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A FUZZY APPROACH TO INCREASE ACCURACY AND PRECISION IN MEASUREMENT SYSTEM ANALYSIS

Abstract. *Due to widely practical application of quality techniques, many decisions in processes and equipment evaluation, control, and improvement are made every day in manufacturing, research, and development. Besides, the philosophy of measurement system analysis (MSA) depends on measurement error which hides true process capability. So it must be implemented prior to any process improvement activities to minimize the measurement errors. Since the capability of each quality system is related to the accuracy of its measurement system, this research developed a novel method for MSA. To increase accuracy and precision of MSA, analyzing measurement systems under fuzziness of its indices have been investigated. In order to obtain much more accurate indices, we develop a new method in measurement system analysis with fuzzy considerations which makes an important contribution within MSA literature. To do so, we propose fuzzy measurement system analysis (FMSA) by considering gauge repeatability and reproducibility (GR&R) index as a triangular fuzzy number. Finally, the applicability of the proposed method has been demonstrated within a case study in automotive parts industry.*

Keywords**:** *Measurement system analysis (MSA); Gauge repeatability and reproducibility (GR&R); Quality techniques; Fuzzy approach.*

JEL Classification: C61

1. Introduction and literature review

Measurement system analysis (MSA) has been implemented as an applicable quality techniques in each process. Nowadays many quality control techniques are used for recognizing reasons of error and preventing their occurrence. Undoubtedly, comprehension of quantifying process performance is essential for successful quality improvement initiatives. One of the most important quality techniques for decreasing process error in factories is analyzing measurement systems. MSA is the process of evaluating an unknown quantity and expressing it into numbers that is usually considered as precedence of any statistical process control. Also, MSA has been provided as one of the main requirements in the old QS9000 Quality Standard, Six Sigma technique, and even new standards such as ISO TS16949. Expanded applications of MSA are due to its various advantages including promotion of the compatibility of the measurement system for the given process and reduction of the contamination of measurement variation in the total process variation (Automotive Industry Action Group (AIAG), 2002).

MSA is based on an important philosophy which believes measurement error lied in any process measurement method. Therefore, it should be considered as the precedent process of any quality measurement system (Harry and Lawson, 2002). MSA quantifies measurement errors via the examination of multiple sources of variation in a process. These variations are consisted of the variation resulting from the measurement system, the operators, and the parts themselves. Since statistical measures are estimated by data which are obtained by sampling, they are usually unreliable (Grubbs, 1973). In this case, it is helpful to think of a measured value as the sum of two variables: (I) the quantity of measured value and (II) its error (e_i) which is formulated as Eq. (1).

$$Y_i(Measured\_value) = X_i(True\_value) + e_i \tag{1}$$

The measurement system increases the total observed variability (σ^2_{obs}) of the measured parts. In any measuring, some of the observed variability is due to variability in the process (σ^2_p), whereas the rest variability is due to the measurement error or gauge variability (σ^2_{msa}). The variance of the total observed measurements can be expressed as Eq. (2). It means that total variability equals to the sum of process variability and measurement variability (Montgomery, 2009).

$$\sigma^2_{obs} = \sigma^2_p + \sigma^2_{msa} \tag{2}$$

The σ_{msa}^2 includes two major types of error which are called repeatability and reproducibility. Repeatability ($\sigma_{\text{Repeatability}}^2$) which can be determined by measuring a part for several times, quantifies the variability in a measurement system resulted from its gauge (Pan, 2006; Smith et al., 2007). Reproducibility ($\sigma_{\text{Reproducibility}}^2$) which is determined from the variability created by several operators measuring a part for several times, quantifies the variation in a measurement system resulted from the operators of the gauge and environmental factors (Tsai, 1989; Burdick et al., 2003). Square root of σ_{msa}^2 is called gauge repeatability and reproducibility (GR&R) that models the all error related to the gauge. It can be shown as Eq. (3).

$$\sigma_{msa}^2 = \sigma_{repeatability}^2 + \sigma_{reproducibility}^2 \quad (3)$$

Foster (2006) proposed some procedures for calculating different indices of MSA in order to calculate GR&R as major output of MSA. In order to distinguish product variance from device variance carried out MSA studies on two or more measurement devices and proposed a procedure for estimating the sensitivity of the measurement devices (Juran and Gyrna, 1993). Senol (2004) statistically evaluated MSA method by the means of designed experiments to minimize α-β risks and n (sample size). A GR&R study which estimates the repeatability and reproducibility components of measurement system variation with the primary objective of assessing whether or not the gauge is appropriate for the intended applications was carried out by Pan (2006). Evaluating measurement and process capabilities by GR&R with four quality measures presented by Al-Refaie and Bata (2010).

