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MODELLING HIGH FREQUENCY TRANSACTION DATA IN FINANCIAL ECONOMICS: A COMPARATIVE STUDY BASED ON SIMULATIONS

Abstract. *This paper considers the modelling of high frequency financial duration data using the class of Autoregressive Conditional Duration (ACD) models. We consider the use of Generalized Gamma innovations and the theory of optimal Estimating functions (EF) as they are very useful in applied financial economics. A simulation study is carried out to compare the performance of the EF estimates with corresponding Maximum likelihood (ML) and Quasi maximum likelihood (QML) estimates. The properties of the optimal EF estimates are investigated. Our results show that the EF and ML estimates are comparable and are very useful in financial economic modeling that are related to duration data.*

Key words: *Financial economics, Duration data, High frequency data, Estimating function, Maximum likelihood.*

JEL Classification: **C53, C58, G17**

1 INTRODUCTION

Engle and Russell (1998) proposed a class of ACD models to analyze high frequency data arise in finance. In their paper, Engle and Russell (1998) used the theory of monotonic hazard functions such as Exponential and Weibull distributions and successfully applied to model the data on transactions of IBM stocks. Due to the fact that these distributions have poor performances in practice, many authors have proposed more flexible distributions in application. For example Grammig and Maurer (2000) introduced the ACD models based on Burr distribution, Hautsch (2001) utilize the Generalized F distribution and Lunde (1999) applied Generalized Gamma distribution to the conditional hazard function. They argued that the property of non-monotonicity in conditional hazard functions of Burr and Generalized Gamma distributions significantly improve the ACD modeling. In their paper, Bauwens et al. (2004) found that Generalized Gamma distribution offer a better choice for the conditional hazard function than that of the Exponential and Weibull distributions in Log-ACD and Fernandes and Grammig (2005) shown that the modelling of ACD with Exponential, Weibull and Burr distributions to the EXXON price duration data do not fit well. On the other hand, Generalized Gamma distribution shows superior performance in modeling. Zhang et al. (2001) proposed the class of threshold ACD model based on Generalized Gamma distribution. Recently, Allen et al. (2009) strongly suggested that Generalized Gamma and Log-normal distributions perform better than the Exponential and Weibull distributions.

Engle and Russel (1998) used the Maximum likelihood (ML) method to estimate the parameters of the ACD model. Further applications on the ML method are available in the literature, see for example Grammig and Maurer (2000), Lunde (1999), Bauwens et al. (2004), Zhang et al. (2001), De Luca and Zuccolotto (2003, 2004). In their papers Fernandes and Grammig (2005) and Allen et al. (2008, 2009), used the Quasi maximum likelihood (QML) methods to estimate ACD parameters. Drost and Werker (2004) considered semi-parametric estimation of ACD models

This paper considers an alternative approach to estimate the ACD parameters using the theory of the Estimating functions (EF). Since the Generalized Gamma distribution performs well in the ACD modeling, we focus on the formulation of EF and the likelihood function based on this distribution. In addition, we investigate the effect of other distributions in estimating of Generalized Gamma ACD parameters. In order to compare these estimation methods, a simulation study has been carried out. In each case, we compare the performance of the ML, QML and EF estimates by computing the mean, bias, standard error (SE) and the mean square error (MSE).

The paper is organized as follows: the Section 2 briefly discusses the ACD model and the methods of parameter estimation ML, QML and EF. Section 3 provides numerical results together with discussions based on simulations to assess the performance of ML, QML and EF methods. Finally, the conclusion remarks of this study are added in Section 4.

2 BASIC PROPERTIES OF ACD MODELS

2.1 ACD MODELS

Let t_i be the time of the i-th transaction and let x_i be the i-th adjusted duration such that $x_i = t_i - t_{i-1}$. Then, the basic ACD model for the variable x_i is defined as

$$x_i = \psi_i \varepsilon_i , \tag{1}$$

where ε_i is a sequence of independently and identically distributed (iid) non-negative random variable's with a known density $f(\cdot)$ and $E(\varepsilon_i)=1$. Further, the conditional duration process, ψ_i satisfies

$$\psi_i = E[x_i \mid x_{i-1},\cdots,x_1] = E[x_i \mid F_{i-1}], \tag{2}$$

where F_{i-1} is the information set available at the $(i-1)$-th trade and ε_i is independent of F_{i-1}.

