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THE COMPUTATIONAL OF STOCK MARKET VOLATILITY FROM THE PERSPECTIVE OF HETEROGENEOUS MARKET HYPOTHESIS

Abstract: *This study investigates interday and intraday time-varying volatility modelling and forecasting based on the heterogeneous market hypothesis. The trading activities of heterogeneous market participants can be categorized into several time durations. These characteristics can be modelled by the autoregressive conditional heteroscedasticity and heterogeneous autoregressive models using the Standard and Poor (S&P500) index as the empirical study. Besides the common sum-of-square intraday realized volatility, we also advocate two power variation realized volatilities to overcome the possible abrupt jumps during the credit crisis with various frequencies. The empirical forecast evaluations consistently show that the realized volatility models are outperformed the interday data models for different frequency data. These empirical findings have implications for financial econometrics modelling, portfolio strategies and risk managements.*

Keywords: *realized volatility, fractionally integrated, heterogeneous autoregressive, market efficiency.*

JEL classification: C22, C52, C58, G14

1. Introduction

The behaviour of high frequency volatility (realized volatility) financial time series has closely linked to informationally market efficiency (Fama, 1998) concept. Over decades, researchers and investors have investigated possible new findings[1] to improve the existence efficient market hypothesis (EMH) in order to

[1] Lo (2004,2005) proposes a new framework called the Adaptive markets hypothesis (AMH) that reconciles market efficiency with behavioural alternatives by applying the principles of evolution such as competition, adaptation and natural selection to financial interactions. This hypothesis emphasises on the counterexamples to economics rationality such as loss aversion, overconfidence, overreaction, mental accounting and other

understand the actual information flow underlies financial markets. Heterogeneous market hypothesis is among the new ideas that recommended non-homogeneous market participants. This concept has been introduced by Muller et.al. (1997), Dacorogna et.al. (1998) and Peters (1994) in the stock and FX markets. Lux and Marchesi (1999) relates this concept using simulation models that include participants with different interest and strategies. Under the normality assumption, their models are able to capture the empirical stylized facts such as heavy-tailed, long-range dependence and scaling law properties. Another related interesting approach introduces by Andersen and Bollerslev (1997) is developed heterogeneity with a mixture of normal distributions. By aggregating distributions with different shape (variance) and location (mean) parameters, the model assembles the market by a different group of participants with dissimilar interest and strategies. This approach is also exhibited heavier tails than a normal distribution.

The heterogeneity of a particular financial market may arise from reaction of market participants to new information entering market. All the market participants may differ from their endowments, interests, risk profiles, degree of information, contractual constraints, motivations of trading and reactions to news.

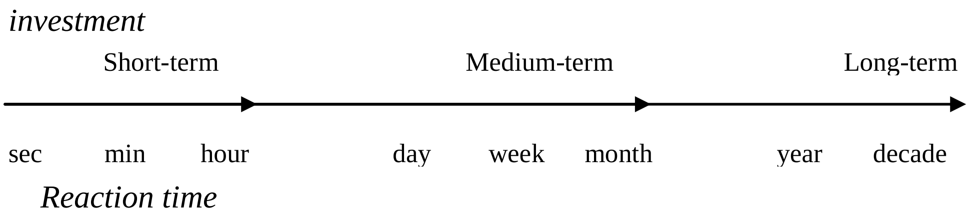

Figure 1. Heterogeneous participants react over different time scales

In this study, we concentrate on the different time scales of market participants in their investment profiles. The time horizons of investments can be characterized by *short*-term, *medium*-term and *long*-term ranging from seconds to decades. Some literatures group the investors in different investments styles (Muller et al., 1997;Lynch and Zumbach,2003). The *short*-term investors may refer to market makers (eg. NASDAQ, KLSE, among others, consists over 500 firms that quote both bid and offer prices for a particular asset) and intraday speculators who trade over very short time horizons (seconds to hours) in order to gain profits (or minimize losses). Next group of investors involve hedge funds and portfolio investments in *medium* time horizon. The former investors trade over a few days or based on daily closing prices whereas the later may take weeks or months to adjust the portfolio according to invested companies conditions and prices in the benchmark indices. For *long*-term investments such as central banks and pension funds may trade over few years and even decades. Central banks often refer to long-term macroeconomics view on FX and money market rates. Pension funds

behavioural biases. However, this concept is still in the early stage of development, therefore, we do not discuss details to it.

investors on the other hand provide a common asset pool to generate stable growth over a very long time horizon which allows them to invest in long term investment such as real estate that normally generate capital gains over time.