One reason that causes indices calculated by sample data are unreliable is the uncertainty of data. Therefore statistical calculations such as standard deviation, point and interval estimation, hypothesis testing and other similar one are used. Besides, unreliability of indices has another reason which is resulted from the impreciseness of data. In the literature to deal with impreciseness usually fuzzy concept (introduced by Zadeh (1965)) is used.

However, in the literature of the quality issues any work with the legend of fuzzy MSA was not found and we have just reviewed the most relevant work such as the application of fuzzy modeling in different quality indices and quality control charts. Lee (2001) and Hong (2004) proposed C_{pk} index estimation using fuzzy numbers. Parchami et al. (2005) using fuzzy specification limits rather than precise ones proposed new fuzzy types of all these process capability indices. Parchami and Mashinchi (2007) introduced a new method that uses confidence interval of capability indices to produce fuzzy number for them. Faraz and Bameni Moghadam (2007) and Gulbay and Kahraman (2006) proposed efficient methods for creating fuzzy quality control charts.

According to what was mentioned, MSA help to judge about compatibility of the measurement system with the given measurement process and provide conditions for more reliable decisions. This study makes important contribution to the MSA literature and develops fuzzy concept on MSA method to create indices of MSA much more accurate.

The rest of the paper is organized as follows. The next section illustrates classical MSA and its different indices along with developing fuzzy concept to create proposed methodology. Section 3 describes a case study in automotive parts industry. Section 4 discusses reasons for this development. Finally, in Section 5 conclusions and future research are given.

2. The developed methodology for MSA

In this section, the MSA method is investigated with the details at first. Then the new developed method is illustrated step by step.

2.1. *Traditional Measurement system analysis*

Measurement system is the collection of instruments or gauges, standards, operations, methods, fixtures, software, personnel, environment and assumption used to quantify a unit of measure or fix assessment, Ford, General Motors and Chrysler (Ford, 1995). Correspondingly, MSA is a collection of statistical methods for the analysis of measurement system capability (Smith et al., 2007). It seeks to describe, categorize, evaluate the quality of measurements; improve the usefulness, accuracy, precision, meaningfulness of measurements; and propose methods for developing better measurement instruments by Montgomery and Runger (1993). Some stated goals of MSA are to estimate components of measurement error, estimate the contribution of measurement error to the total variability of a process or equipment parameter, determine stability of a metrology tool over time, and to compare and correlate multiple metrology tools. Measurement process is a kind of production process that its output is number. Fig.1 presents measurement process with its inputs and outputs (Ford, 1995).

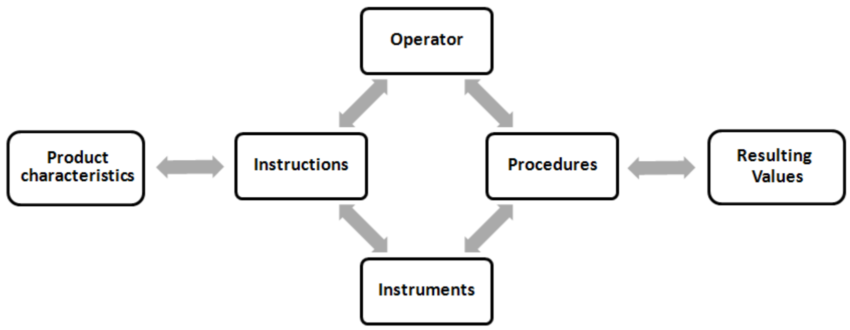

Fig.1. Measurement system analysis process

According to the type of data, MSA has two categories of measurements; quantitative measurement and qualitative one. In this paper, quantitative measurements are discussed. In the rest of the section, we illustrate the steps required to execute the MSA.

