor to the buyer (a decision variable)
r	Continuous discounting rate

2.2 Assumptions

- A supply chain of a single vendor and single buyer is considered.
- An inventory system deals with a single item.
- The deterioration rates of items in the vendor's and buyer's inventory are same and proportional to on hand stock in inventory. There is no repair or replacement of deteriorated units during a cycle time.
- The demand rate is trapezoidal. Its functional form is

$$R(t)=\begin{cases} f(t) & ;0\le t\le u_1 \\ D_0 & ;u_1\le t\le u_2 \\ g(t) & ;u_2\le t\le T \end{cases}$$

where $f(t)$ is linear in t,

$D_0 = f(u_1) = g(u_2)$, and $g(t)$ is exponentially decreasing in t. (say)

$$R(t) = \begin{cases} a(1+b_1 t) & ;0 \le t \le u_1 \\ a(1+b_1 u_1) & ;u_1 \le t \le u_2 \\ a(1+b_1 u_1)e^{-b_2(-u_2+t)} & ;u_2 \le t \le T \end{cases}$$

where a denotes scale demand, $0 < b_1, b_2 < 1$ denotes rates of change of demand. (See Figure 1)

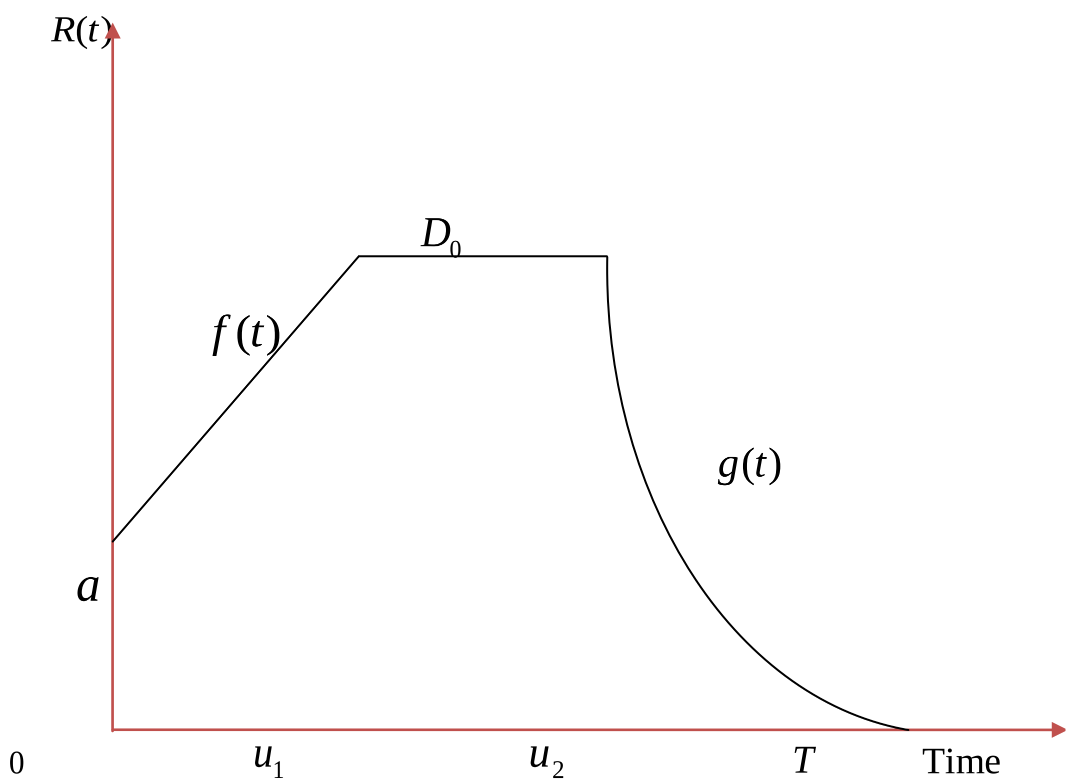

Figure 1. Trapezoidal demand

- The lead time is zero and shortages are not allowed.
- The credit period is offered for settling the accounts due against purchases to attract the buyer to opt a joint decision policy.