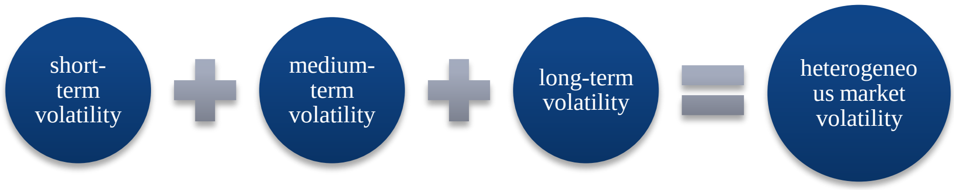

Figure 2. Struture of heterogenous market

The non-homogeneous market participants interpret same information differently according to their trading opportunities. Each time horizon trading activities creates a unique volatility under the fluctuating price movements. Thus, the financial markets which compose by participants with different reaction times to news have created volatility cascade ranging from low to high frequencies. The combinations of these dissimilar volatilities (due to reaction times) are believed to produce hyperbolic autocorrelation decays or long-range dependence property in financial markets. In short, the structure of heterogeneous market volatility can be illustrated in Figure 2.

Based on the aforementioned structure of heterogeneous market, high frequency data have been widely used to measure the market volatility. Accurate volatility estimations are important in assisting investment portfolio management and market risk management. Realized volatility is one of the famous model-free measures of latent volatility that normally cannot be observed directly from financial time series. The interest of high frequency volatility estimation has steadily increased after it has been proven (Andersen and Bollerslev, 1998; Blair et al.,2001) significantly improve the modelling and forecast performance in foreign exchange and stock markets. There are two major research directions in realized volatility literature. The first group of researchers incorporate intraday information with the GARCH model as the conditional variance regressor (Marten & Djik, 2007; Taylor & Xu,1997) while the other groups advocate the intraday information directly for econometric modeling and forecasting (Corsi, 2009; Engle and Gallo,2006).

This study aims to further investigate the clustering volatility and long memory in realized volatility. For former stylized fact, Corsi et al. (2008) and Cheong et al. (2007) have considered the GARCH model to cope time-dependent conditional heteroskedasticity in the realized volatility. The latter long memory behaviour is commonly captured by ARFIMA (Andersen et al. 2003; Baillie et al., 1996) model. This stylized fact is well explained by the concept underlies heterogeneous market hypothesis (Dacorogna et al., 2001). In this research, we include both the clustering volatility and long memory behaviour in realized volatility model which based on the framework of HAR (Corsi, 2009) and ARFIMA (Andersen et al., 2003). In addition, we also consider two realized volatility estimators which are immune to abrupt jumps namely the realized power variation and realized bi-power variation (Barndorff-Nielsen and Shephand, 2004) to avoid possible abrupt jumps during the subprime mortgage crisis in year 2008. Both the estimators are examined using 5-minute and 15-minute intervals. Although the error disturbances are commonly assumed to be normally distributed, our empirical analysis has used a non-Gaussian conditional distribution namely the generalized error distribution (Nelson, 1991) which, to our knowledge, has not yet been used in the literature of realized volatility modelling.

Overall, there are 12 realized volatility models based on three types of intraday data. The best in-sample forecasts among the models are selected based on three information criteria. Next, rolling one-day-ahead out-of-sample forecasts are conducted for the duration of six months trading days in year 2009. Three loss functions are used for forecast evaluations. For the purpose of comparison, the exponential GARCH (Nelson, 1991) and fractionally integrated exponential GARCH (Bollerslev and Mikkelsen, 1996) are also considered in forecast evaluations. The paper is organized as follows. Section 2 describes the data source and return definition. Section 3 presents the time-varying HAR and ARFIMA model, estimation, diagnostics and forecast evaluations. Section 4 discusses the empirical results, and Section 5 concludes this study.