2.1.1. Bias

The difference between the observed average of measurements and the master average of the same parts using precision instruments is defined as bias metric (Ford, 1995). Actually, bias is a measure that represents difference between the averages value of the measurement and certified value of a specific part.

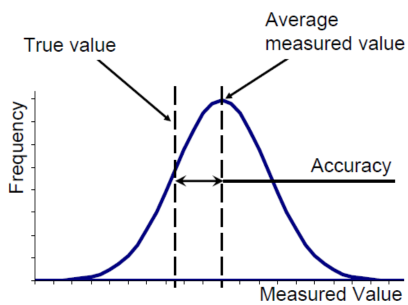

Fig.2. Scheme of bias (Ford, 1995)

Fig.2 represents bias concept schematically. For computing this index, we measure a part with an instrument for at least ten times, and then we should acquire average of these observations and compare with true value of the part. We can obtain value of bias by Eq. (3).

$$B = \bar{x}_g - x_m \tag{4}$$

2.1.2. Capability

Capability is a measure of process' ability to consistently produce a result that meets the specification requirements. The term process capability was synonymous with process variation measures such as standard deviation or range of the observed data. However these measures do not consider customer requirements and it is not suitable for general comparison among processes (Leung and Spiring, 2007). Capability indices C_p, C_{pk}, C_{pm}, and C_{pmk} have been proposed in the manufacturing and service industries, providing numerical measures on whether a process is capable of reproducing items within the specification limits (Shishebori and Hamadani, 2010). Similarly, C_p, C_{pk} are used to show the capability of the measurement gauge. The indices C_g , C_{gk} is defined by Eq. (5) and (6).

$$C_g = \frac{0.2T}{6S_g} \tag{5}$$

$$C_{gk} = \frac{0.1T - \left|\bar{x}_g - x_m\right|}{3S_g} \tag{6}$$

Where T is part tolerance and S_g shows standard deviation of observed values using measurement instrument. Minimum acceptance criteria for C_g, C_{gk} is equal to 1.33, [24].

2.1.3. Gauge Repeatability and Reducibility (GR&R)

Thereof, total measurement variation is sum of variation due to repeatability and reproducibility (3). Repeatability and reproducibility can influence the precision and accuracy respectively.

2.1.3.1. Repeatability

The same characteristic of the product should be measured repeatedly in order to determine the sensitivity of the measurement process (Foster, 2006). When an inspector uses the same gauge to measure a product several times under the same conditions, several different values of measurement may occur. This error, called repeatability, comes from the gauge itself (Montgomery and Runger, 1993). Repeatability is computed as Eq. (7).

$$EV = 5.15\frac{\overline{R}}{d_2^*} \tag{7}$$

Where $\overline{R}$ is average of variation range, d_2^* obtained from a specific table, and *EV* is tool variation. Also 5.15σ interval involve 99 percents of data in normal distribution. Fig.3 represents the variation among successive measurements of the same characteristic, by the same person using the same instrument (Ford, 1995).

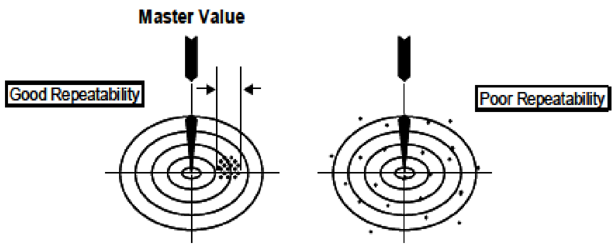

Fig.3. Precision of Repeatability (Ford, 1995)

2.1.3.2. Reproducibility

This error occurs when different inspectors measure a product under the same condition. Practically, it is due to deficient trained inspectors or out of standard measuring methods (Montgomery and Runger, 1993). It is computed as Eq. (8).