3. Mathematical Model

Figure 2 depicts the time-varying inventory status of the vendor and the buyer.

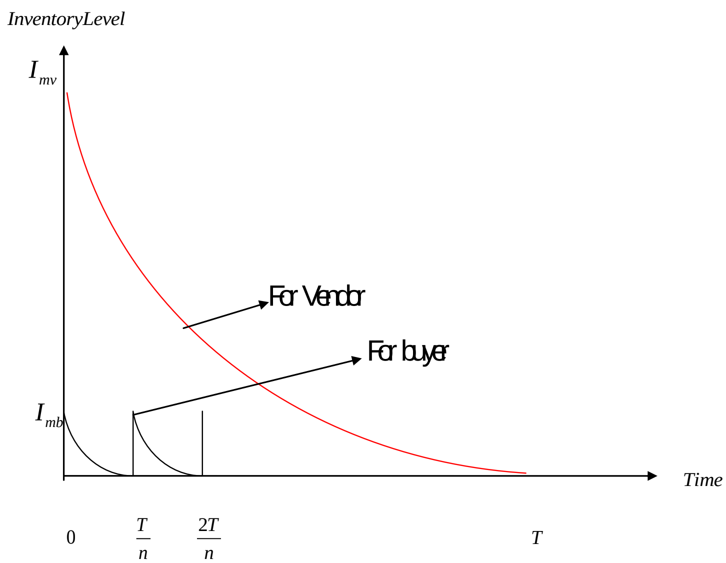

Figure 2. Vendor –Buyer Inventory Status

The inventory changes due to trapezoidal demand for both vendor and buyer. The rate of change of inventory for both the players is governed by the differential equations:

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -R(t), 0 \le t \le T_b \tag{1}$$

$$\frac{dI_v(t)}{dt} + \theta I_v(t) = -R(t), 0 \le t \le T \tag{2}$$

with the boundary conditions $I_b(T_b)=0$, $I_v(T)=0$ and initial conditions $I_b(0)=I_{mb}$, $I_v(0)=I_{mv}$.

The solutions of the differential equations are

$$I_b(t)=\begin{cases} a\left[-\frac{1+b_1t}{\theta}+\frac{b_1}{\theta^2}+\frac{1+b_1u_1}{\theta}e^{\theta u_1-\theta t}-\frac{b_1}{\theta^2}e^{\theta u_1-\theta t}\right]+ \\ \frac{a(1+b_1u_1)}{\theta}\left[e^{\theta u_2-\theta t}-e^{\theta u_1-\theta t}\right]+ \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-\frac{b_2T}{n}+\frac{\theta T}{n}-\theta t}-e^{-b_2u_2+\theta u_2-\theta t}\right] & ;0\le t\le u_1 \\ \\ \frac{a(1+b_1u_1)}{\theta}\left[-1+e^{\theta u_2-\theta t}\right]+ \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-\frac{b_2T}{n}+\frac{\theta T}{n}-\theta t}-e^{-b_2u_2+\theta u_2-\theta t}\right] & ;u_1\le t\le u_2 \\ \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-\frac{b_2T}{n}+\frac{\theta T}{n}-\theta t}-e^{-b_2t}\right] & ;u_2\le t\le T_b \end{cases}$$

(3)

$$I_v(t)=\begin{cases} a\left[-\frac{1+b_1t}{\theta}+\frac{b_1}{\theta^2}+\frac{1+b_1u_1}{\theta}e^{\theta u_1-\theta t}-\frac{b_1}{\theta^2}e^{\theta u_1-\theta t}\right]+ \\ \frac{a(1+b_1u_1)}{\theta}\left[e^{\theta u_2-\theta t}-e^{\theta u_1-\theta t}\right]+ \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-b_2T+\theta T-\theta t}-e^{-b_2u_2+\theta u_2-\theta t}\right] & ;0\le t\le u_1 \\ \\ \frac{a(1+b_1u_1)}{\theta}\left[-1+e^{\theta u_2-\theta t}\right]+ \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-b_2T+\theta T-\theta t}-e^{-b_2u_2+\theta u_2-\theta t}\right] & ;u_1\le t\le u_2 \\ \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-b_2T+\theta T-\theta t}-e^{-b_2t}\right] & ;u_2\le t\le T \end{cases}$$

(4)

Using $I_b(0)=I_{mb}$, $I_v(0)=I_{mv}$ the maximum procurement quantities for the buyer and the vendor are

$$I_{mb}=\left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-\frac{b_2T}{n}+\frac{\theta T}{n}}-1\right]$$

$$I_{mv}=\left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta-b_2}\right]\left[e^{-b_2T+\theta T}-1\right]$$

respectively.