From Equation (1), it is clear that a vast set of ACD models can be defined by allowing different distributions for ε_i and specifications of ψ_i.

The general class of ACD models of order (p,q) or ACD (p,q) is given by

$$\psi_i = \omega + \textstyle\sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{k=1}^{q} \beta_k \psi_{i-k}\ ,\ (p \ge 1, q \ge 0), \tag{3}$$

where $\omega > 0$, $\alpha_j, \beta_k > 0$ and $\sum_{j=1}^{r}(\alpha_j + \beta_j) < 1$, and $r = \max(p,q)$. Note that $\alpha_j = 0$ for $j > p$ and $\beta_k = 0$ for $k > q$.

Now we consider the ML, QML and EF approaches to estimate the parameters of ACD models.

2.2 ML AND QML METHODS IN ACD MODELLING

In the literature, the parameters of the ACD models can be estimated by popular ML and QML methods. In order to estimate the parameter, let $l(\boldsymbol{\lambda})$ be the log-likelihood function with parameter vector $\boldsymbol{\lambda}$ so that

$$l(\boldsymbol{\lambda}) = \textstyle\sum_{i=1}^{T} \log f(x_i, \boldsymbol{\lambda})\,. \tag{4}$$

Then the ML estimator, $\hat{\boldsymbol{\lambda}}$, of $\boldsymbol{\lambda}$ is given by

$$\hat{\boldsymbol{\lambda}} = \arg\max_{\boldsymbol{\lambda}\in\Lambda} l(\boldsymbol{\lambda})\,. \tag{5}$$

Note that the log-likelihood function in each case is given by Equation (4) with $f(x_i, \boldsymbol{\lambda})$ replaced by the appropriate density function. However, in practice, the true distribution of ε_i is seldom known, and the corresponding estimator such that $\hat{\boldsymbol{\lambda}}$, as defined in Equation (5) will be the QML estimator rather than the ML estimator. In this paper, several popular choices of innovation standardized distributions have been used in practice that are Exponential, Lognormal, Weibull and the Generalized Gamma distributions. The corresponding standardized density functions are given below:

a) Exponential distribution: $f_1(\varepsilon_i) = \exp(-\varepsilon_i)$.

b) Lognormal distribution: $f_2(\varepsilon_i) = \dfrac{1}{\varepsilon_i \sigma \sqrt{2\pi}} \exp\left(-\dfrac{1}{2}\left(\dfrac{\log(\varepsilon_i) + \sigma^2/2}{\sigma} \right)^2 \right)$.

c) Weibull distribution with parameter $\gamma > 0$:

$$f_3(\varepsilon_i) = \gamma \left[\Gamma(1+1/\gamma)\right]^{\gamma} \varepsilon_i^{\gamma-1} \exp\left\{-\left[\Gamma(1+1/\gamma)\,\varepsilon_i\right]^{\gamma}\right\}.$$

d) Generalized Gamma distribution with parameters $\kappa > 0$ and $\gamma > 0$:

$$f_4(\varepsilon_i) = \frac{\gamma \varepsilon_i^{\kappa\gamma-1}}{\theta^{\kappa\gamma}\Gamma(\kappa)} \exp\left(-\left(\frac{\varepsilon_i}{\theta} \right)^{\gamma} \right) \text{ where } \theta = \Gamma(\kappa)/\Gamma(\kappa+1/\gamma).$$

Now we consider the theory EF as an alternative approach for parameter estimation.

2.3 THE EF APPROACH

Let $h_i(\cdot)$ be a real valued function of both $\mathbf{x}_i = \{x_1, x_2, \cdots, x_i\}$ and the parameter $\boldsymbol{\theta}$ such that

$$E_{i-1,F}[h_i(\mathbf{x}_i;\boldsymbol{\theta})] = 0 \quad \text{for all } F \in \Theta, \tag{6}$$

and

$$E(h_i h_j) = 0, \ (i \neq j), \tag{7}$$

where $E_{i-1,F}(\cdot)$ denotes the conditional expectation holding the first $i-1$ values $\mathbf{x}_{i-1} = \{x_1, x_2, \cdots, x_{i-1}\}$ fixed, $E_{i-1,F}(\cdot) \equiv E_{i-1}$, $E_F(\cdot) \equiv E(\cdot)$ (unconditional mean).