$$AV = \sqrt{(5.15\frac{\overline{X}_{DIF}}{d_2^*})^2 - \frac{(EV)^2}{n.r}} \tag{8}$$

where $\overline{X}_{DIF} = max(\overline{X}_i) - min(\overline{X}_i)$ that i=1,2,…,numbers of operator, d_2^* obtained from same table within previous subsection with *g=1* , *m* is number of operators, $\frac{\overline{x}_{DIF}}{d_2^*}$ is standard deviation of reproducibility, *EV* is repeatability value, *AV* is appraiser variation, *n* is number of used parts and *r* is number of trials that each piece is measured. $5.15\frac{\overline{X}_{DIF}}{d_2^*} >> \frac{(EV)^2}{n.r}$ then $AV = \frac{5.15\overline{X}_{DIF}}{d_2^*}$ and otherwise reproducibility value is equal to zero.

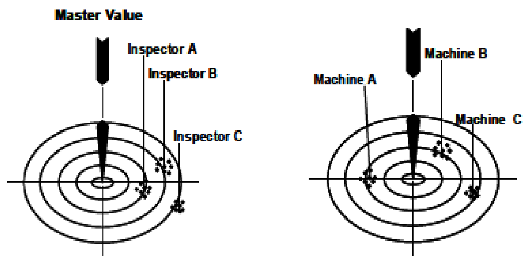

Fig.4. Precision of Reducibility (Ford, 1995)

Fig.4 represents the standard deviation of the averages of the measurements made by different persons, machines, tools, when measuring the identical characteristic on the same part (Ford, 1995).

2.1.3.3. GR&R

A GR&R study is a method of determining the suitability of a gauge system for measuring a particular process. Every measurement has some associated error, and if this error is large compared to the allowable range of values (the tolerance band), the measuring device will frequently accept bad parts and reject good ones (AIAG, 2002).

Total GR&R is the estimate of the combined estimated variation from repeatability and reproducibility. In a GR&R study, we try to quantify measurement variation as a percent of process variation. GR&R index is computed as Eq. (9).

$$R\&R=\sqrt{EV^2+AV^2} \tag{9}$$

An ideal measurement system should not have any variation. However, this is impossible and we have to be satisfied with a measurement system that has variation less than 10% of the process variation. As the portion of variation due to measurement system increases, the value of measurement system reduces. If this proportion is more than 30%, the measurement system is unacceptable. Table (1) summarizes system status for obtained %GR&R (AIAG, 2002).

Table 1

System status after computing GR&R

%GR&R	Decision Guideline
<%10	Acceptable measurement system
%10 to %30	This needs to be agreed with the customer
>%30	Unacceptable measurement system

2.2. Fuzzy measurement system analysis (FMSA)

In this section, some basic concept of fuzzy sets and fuzzy numbers are reviewed. Fuzzy set theory which was introduced by Zadeh (1965) is a typical method for encountering with ambiguity and imprecision. Since most practical and industrial methods and problems are encountered with imprecise data or lack of data considering fuzzy techniques help us to make our methods much more accurate.

Nowadays we deal with different quality problems that all of them are preceded by MSA. In this situation, if the quality of measurement system is low, it can be expected that process analysis will not be valid. Therefore, we consider MSA with fuzzy numbers to have a more precise and accurate measurement system and data analysis. This precise data analysis will lead to a more accurate decision making and quality system. Experimental results of a case study will represent performance of this new type of MSA in an industrial real world instance and show how MSA executed with fuzzy calculations in detail. Meanwhile, first some mathematical operations in fuzzy concept is reviewed, then they are expanded on different indices in MSA in the rest of this section.

Fuzzy calculations are handled with fuzzy numbers. A fuzzy number is a convex fuzzy subset of the real line *R* and is completely defined by its membership function. Different type of membership functions are considered in the literature of fuzzy numbers. This paper uses triangular membership function for different indices of MSA and shows detailed calculations of MSA in the environment of fuzzy concept. Rest of the section illustrated all of these calculations.