During the cycle time $\left[0,T\right]$, the buyer's

- Purchase Cost ; $PC_b=nC_bI_{mb}$

- Holding Cost ; $HC_b = nC_b I_b \int_0^{T_b} I_b(t)dt$
- Ordering Cost ; $OC_b = nA_b$

Hence, the buyer's total cost; K_b per unit time is

$$K_b = \frac{1}{T}\left[PC_b + HC_b + OC_b\right] \tag{5}$$

The vendor's inventory is the difference between the vendor-buyer combined inventory and the buyer's inventory during n-orders. This is known as the joint two-echelon inventory model. The vendor's

- Purchase Cost ; $PC_v = C_v I_{mv}$
- Holding Cost ; $HC_v = C_v I_v \left[\int_0^{T} I_v(t)dt - n\int_0^{T_b} I_b(t)dt\right]$
- Ordering Cost ; $OC_v = A_v$

Hence, the vendor's total cost; K_v per unit time is

$$K_v = \frac{1}{T}\left[PC_v + HC_v + OC_v\right] \tag{6}$$

The joint total cost K is the sum of K_b and K_v where $T_b = \frac{T}{n}$

Thus K is the function of discrete variable n and continuous variable T .

5. Computational Procedure

There are two cases to be analyzed

.

Case 1: When the vendor and the buyer take decision independently.

For given value of n, differentiate K_b with respect to T_b (equivalently, T) and solve $\frac{\partial K_b}{\partial T_b} = 0$. This n and T_b minimizes K_v provided

$$K_v(n-1) \geq K_v(n) \leq K_v(n+1) \tag{7}$$

satisfies.

Here, the total cost per unit time with independent decision; K_{NJ} is given by

$$K_{NJ} = [\min_{n} K_b + K_v] \quad (8)$$

Case 2: When vendor and the buyer make decisions jointly.

The optimum value of T and n must satisfy the following conditions simultaneously:

$$\frac{\partial K_J}{\partial T} = 0 \quad \text{and} \quad K_J(n-1) \geq K_J(n) \leq K_J(n+1) \quad (9)$$

Then, the total joint cost is

$$K_J = \min_{n,T}[K_b + K_v] \quad (10)$$

It is obvious that $K_J \leq K_{NJ}$. Hence, total cost savings Sav_J is defined as $Sav_J = K_{NJ} - K_J$. Now define buyer's cost saving, Sav_b as $Sav_b = \alpha\ Sav_J$, where $0 \leq \alpha \leq 1$ is the negotiation factor. When negotiation factor equals to 0.5, saving gets equally distributed between two players; when it is equal to zero, all saving is in the vendor's pocket and when it is equal to 1, it is in favor of buyer.

The present value of the unit after a time interval M is e^{-rM}, where r is discounting rate. Solving the following equation

$$R(t)C_b(1-e^{-rM}) = Sav_b \quad (11)$$

the buyer's credit period is given by

$$M = \frac{1}{r}\ln\left[\frac{C_b R(t)}{C_b R(t) - Sav_b}\right] \quad (12)$$

5. Numerical Example and Sensitivity Analysis

Consider following inventory parameters values in proper units:

$[a \quad b_1 \quad b_2 \quad A_b \quad A_v \quad C_b \quad C_v \quad I_b \quad I_v \quad \theta \quad r]$

=[40000 0.04 0.02 600 3000 10 6 0. 11 0.10 .08 0.06]

Let $\alpha = 0.5$.
The optimal solution is listed in Table 1 for independent and joint decisions.