Any real valued function $g(\mathbf{x};\boldsymbol{\theta})$ of the vector of random variate $\mathbf{x}$ and the parameter $\boldsymbol{\theta}$, that can be used to estimate $\boldsymbol{\theta}$ is called an estimating function. Under certain regularity conditions, the function $g(\mathbf{x};\boldsymbol{\theta})$ is called a regular unbiased estimating function if $E[g(\mathbf{x};\boldsymbol{\theta})] = 0$ for all $\boldsymbol{\theta}$ (see Godambe, 1985).

Among all regular unbiased estimating functions $g(\mathbf{x};\boldsymbol{\theta})$, $g^*(\mathbf{x};\boldsymbol{\theta})$ is said to be optimum if

$$E[g^2(\mathbf{x};\boldsymbol{\theta})] / \left\{E\left[\partial g(\mathbf{x};\boldsymbol{\theta})/\partial\boldsymbol{\theta}\right]\right\}^2 \tag{8}$$

is minimized for all $F \in \Theta$ at $g(\mathbf{x};\boldsymbol{\theta}) = g^*(\mathbf{x};\boldsymbol{\theta})$.

Then, we estimate $\boldsymbol{\theta}$ by solving the optimum estimating equation

$$g^*(\mathbf{x};\boldsymbol{\theta}) = 0.$$

Main Results

We consider the estimating functions $g(\mathbf{x};\boldsymbol{\theta})$ of the linear form given by

$$g(\mathbf{x};\boldsymbol{\theta}) = \sum_{i=1}^{n} h_i a_{i-1} ,$$

where the functions h_i are as defined before and a_{i-1} is a function of the random variates $\mathbf{x}_{i-1} = \{x_1, x_2, \cdots, x_{i-1}\}$ and the parameter $\boldsymbol{\theta}$ for all $i = 1,2,\cdots,n$. We consider the class of linear estimating functions L generated by $g(\mathbf{x};\boldsymbol{\theta})$. Note that $g(\mathbf{x};\boldsymbol{\theta})$ being linear in h_i, the class L corresponds to linear functions in Gauss-Markov set-up for linear models.

The following theorem due to Godambe (1985) establishes the optimum EF.

Theorem

In the class L of linear unbiased estimating functions $g(\mathbf{x};\boldsymbol{\theta})$, the function $g^*(\mathbf{x};\boldsymbol{\theta})$ minimizing Equation (8) is given by

$$g^*(\mathbf{x};\boldsymbol{\theta}) = \sum_{i=1}^{n} h_i a_{i-1}^* ,$$

where

$$a_{i-1}^* = E_{i-1}\left[\partial h_i / \partial \boldsymbol{\theta}\right] / E_{i-1}[h_i^2] .$$

Proof: See (Godambe, 1985).

The Section 3 considers a large scale simulation study to investigate the sensitivity of parameter estimates obtained by the ML, QML and EF methods.

3 A SIMULATION STUDY

3.1 A COMPARISON OF ESTIMATION METHODS

Consider the following ACD (1,1) as a special case of Equation (3) given by

$$x_i = \psi_i \varepsilon_i , \tag{9}$$

and

$$\psi_i = \omega + \alpha_1 x_{i-1} + \beta_1 \psi_{i-1} , \tag{10}$$

where $\{\varepsilon_i\}$ is a sequence of iid positive random variables and $\boldsymbol{\theta} = (\omega, \alpha_1, \beta_1)$ is the vector of parameters.

For the sake of illustration of our methods, suppose $\{\varepsilon_i\}$ follows the standardized Generalized Gamma distribution, it can be seen that the mean and variance of the corresponding conditional distribution of x_i are respectively $\mu_i = \psi_i$ and $\sigma_i^2 = \psi_i^2 \sigma_\varepsilon^2$. That is

$$x_i \mid F_{i-1} \sim (\psi_i, \psi_i^2 \sigma_\varepsilon^2)$$

where σ_ε^2 is the variance of ε_i.

The parameters of the ACD models based on ML or QML methods are obtained by maximizing the log-likelihood functions as defined in Equation (5), based on Generalized Gamma distribution and the log likelihood function defined in Equation (4).