2.0 Data source

This study calculates the daily volatility from sample variance of intraday return using two different frequencies, namely 5- and 15-minute intervals to battle microstructure problem and noisy estimation issue of realized volatility. The empirical data consists of the S&P500 stock exchange index from January 2005 to June 2009 (1131 trading days with 440700 observations) with trading hours from 09.30 to 16.00. In general, intraday returns are calculated as the difference between successive close to close log prices and express in percentages as follows:

$$r_{t,i} = 100\left(\ln P_{t,i}^{close} - \ln P_{t,i-1}^{close}\right) \quad (1)$$

i = 2, …, M–1 and t = 1, …, T. Thus, a full trading day for 5-minute interval consists of $M\times\delta$ = 78 minutes with M equally-spaced subintervals of length δ. For 15-minute interval, M denotes 26 intraday returns. The intraday daily return with M=1 becomes

$$r_t = \sum_{i=1}^{M} r_{t,j} = 100\left(\ln P_{t,M}^{close} - \ln P_{t,1}^{close}\right). \quad (2)$$

3.0 Methodology

3.1 The autoregressive conditional heteroscedasticity model

The autoregressive conditional heteroscedasticity (ARCH) model is introduced by Engle (1982) in the study of United-Kingdom inflation uncertainty. One of the well known extensions of ARCH is advocated fractionally integrated differencing parameter (Granger and Joyeux,1980) in the ARCH framework or commonly known as the FIGARCH (Baillie, Bollerslev and Mikkelsen,1996) model with the following specification

(1)

where and are lag polynomials and $(1-B)^d = \sum_{i=1}^{\infty}(-1)^i \binom{d}{i} L^i$ with $\binom{d}{i} = \frac{d(d-1)...(d-i+1)}{i!}$. To avoid covariance-nonstationary and nonnegativity issues, Bollerslev and Mikkelsen (1996) used logarithmic log nested with Nelson (1991) specification in FIEGARCH model

(2)

with leverage effect if <0. The estimation results of interday ARCH model are mainly for comparison purposes with the intraday realized volatility models.

3.2 Long memory time-varying realized volatility models

This study considers the heterogeneous autoregressive (HAR) and autoregressive fractionally integrated moving average (ARFIMA) models in the realized volatility modelling. For ARFIMA-GARCH[2] time-dependent heteroskedasticity model, the specification can be written as:

, (3)

where denotes the fractional differencing operator, and are lag polynomials. In this study, the is assumed to be a generalized error distribution (GED) and RV_i represents the type of logarithmic realized volatility (Andersen et al., 2003). Another model which is capable to capture this stylized fact is the heterogeneous autoregressive realized volatility model (HAR-RV) with linear cascading of different time horizon realized volatility components. This model provides a simple autoregressive structure where the current volatility is dependence by previous daily, weekly and monthly realized volatilities. Later, Cheong et al. (2007) and Corsi,et al. (2008) extended the HAR with the inclusion of time-varying volatility in realized volatility. The modified HAR-GARCH(1,1) model can be written as:

, (4)

[2] Baillie et al. (1996) in inflation.

where follows a conditional density with time-varying variance. For HAR components, – and — . For both the models, the RV_i is either sum squared of intraday realized volatility (Andersen, et al.,1999), realized power variation (Barndorff-Nielsen and Shephand, 2004) or realized bipower variation (Barndorff-Nielsen and Shephand, 2004) as follows:

(7)

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–

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(8)

– (5)

where M is the total interval within a day and $0 < z < 2$. All the realized volatilities are examined using frequency 5-minute and 15-minute intervals. When very high frequency is used, M approaches ∞ and converges in probability to the continuous part of the price process. Under this condition, both the volatility estimators are immune to abrupt jumps. Both the estimators are based on absolute return where it is more persistence (Ding et al., 1993) than other counterparts such as squared return. In practical applications, 5-minute data are normally used to lessen the impact of market noise (ABDL,2003) whereas 15-minute data are recommended by ABDL (2000) in order to reduce the biasness issue in estimation.