Table 1: Optimal Solution for Independent and Joint Decisions

	Case 1 Independent Decision	**Case 2 Joint Decision**
n	3	2
T_b	0.132530333	0.185847
T	0.397591	0.371694
K_b	410038	410561
K_v	254216	253600
$K(=K_{NJ}\ or\ K_J)$	K_{NJ} =664254	K_J =664161
$PJCR$	-	0.014002629
$M(days)$	-	1.905565479

(Where; PJCR = Percentage Change in Joint Cost Reduction)

The buyer's cost and cycle time increase in joint decisions. The vendor gains \$616 and the buyer loses \$523. This hinders the buyer to agree for joint decision. To entice the buyer to joint decision, the vendor offers the buyer a credit period of (1.9$\approx$) 2 days with equal sharing of cost savings. This reduces the joint total cost PJCR by 0.050075188 %, where PJCR is defined as $\frac{K_{NJ}-K_J}{K_J}\times 100$.

The convexity of total integrated cost and independent costs are shown in Figure 3.

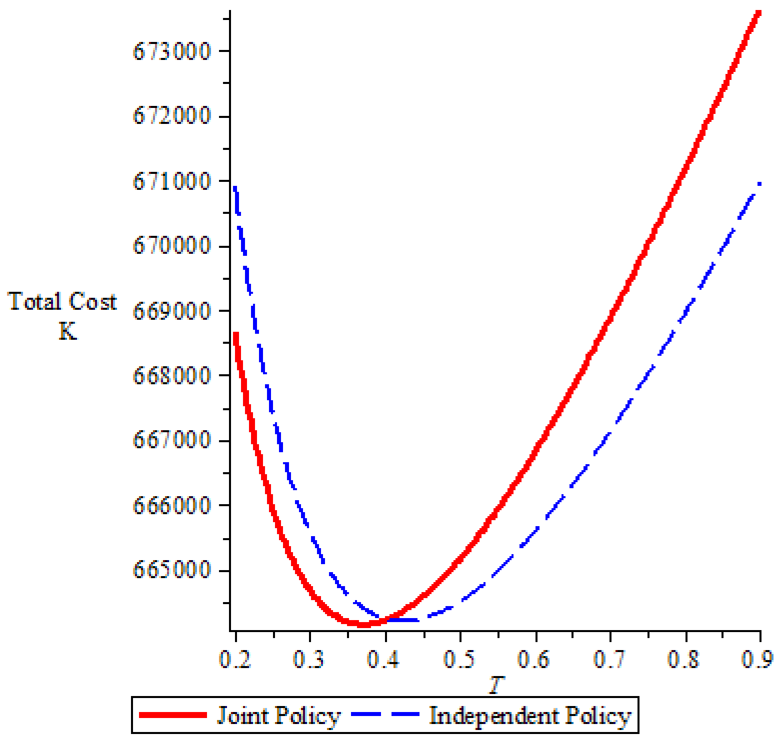

Figure 3. Total Cost for Independent Vs Joint Vendor-Buyer Inventory System

Table 2. Sensitivity Analysis of Demand Rate

a	24000	32000	40000	48000	56000
K_{NJ}	402515	533546	664254	794733	925037
K_J	402443	533463	664161	794630	924926
$PJCR$	0.017890732	0.015558717	0.014002629	0.012962007	0.01200096
$M\,(days)$	1.907466134	1.9026284	1.905565479	1.925765213	1.920809473

Observations

- The increase in fixed demand a, decreases the percentage of cost reduction and the delay period or decrease delay period.

Table 3. Sensitivity Analysis of Linear Rate of Change of Demand

b_1	**0.024**	**0.032**	**0.04**	**0.048**	**0.056**
K_{NJ}	663977	664116	664254	664393	664531
K_J	663883	664022	664161	664300	664438
$PJCR$	0.014159121	0.014156157	0.014002629	0.013999699	0.013996791
$M(days)$	1.928006858	1.927097216	1.905565479	1.904667334	1.903770035

Observations

- Increase in linear rate of change of demand b_1 decreases the percentage of cost reduction and delay period

Table 4. Sensitivity Analysis of Exponential Rate of Change of Demand

b_2	**0.012**	**0.016**	**0.02**	**0.024**	**0.028**
K_{NJ}	664539	664399	664254	664105	663951
K_J	664470	664318	664161	663998	663831
$PJCR$	0.010384216	0.012192956	0.014002629	0.016114506	0.01807689
$M(days)$	1.454911612	1.684042533	1.905565479	2.159883974	2.384997417

Observations

- Increase in exponential rate of change of demand b_2, increases percentage of cost reduction and delay period.