Denoting the triangular fuzzy number $\tilde{M}$ by a triplet (a,b,c) and $\tilde{N}$ by a triplet (d,e,f), the addition, subtraction, multiple, and division of the two triangular fuzzy numbers can be shown as follow (Zadeh, 1965).

$$\begin{aligned}
\tilde{M} + \tilde{N} &= (a,b,c) + (d,e,f) = (a+d, b+e, c+f) \\
\tilde{M} - \tilde{N} &= (a,b,c) - (d,e,f) = (a-f, b-e, c-d) \\
\tilde{M} \times \tilde{N} &= (a,b,c) \times (d,e,f) = (a \times d, b \times e, c \times f) \\
\tilde{M} \div \tilde{N} &= (a,b,c) \div (d,e,f) = (\frac{a}{f}, \frac{b}{e}, \frac{c}{d})
\end{aligned} \tag{10}$$

Our method for making a fuzzy triangular number is defined in Eq. (11).

$$\tilde{X}_b = (\tilde{X}_b \sigma_1, \tilde{X}_b, \tilde{X}_b \sigma_2); \quad \sigma_1 = 0.999, \sigma_2 = 1.001 \tag{11}$$

Corresponding shape of the fuzzy number is plotted as Fig.5.

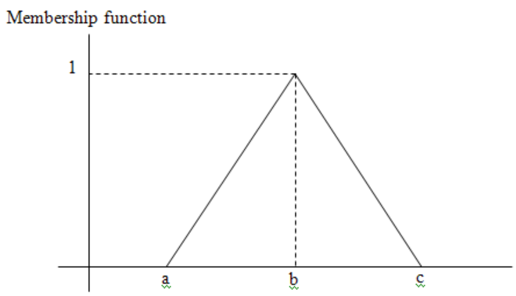

Fig.5. Membership function of *b* fuzzy number

It should be mentioned that in the paper for ranking two fuzzy numbers like (a,b,c) and (d,e,f) third member of each set consider as the criteria of comparison. For instance among fuzzy *M* and *N*, since *max (11,12)=12*, fuzzy M is selected as follows:

$$\tilde{N} = (6,8,11) \;\&\; \tilde{M} = (5,9,12) \tag{12}$$

The indices of fuzzy measurement system are as follow. Equations (13)-(17) are fuzzy indices (4), (7), (8), and (9) respectively. For the sake of completeness, we have given complete set of equations.

$$\begin{aligned}\tilde{B} &= \tilde{\bar{x}}_g - x_m \\ &= (\tilde{\bar{x}}_a - x_m, \tilde{\bar{x}}_b - x_m, \tilde{\bar{x}}_c - x_m) \\ &= (\tilde{B}_a, \tilde{B}_b, \tilde{B}_c)\end{aligned} \tag{13}$$

$$\begin{aligned}E\tilde{V} &= 5.15\frac{\tilde{\bar{R}}}{d_2^{\,*}} \\ &= 5.15\frac{(\tilde{\bar{R}}_a, \tilde{\bar{R}}_b, \tilde{\bar{R}}_c)}{d_2^{\,*}} \\ &= (\frac{5.15}{d_2^{\,*}}\tilde{\bar{R}}_a, \frac{5.15}{d_2^{\,*}}\tilde{\bar{R}}_b, \frac{5.15}{d_2^{\,*}}\tilde{\bar{R}}_c) \\ &= (E\tilde{V}_a, E\tilde{V}_b, E\tilde{V}_c)\end{aligned} \tag{14}$$

Then, according to corresponding ranking approach results are as follow, for example we suppose that i=1 is the largest member of a fuzzy number and i=2 is the smallest member of a fuzzy number.

$$\begin{aligned}
\tilde{\bar{X}}_{DIFF} &= max(\tilde{\bar{X}}_i) - min(\tilde{\bar{X}}_i) \\
&= max(\tilde{\bar{X}}_i^{\,a}, \tilde{\bar{X}}_i^{\,b}, \tilde{\bar{X}}_i^{\,c}) - min(\tilde{\bar{X}}_i^{\,a}, \tilde{\bar{X}}_i^{\,b}, \tilde{\bar{X}}_i^{\,c}) \\
&= (\tilde{\bar{X}}_1^{\,a}, \tilde{\bar{X}}_1^{\,b}, \tilde{\bar{X}}_1^{\,c}) - (\tilde{\bar{X}}_2^{\,a}, \tilde{\bar{X}}_2^{\,b}, \tilde{\bar{X}}_2^{\,c}) \\
&= (\tilde{\bar{X}}_1^{\,a} - \tilde{\bar{X}}_2^{\,c}, \tilde{\bar{X}}_1^{\,b} - \tilde{\bar{X}}_2^{\,b}, \tilde{\bar{X}}_1^{\,c} - \tilde{\bar{X}}_2^{\,a}) \\
&= (\tilde{\bar{X}}_{DIFF}^{\,a}, \tilde{\bar{X}}_{DIFF}^{\,b}, \tilde{\bar{X}}_{DIFF}^{\,c})
\end{aligned} \tag{15}$$