In order to obtain the EF estimates, we will use the h_i function given by Peiris et al. (2007), Pathmanathan et al. (2009) and Allen et al. (2012) given by $h_i = \psi_i - x_i$. We can easily to verify the following:

- $E(h_i) = E[E(\psi_i - x_i \mid F_{i-1})]$
  $= E(\psi_i) - E(E(x_i \mid F_{i-1}))$
  $= E(\psi_i) - E(\psi_i)$
  $= 0$

and

- For $i < j$,
  $E(h_i h_j) = E[E\{(\psi_i - x_i)(\psi_j - x_j) \mid F_{j-1}\}]$
  $= E[E\{\psi_i \psi_j - x_i \psi_j - \psi_i x_j + x_i x_j \mid F_{j-1}\}]$
  $= E(\psi_i \psi_j) - E[\psi_j E(x_i \mid F_{j-1})] - E[\psi_i E(x_j \mid F_{j-1})] + E[E(x_i x_j \mid F_{j-1})]$
  $= E(\psi_i \psi_j) - E(\psi_j x_i) - E(\psi_i \psi_j) + E[x_i E(x_j \mid F_{j-1})]$
  $= E(\psi_i \psi_j) - E(\psi_j x_i) - E(\psi_i \psi_j) + E(x_i \psi_j)$
  $= 0$

Therefore, h_i is an unbiased and mutually orthogonal estimating function.

Using the above theorem of Godambe (1985), the corresponding a_{i-1}^* is

$$a_{i-1}^* = \frac{E_{i-1}[\partial h_i / \partial \boldsymbol{\theta}]}{\psi_i^2 \sigma_\varepsilon^2},$$

where

- $E_{i-1}\left[\frac{\partial h_i}{\partial \omega}\right] = E_{i-1}\left[\frac{\partial \psi_i}{\partial \omega}\right] = 1 + \beta_1 \frac{\partial \psi_{i-1}}{\partial \omega}$,
- $E_{i-1}\left[\frac{\partial h_i}{\partial \alpha_1}\right] = E_{i-1}\left[\frac{\partial \psi_i}{\partial \alpha_1}\right] = x_{i-1} + \beta_1 \frac{\partial \psi_{i-1}}{\partial \alpha_1}$,
- $E_{i-1}\left[\frac{\partial h_i}{\partial \beta_1}\right] = E_{i-1}\left[\frac{\partial \psi_i}{\partial \beta_1}\right] = \psi_{i-1} + \beta_1 \frac{\partial \psi_{i-1}}{\partial \beta_1}$.

Now we obtain the optimal set of estimates by solving the system of equations

$$\sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}E_{i-1}\left[\frac{\partial h_i}{\partial \boldsymbol{\theta}}\right](\psi_i - x_i) = 0 \tag{11}$$

for the parameter vector $\boldsymbol{\theta} = (\omega, \alpha_1, \beta_1)$.

The asymptotic variance-covariance matrix of the coefficient vector $\hat{\boldsymbol{\theta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\beta}_1)$ may be written as the 3 by 3 matrix V^{-1}, where

$$V = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix}$$

has elements given by

$$V_{11} = E_{i-1}\left[\frac{\partial g_1^*}{\partial \omega}\right] = \sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}\left(\frac{\partial \psi_i}{\partial \omega}\right)^2, V_{22} = E_{i-1}\left[\frac{\partial g_2^*}{\partial \alpha_1}\right] = \sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}\left(\frac{\partial \psi_i}{\partial \alpha_1}\right)^2,$$

$$V_{12} = V_{21} = E_{i-1}\left[\frac{\partial g_1^*}{\partial \alpha_1}\right] = \sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}\left(\frac{\partial \psi_i}{\partial \omega}\right)\left(\frac{\partial \psi_i}{\partial \alpha_1}\right),$$

$$V_{13} = V_{31} = E_{i-1}\left[\frac{\partial g_1^*}{\partial \beta_1}\right] = \sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}\left(\frac{\partial \psi_i}{\partial \omega}\right)\left(\frac{\partial \psi_i}{\partial \beta_1}\right),$$

$$V_{23} = V_{32} = E_{i-1}\left[\frac{\partial g_2^*}{\partial \beta_1}\right] = \sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}\left(\frac{\partial \psi_i}{\partial \alpha_1}\right)\left(\frac{\partial \psi_i}{\partial \beta_1}\right) \text{ and}$$

$$V_{33} = E_{i-1}\left[\frac{\partial g_3^*}{\partial \beta_1}\right] = \sum_{i=1}^{n}\frac{1}{\psi_i^2\sigma_\varepsilon^2}\left(\frac{\partial \psi_i}{\partial \beta_1}\right)^2.$$

The diagonal entries of V^{-1} are the variance of the estimate $\hat{\boldsymbol{\theta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\beta}_1)$. A similar technique can be extended to the general ACD (p,q) model (see Allen et.al (2012)).