Table 5. Sensitivity Analysis of Buyer's Ordering Cost

A_b	**360**	**480**	**600**	**720**	**840**
K_{NJ}	663036	663546	664254	665043	665861
K_J	662830	663505	664161	664797	665417
$PJCR$	0.031078859	0.006179305	0.014002629	0.037003777	0.066725076
$M(days)$	4.484612716	0.865010403	1.905565479	4.904354703	8.625591441

Observations

- Increase in buyer's ordering cost A_b , may increase or decreases percentage of cost reduction and delay period.

Table 6. Sensitivity Analysis of Vendor's Ordering Cost

A_v	**1800**	**2400**	**3000**	**3600**	**4200**
K_{NJ}	661236	662745	664254	665763	667272
K_J	660662	662484	664161	665721	667187
$PJCR$	0.086882551	0.039397178	0.014002629	0.006308949	0.012740056
$M(days)$	13.91984492	5.775957682	1.905565479	0.805234181	1.537028651

Observations

- Increase in Vendor's Ordering Cost A_v, may increase or decrease percentage of cost reduction and delay period.

Table 7. Sensitivity Analysis of Buyer's Purchase Cost

C_b	**6**	**8**	**10**	**12**	**14**
K_{NJ}	501893	583024	664254	745505	826746

K_J	501066	582656	664161	745593	826965
$PJCR$	0.165048117	0.06315905	0.014002629	-0.01180269	-0.02648238
$M(days)$	24.93043163	8.88783176	1.905565479	-1.58399515	-3.54418435

Observations

- Increase in Buyer's purchase cost C_b, decreases percentage of cost reduction and delay period significantly.

Table 8. Sensitivity Analysis of Vendor's Purchase Cost

C_v	**3.6**	**4.8**	**6**	**7.2**	**8.4**
K_{NJ}	565586	614920	664254	713588	762922
K_J	565854	615032	664161	713246	762292
$PJCR$	-0.04736204	-0.01821043	0.014002629	0.047949796	0.082645495
$M(days)$	-4.98673562	-2.19193832	1.905565479	7.310029726	14.00230793

Observations

- Increase in Vendor's purchase cost C_v, increases percentage of cost reduction and delay period significantly.

Table 9. Sensitivity Analysis of Inventory Carrying Charge Fraction of Buyer

I_b	**0.066**	**0.088**	**0.11**	**0.132**	**0.154**
K_{NJ}	662932	663581	664254	664935	665615

K_J	662456	663325	664161	664967	665746
$PJCR$	0.071853829	0.03859345	0.014002629	-0.00481227	-0.01967717
$M(days)$	9.027463347	5.054111299	1.905565479	-0.67881087	-2.87026687

Observations

- Increase in buyer's inventory Carrying Charge Fraction I_b, decreases percentage of cost reduction and delay period significantly.

Table 10: Sensitivity Analysis of Inventory Carrying Charge Fraction of Vendor

I_v	**0.06**	**0.08**	0.1	**0.12**	**0.14**
K_{NJ}	662980	663617	664254	664891	665528
K_J	663249	663710	664161	664603	665037
$PJCR$	-0.04055792	-0.01401214	0.014002629	0.043334141	0.073830479
$M(days)$	-5.28841086	-1.86736819	1.905565479	6.017806602	10.45493491

Observations

- Increase in Vendor's Inventory Carrying Charge Fraction I_v, increases percentage of cost reduction and delay period significantly.

The following figures show variations in total savings and delay payment period with respect to inventory parameters.