$$\begin{aligned}
A\tilde{V} &= \sqrt{(5.15\frac{\bar{X}_{DIF}}{d_2^{*}})^2 - \frac{(EV)^2}{n.r}} \\
&= \sqrt{(5.15\frac{(\tilde{\bar{X}}_{DIFF}^{\,a}, \tilde{\bar{X}}_{DIFF}^{\,b}, \tilde{\bar{X}}_{DIFF}^{\,c})}{d_2^{*}})^2 - \frac{(E\tilde{V}_a, E\tilde{V}_b, E\tilde{V}_c)^2}{n.r}} \\
&= (\sqrt{\frac{5.15}{d_2^{*}}\tilde{\bar{X}}^2_{DIFF}{}^{a} - \frac{5.15}{n.r.d_2^{*}}\tilde{\bar{R}}^2}, \\
&\quad \sqrt{\frac{5.15}{d_2^{*}}\tilde{\bar{X}}^2_{DIFF}{}^{b} - \frac{5.15}{n.r.d_2^{*}}\tilde{\bar{R}}_b^{\,2}}, \\
&\quad \sqrt{\frac{5.15}{d_2^{*}}\tilde{\bar{X}}^2_{DIFF}{}^{c} - \frac{5.15}{n.r.d_2^{*}}\tilde{\bar{R}}_a^{\,2}}) \\
&= (A\tilde{V}_a, A\tilde{V}_b, A\tilde{V}_c)
\end{aligned} \tag{16}$$

Finally, fuzzy GR&R is defined as (18). All of the membership functions of these indices are represented in next section within a case study.

$$\begin{aligned}
\tilde{R}\,\&\,\tilde{R} &= \sqrt{E\tilde{V}^2 + A\tilde{V}^2} \\
&= \sqrt{(E\tilde{V}_a, E\tilde{V}_b, E\tilde{V})^2 + (A\tilde{V}_a, A\tilde{V}_b, A\tilde{V}_c)} \\
&= (\tilde{R}\,\&\,\tilde{R}^a, \tilde{R}\,\&\,\tilde{R}^b, \tilde{R}\,\&\,\tilde{R}^c)
\end{aligned} \tag{17}$$

3. Implementing FMSA and comparison with traditional MSA

As a case study, which was investigated housing clutch in automotive parts industry in Kachiran Company in Asia within crisp environment, is considered to illustrate the proposed procedure for assessing the better performance and decision making. In the case, we have 10 parts, 3 operators, 2 trials and the tolerance of corresponding part is 62.2 ± 0.1. Then X_m is 62.2 and T is 0.2. Also, the coefficient for right hand side and left hand side corresponding fuzzy number is 0.001. The Table (2) shows case study's data and Tables (3) and (4) represent results of MSA's indices with fuzzy number. It should be mentioned that stability and linearity of the data had tested in an exact environment before the data become fuzzy. It means that our non fuzzy data had the both basic features which are stability and linearity.