Due to the fact that the true distribution is unknown in many practices, thus we use the Generalized Gamma distribution as it is more flexible than any other distribution in ACD modeling. For comparison, we apply all these methods of estimation using this Generalized Gamma distribution.

3.2 RESULTS AND DISCUSSIONS

Following the theory in Section 3.1, we generate the time series data x_i of length $n = 500$ from Equations (9) and (10) with known values for $\boldsymbol{\theta} = (\omega, \alpha_1, \beta_1)$. Now treating the parameters are unknown of an ACD (1,1) model, estimate them using the methods discussed in Sections 2.2 and 2.3. Finally, the above steps are repeated for $N = 2000$ times and compute the mean, bias, standard error and the mean square error for each parameter. These steps are repeated for series of length $n = 2000$.

Tables 1 to 4 show the simulation results for each set of parameter values using the ML, QML and EF methods. The Tables 1 and 3 show the results for the sample size, $n = 500$. It is clear from our simulations that when the true distribution follows the Generalized Gamma distribution, the ML in general produces slightly smaller standard errors than the EF approach for all estimates. We also observe that QML based on Exponential likelihood function performs very well giving comparable estimates based on the ML and EF methods. The simulation results also show that likelihood functions based on Lognormal and Weibull distributions produce large values of the estimated bias and mean squares errors in their parameter estimates. If we refer to the parameter estimate of ω in Tables 1 and 3, then it is clear that the standard errors and mean squares errors are inconsistent. In the case of the true distribution is unknown, we may consider using alternative simple likelihood functions to estimate the model parameters. Simulation results in Tables 1 and 3 shows that the incorrect likelihood functions will affect the parameter estimates in terms of their means and the standard errors while the EF are robust. Therefore, we conjecture the EF approach is an efficient alternative method for parameter estimation.

From Tables 2 and 4, it is clear that the MSE for all theree estimates have been reduced with increase of sample size from $n = 500$ to $n = 2000$. Even though the sample size has increased, the QML using the Lognormal likelihood function performs poorly than the other two methods.

In addition to the above simulation study, the theoretical results of standard errors of each parameter based on the EF method for ACD (1,1) model with $n = 2000$ can be obtained from the entries of the matrix V^{-1} as shown in Section 3.1. For the ACD (1,1) model when the error of distribution follows a Generalized Gamma distribution, the asymptotic standard errors of $\hat{\omega}, \hat{\alpha}_1$ and $\hat{\beta}_1$ respectively are 0.0141, 0.0224 and 0.0400 and those are closer to the estimated standard errors. The accuracy of these estimated SE's increase as the sample size increases. Table 4 shows that the estimated standard errors are close to the asymptotic SE's given above.

From Figures 1 to 3, one observes that the asymptotic distribution of $\hat{\boldsymbol{\theta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\beta}_1)$ follows an approximate normal distribution, $\hat{\boldsymbol{\theta}} \sim N(\boldsymbol{\theta}, V^{-1})$ when the error of distribution is Generalized Gamma. Although these results show that the ML, QML and the EF estimates are comparable, it is important to note that the computation time for the EF method is at least 5 times shorter than the ML or QML methods to obtain the solution (See Allen et al. (2012)).

Table 1: Estimation results based on the standardized Generalized Gamma ACD (1,1) with $\kappa = 2.0$ and $\gamma = 1.2$ ($n = 500$; $\omega = 0.30$, $\alpha_1 = 0.20$, $\beta_1 = 0.70$ and $\psi_1 = 0.50$).