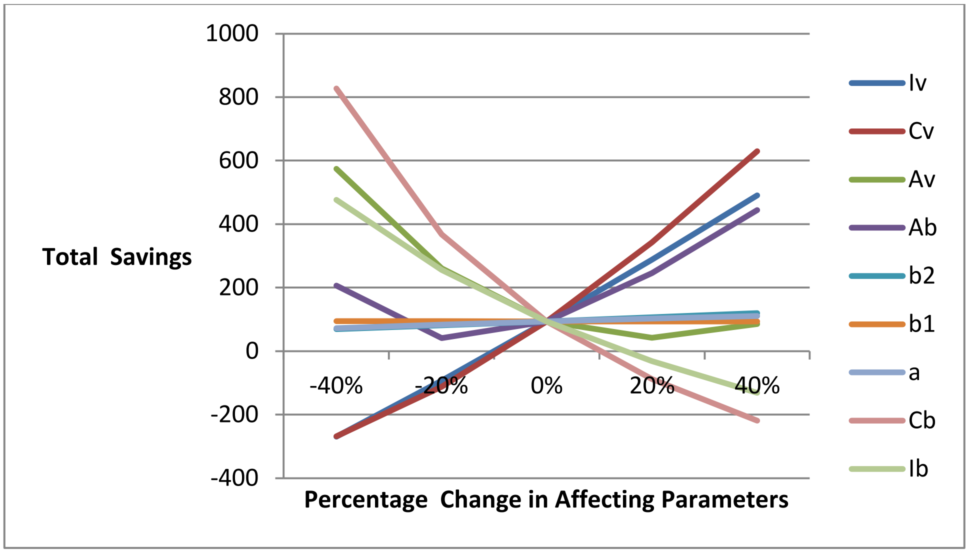

Figure 4. Total Savings Vs. Percentage of Changes in Affecting Parameters

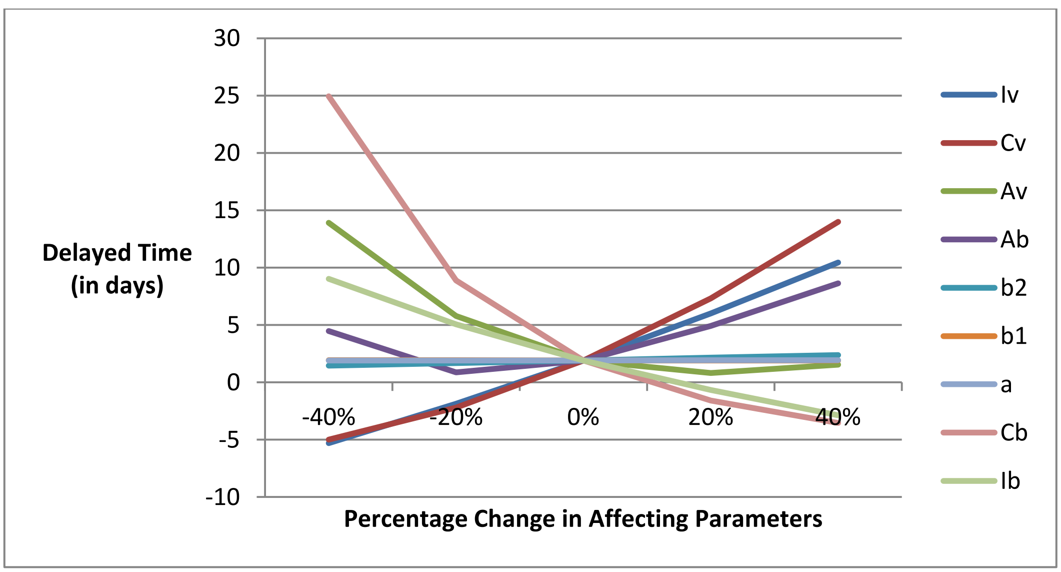

Figure 5. Delayed Time in Days Vs. Percentage of Changes in Affecting Parameters

6. Conclusions

In this article, a mathematical model is proposed to illustrate an optimal ordering policy for a supply chain of vendor-buyer joint inventory system when demand is trapezoidal. The deterioration rate of units in the system is considered to be constant. It is established that the joint decision lowers the total cost of the inventory system, even though the buyer's cost increases significantly. To attract the buyer for a joint decision, a credit period is offered by the vendor to the buyer to settle the account.

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