Table 2
Case study data

Part Nr.	Operator 1 Measurements (*mm*)		Operator 2 Measurements (*mm*)		Operator 3 Measurements (*mm*)	
	M1	M2	M1	M2	M1	M2
1	(62.14,62.2,62.26)	(62.1,62.16,62.22)	(62.09,62.15,62.21)	(62.08,62.14,62.2)	(62.09,62.15,62.21)	(62.1,62.16,62.22)
2	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.14,62.2,62.26)
3	(62.05,62.11,62.17)	(62.06,62.12,62.18)	(62.05,62.11,62.17)	(62.04,62.10,62.16)	(62.04,62.1,62.16)	(62.05,62.11,62.17)
4	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)
5	(62.19,62.25,62.31)	(62.19,62.25,62.31)	(62.19,62.25,62.31)	(62.20,62.26,62.32)	(62.19,62.25,62.31)	(62.19,62.25,62.31)
6	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)
7	(62.07,62.13,62.19)	(62.08,62.14,62.2)	(62.08,62.14,62.20)	(62.07,62.13,62.19)	(62.07,62.13,62.19)	(62.07,62.13,62.19)
8	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)
9	(62.24,62.3,62.36)	(62.24,62.3,62.36)	(62.24,62.3,62.36)	(62.23,62.29,62.35)	(62.23,62.29,62.35)	(62.24,62.3,62.36)
10	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.21,62.27,62.33)

Table 3
Result in fuzzy environment

Index	Operator 1	Operator 2	Operator 3
$\tilde{\bar{X}}_1$	(62.13,62.19,62.26)	(62.13,62.19,62.25)	(62.13,62.19,62.25)
$\tilde{\bar{B}}_1$	(-0.07,-0.01,0.06)	(-0.07,-0.01,0.05)	(-0.07,-0.01,0.05)
$\tilde{\bar{R}}_1$	(-0.12,0.01,0.13)	(-0.12,0,0.13)	(-0.12,0.01,0.13)

Table 4
MSA indices with fuzzy numbers

Indices	Fuzzy Number
$\tilde{\tilde{R}}$	(-0.12,0.01,0.13)
$E\tilde{V}$	(-0.53,0.03,0.6)
$\tilde{\tilde{X}}_{DIFF}$	(-0.13,0,0.12)
$A\tilde{V}$	(0,0.01,0.29)
$\tilde{R}$ & $\tilde{R}$	(0,0.04,0.67)

4. Discussion

This paper propose a new method in MSA using tranguilar fuzzy number. For increasing accuracy and precision of our decision making method, we utilize fuzzy concept. Therefore, since fuzzy indices provide expanded area for decision makers, through this method the decision is made with more accuracy. Although fuzzy MSA can be a considerable issue, in the literature any corresponding paper has not been found. To clarify concept of fuzzy indices in MSA, Fig. 6-9 plots the membership functions of MSA indices.