		QML			ML	EF
		Exponential	Lognormal with $\sigma = 0.5$	Weibull with $\gamma = 1.5$	Generalized Gamma with $\kappa = 2.0$ $\gamma = 1.2$	
$\hat{\omega}$	Mean	0.3726	0.3896	0.3763	0.3718	0.3726
	Bias	0.0726	0.0896	0.0763	0.0718	0.0726
	SE	0.1250	0.1638	0.1354	0.1241	0.1251
	MSE	0.0209	0.0349	0.0242	0.0206	0.0209
$\hat{\alpha}_1$	Mean	0.2025	0.2077	0.2026	0.2028	0.2025
	Bias	0.0025	0.0077	0.0026	0.0028	0.0025
	SE	0.0381	0.0450	0.0389	0.0381	0.0382
	MSE	0.0015	0.0021	0.0015	0.0015	0.0015
$\hat{\beta}_1$	Mean	0.6726	0.6710	0.6715	0.6726	0.6726
	Bias	-0.0274	-0.0029	-0.0285	-0.0273	-0.0274
	SE	0.0640	0.0790	0.0686	0.0638	0.0641
	MSE	0.0049	0.0062	0.0055	0.0048	0.0049

Table 2: Estimation results based on the standardized Generalized Gamma ACD (1,1) with $\kappa = 2.0$ and $\gamma = 1.2$ ($n = 2000$; $\omega = 0.30$, $\alpha_1 = 0.20$, $\beta_1 = 0.70$ and $\psi_1 = 0.50$).

		QML			ML	EF
		Exponential	Lognormal with $\sigma = 0.5$	Weibull with $\gamma = 1.5$	Generalized Gamma with $\kappa = 2.0$ $\gamma = 1.2$	
$\hat{\omega}$	Mean	0.3200	0.3315	0.3229	0.3200	0.3210
	Bias	0.0200	0.0315	0.0229	0.0200	0.0210
	SE	0.0547	0.0646	0.0604	0.0547	0.0554
	MSE	0.0034	0.0052	0.0042	0.0034	0.0035
$\hat{\alpha}_1$	Mean	0.2012	0.2072	0.2025	0.2013	0.2010
	Bias	0.0012	0.0072	0.0025	0.0013	0.0010
	SE	0.0186	0.0215	0.0192	0.0186	0.0183
	MSE	0.0003	0.0005	0.0004	0.0003	0.0003
$\hat{\beta}_1$	Mean	0.6918	0.6916	0.6904	0.6917	0.6916
	Bias	-0.0082	-0.0084	-0.0096	-0.0083	-0.0084
	SE	0.0300	0.0341	0.0318	0.0301	0.0301
	MSE	0.0001	0.0012	0.0011	0.0001	0.0001

Table 3: Estimation results based on the standardized Generalized Gamma ACD (1,1) with $\kappa = 2.0$ and $\gamma = 1.2$ ($n = 500$; $\omega = 0.10$, $\alpha_1 = 0.30$, $\beta_1 = 0.50$ and $\psi_1 = 0.50$).

		QML			ML	EF
		Exponential	Lognormal with $\sigma = 0.5$	Weibull with $\gamma = 1.5$	Generalized Gamma with $\kappa = 2.0$ $\gamma = 1.2$	
$\hat{\omega}$	Mean	0.1073	0.1119	0.1072	0.1082	0.1073
	Bias	0.0073	0.0119	0.0072	0.0082	0.0073
	SE	0.0290	0.0357	0.0304	0.0299	0.0309
	MSE	0.0009	0.0014	0.0010	0.0010	0.0010
$\hat{\alpha}_1$	Mean	0.3008	0.3078	0.3017	0.2999	0.2995
	Bias	0.0008	0.0078	0.0017	-0.0001	-0.0005
	SE	0.0460	0.0548	0.0479	0.0461	0.0454
	MSE	0.0021	0.0031	0.0023	0.0021	0.0021
$\hat{\beta}_1$	Mean	0.4839	0.4836	0.4857	0.4832	0.4854
	Bias	-0.0161	-0.0164	-0.0143	-0.0168	-0.0146
	SE	0.0833	0.0975	0.0866	0.0851	0.0854
	MSE	0.0072	0.0098	0.0077	0.0075	0.0075

Table 4: Estimation results based on the standardized Generalized Gamma ACD (1,1) with $\kappa = 2.0$ and $\gamma = 1.2$ ($n = 2000$; $\omega = 0.10$, $\alpha_1 = 0.30$, $\beta_1 = 0.50$ and $\psi_1 = 0.50$).