3.3 Estimation, diagnostic and model selection

In this study, the error ε_t are assumed to be followed a generalized error distribution (Nelson,1991) under the maximum likelihood estimation to capture the heavy-tail property that often exhibited in financial time series with the density function

$$f(z_t;v) = \frac{v \exp^{\left(-\frac{1}{2}\left|\frac{z_t}{\lambda}\right|^v\right)}}{\lambda 2^{\left(\frac{1+v}{v}\right)} \Gamma\left[\frac{1}{v}\right]} \,, \qquad (6)$$

where $\Gamma[\cdot]$ is the gamma function and $\lambda = \left(\frac{2^{-2/v}\Gamma[v^{-1}]}{\Gamma[3v^{-1}]}\right)^{0.5}$ with v<2 for heavier tail compared to normal distribution v=2. The log-likelihood for normal and GED are

$$L_{T,GED} = \sum_{t=1}^{T}\left(\ln\left(\frac{v}{\lambda}\right) - \frac{1}{2}\left|\frac{z_t}{\lambda}\right|^v - \left(1+\frac{1}{v}\right)\ln 2 - \ln\Gamma\left[\frac{1}{v}\right] - \frac{1}{2}\ln\sigma_t^2\right) \qquad (7)$$

For faster and easier computation, we use the Marquardt (1963) method where only the outer products of the gradient vectors are computed in the numerical analysis estimations. In the model diagnostic, the Ljung-Box serial correlation and

Engle ARCH tests are used to examine the standardized and squared standardized residuals under the null hypothesis that the noise terms are serially uncorrelated or random. After that, the model selections are based on the Akaike information criterion (AIC), Schwarz information criterion (SIC) and Hannan-Quinn information criterion (HIC) which evaluated from the adjusted (penalty function due to additional number estimated parameters) average log likelihood function (L_T) are selected for the estimation evaluation. The information criteria can be expressed as:

$$ - \quad -, \qquad (8) $$

$$ - \quad \rule{2em}{0.4pt} $$

$$ - \quad \rule{3em}{0.4pt} \qquad (9) $$

where k is the number of estimated parameters.

3.4 Forecast evaluations

Each volatility model is estimated H times based on fixed interval of 1007 observations (Jan 2005 until Jan 2009). The in-sample estimation after the structural change contains observations from t=1 to t=1007. A rolling parameter estimations is implemented, for example, the first one-day ahead forecast at t=1008, is using the estimation from t=1 to 1007 while the estimation from t=2 to 1008 is used to forecast the volatility at t=1009. Therefore, H (Feb 2009 until Jun 2009) one-day ahead volatility forecasts can be obtained by using the rolling estimations procedures for $\hat{\sigma}^2_{(h),t}$, where h=1007,…, H. Three loss functions are used to evaluate the predictive accuracy:

$$L_{1:} \quad \text{RMSE} \quad = \quad \sqrt{\frac{1}{H}\sum_{h=t+1}^{t+H}\left(\sigma^2_{RV,h} - \sigma^2_{Forecast,h}\right)^2};$$

$$L_2: \quad \text{MAE} \quad = \quad \frac{1}{H}\sum_{h=t+1}^{t+H}\left|\sigma^2_{RV,h} - \sigma^2_{Forecast,h}\right|;$$

$$L_3: \quad \text{TIC} \quad = \quad \frac{\sqrt{1/H\sum_{h=t+1}^{t+H}\left(\sigma^2_{RV,h} - \sigma^2_{Forecas,h}\right)^2}}{\sqrt{1/H\sum_{h=t+1}^{t+H}\left(\sigma^2_{RV,h}\right)^2} + \sqrt{1/H\sum_{h=t+1}^{t+H}\left(\sigma^2_{Forecast,h}\right)^2}} \qquad (10)$$

where the *actual* and *forecast* represented their respective RV and forecasted volatility respectively. Root mean square error (RMSE), mean absolute error (MAE), and Theil inequality coefficient (TIC) are the common loss functions in forecasting evaluations. The three loss functions report the evaluations directly based on the deviations among the forecasts and realizations. TIC on the other hand states that the value lies between 0 and 1 with a perfect fit if the score value is zero.