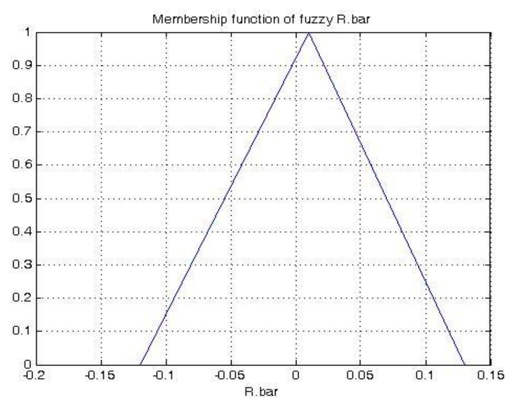

Fig.6. Membership function of $\tilde{\tilde{R}}$

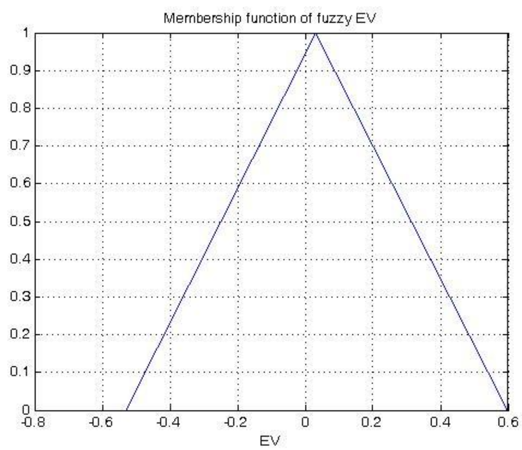

Fig.7. Membership function of $E\tilde{V}$

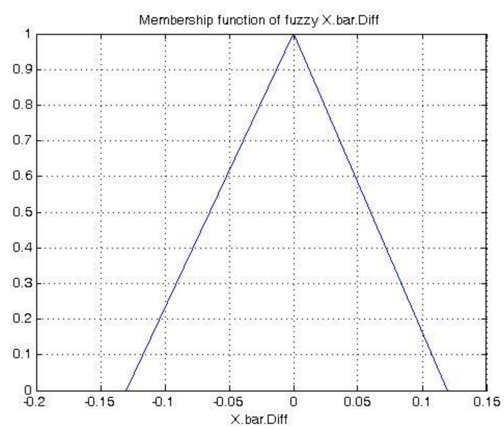

Fig.8. Membership function of $\tilde{\bar{\bar{X}}}_{DIFF}$

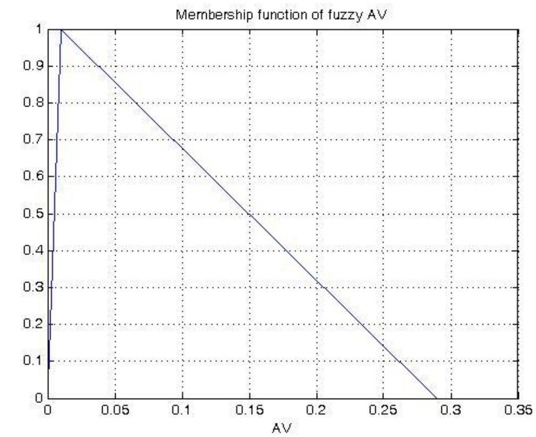

Fig.9. Membership function of $A\tilde{V}$

Finally, in Fig.10 we introduce membership function of fuzzy GR&R as triangular fuzzy number. Considering MSA indices as fuzzy numbers improves quality of measurement system and causes corresponding decisions being made with more information.

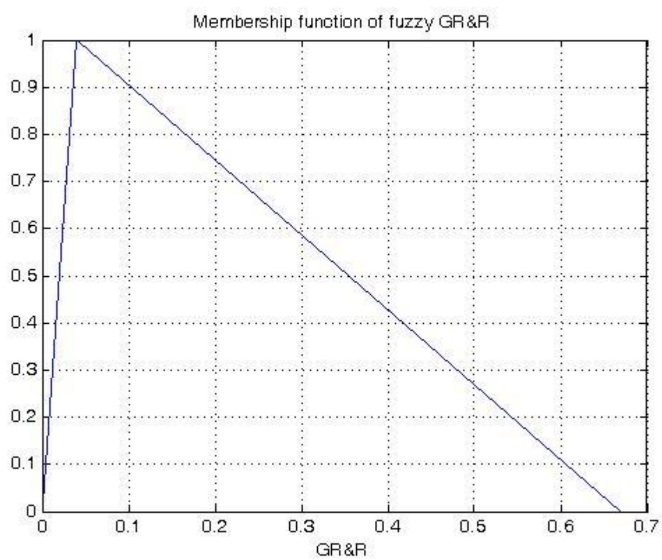

Fig.10. Membership function of $\tilde{R}\,\&\,\tilde{R}$

5. Conclusion and directions for future researches

This research investigates one of the most practical quality techniques namely MSA which is a collection of statistical methods for the analysis of measurement systems. Since most practical problems encountered with imprecise data considering fuzzy concept help us to make methods much more accurate. Usually, the underlying data are assumed to be precise numbers, but it is much more realistic in general to consider fuzzy values which are imprecise numbers. However, fuzzy MSA can be a considerable issue; in the literature any corresponding paper has not been found. Thus, in this article MSA was considered in fuzzy environment and triangular fuzzy numbers are introduced for MSA's indices. Finally, a real-world example taken from a housing clutch manufacturing process was examined to explain efficient performance of FMSA more explicitly. For future research, we will use other types of membership functions that can leads to better results. Furthermore, other aspects of MSA are qualitative measurement data, so we can expand this method for qualitative data. Other index of MSA such as fuzzy stability or fuzzy linearity can also be considered. In addition to the ranking approach in this paper, one can develop a new ranking approach.

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