		QML			ML	EF
		Exponential	Lognormal with $\sigma = 0.5$	Weibull with $\gamma = 1.5$	Generalized Gamma with $\kappa = 2.0$ $\gamma = 1.2$	
$\hat{\omega}$	Mean	0.1019	0.1055	0.1021	0.1019	0.1019
	Bias	0.0019	0.0055	0.0021	0.0019	0.0019
	SE	0.0134	0.0159	0.0136	0.0133	0.0137
	MSE	0.0002	0.0003	0.0002	0.0002	0.0002
$\hat{\alpha}_1$	Mean	0.3002	0.3093	0.3013	0.3003	0.2997
	Bias	0.0002	0.0093	0.0013	0.0003	-0.0003
	SE	0.0223	0.0261	0.0228	0.0223	0.0231
	MSE	0.0005	0.0008	0.0005	0.0005	0.0005
$\hat{\beta}_1$	Mean	0.4957	0.4953	0.4955	0.4957	0.4967
	Bias	-0.0043	0.0047	0.0045	0.0043	0.0033
	SE	0.0386	0.0445	0.0394	0.0385	0.0393
	MSE	0.0015	0.0020	0.0016	0.0015	0.0016

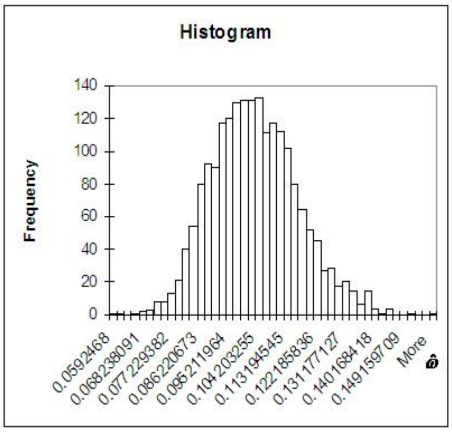

Figure 1: The histogram for the $\hat{\omega}$ in the ACD (1,1) model with a standardized Generalized Gamma distribution by the EF method when the sample size of $n = 2000$.

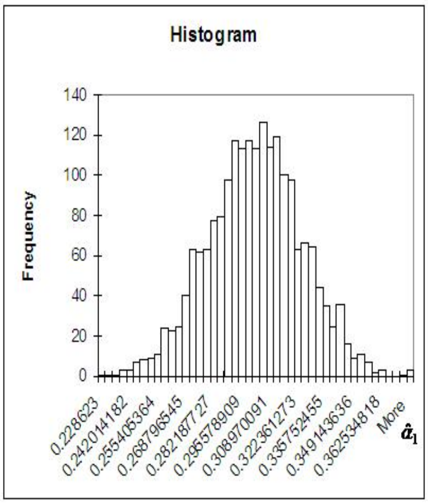

Figure 2: The histogram for the $\hat{\alpha}_1$ in the ACD (1,1) model with a standardized Generalized Gamma distribution by the EF method when the sample size of $n = 2000$.

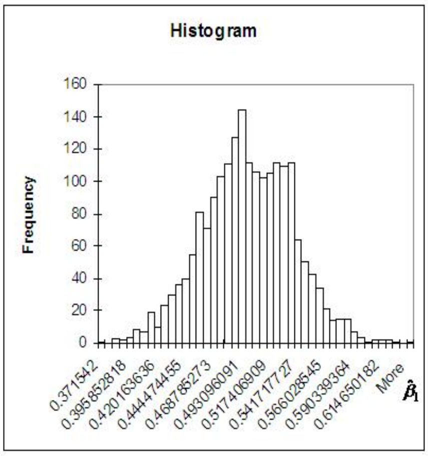

Figure 3: The histogram for the $\hat{\beta}_1$ in the ACD (1,1) model with a standardized Generalized Gamma distribution by the EF method when the sample size of $n = 2000$.

4 CONCLUSION REMARKS

This paper justifies the usefulness of the EF approach in estimation of parameters of ACD models in financial economic applications. Based on our simulation study, we have shown that the EF approach is comparable with the traditional ML or QML methods. However, the EF method is more computationally efficient and easy to apply in practice than the existing ML or QML methods. The ML estimates are slightly better than the EF estimates when the true distribution is known. Since the true distribution is seldom known in practice, the EF approach gives good, very reliable estimates in many economic applications. In addition, newly derived asymptotic normality properties of $\hat{\boldsymbol{\theta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\beta}_1)$ can be used in related hypothesis testing problems.

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