4.0 Empirical results

Table 1. Descriptive statistics for natural logarithm non-parametric volatility estimator

Statistic	$\sigma^2_{RV\,05}$	$\sigma^2_{RV\,15}$	$\sigma^2_{RBP\,05}$	$\sigma^2_{RBP\,15}$	$\sigma^2_{RPV\,05}$	$\sigma^2_{RPV\,15}$
Mean	-0.702746	-0.744635	-0.821952	-0.903474	-0.057599	-0.085946
Std. Dev.	1.207053	1.234885	1.219173	1.245823	1.088766	1.111339
Skewness	1.067485	0.991685	1.101592	0.975037	1.069695	0.991666
Kurtosis	4.054516	4.018768	4.077248	3.943726	4.052802	4.007760
JB test	238.1442*	208.8089*	252.6081*	197.1231*	238.7860*	207.8658*
Hurst	0.726 (0.976)	0.740 (0.980)	0.708(0.973)	0.726 (0.978)	0.764 (0.983)	0.776 (0.985)

*Note: Jacque-Bera test, H_0: normality; Hurst parameter: rescale-range method (long range dependence if 0.5<H<1.0) with coefficient of determination in parenthesis. * Significant at 5% level.*

Table 1 shows the descriptive statistics of three unconditional realized volatilities with 5-minute and 15-minute frequencies. All the estimators are deviated from normal distribution according to the Jarque-Bera test. Next the degree of persistence of the estimators is measured by the rescaled-range method where the time series is long memory if the Hurst value lies from 0.5 to 1.0. Overall, the intensity of persistence are almost the same for all the estimators (average 0.730), however, the 15-minute estimators are consistently greater than 5-minute. In other words, the estimator based on 15-minute interval consists of more predictability component for future volatility. As a conclusion, non-Gaussian and long memory behaviours should be taken account in model specification.

Table 2 and **Table 3** reports the maximum likelihood estimations for both the HAR-GARCH(1,1) and ARFIMA(1,d,0)-GARCH(1,1) models with GED distributed error. First, the tail parameter, v with value less than two convinces the inadequacy of normality assumption for both models. Second, both type of models indicate the time-dependent heteroskedasticity volatility have been eliminated by the GARCH(1,1) in realized volatility. Third, the fractional difference parameter, d for ARFIMA models indicate the presence of long memory volatility whereas the additive volatility cascade of different time horizons of HAR are all statistically significant different from zero. For HAR models, the impact of volatility components are the strongest for past daily volatility, follows by weekly and monthly volatility. This is a common fact where the nearest historical information has the highest influence to the recent volatility movements.

In **Table 2**, the in-sample forecast using time-dependent conditional volatility model, HAR-GARCH (GED) shows improvement in goodness of fit, measured by AIC, BIC and HIC criteria over the HAR and HAR-GARCH (Normal). The HAR indicates significant heteroskedasticity effect under the Ljung-Box correlation and ARCH-LM tests. Although the conditional heteroskedastic effect can be removed by HAR-GARCH (normal), the HAR-GARCH (GED) seems to provide better goodness of fit tests. Similar results also have been observed[3] in the ARFIMA-GARCH (GED) estimations.

The in-sample forecast evaluation can be analyzed in two ways. First, overall the ARFIMA-type models are slightly outperformed the HAR-type models by referring to their AIC, BIC and HQC criteria. Although the HAR model is more preferable (in terms of structurally inline with heterogeneous market hypothesis), the ARFIMA model has the advantage of parsimonious structure with less number of parameters to be estimated (three against four). As a comparison, the realized power variation indicates the best fitting results for both the HAR and ARFIMA models. This follows by the sum of squared realized volatility and finally the realized bipower variation. Second, overall the 5-minute interval volatility estimators perform better than 15-minute. These results suggest that the microstructure noise problem (ABDL,2003) is more severe than the biasness issue(ABDL,2000) in the S&P500 market.

Finally, Table 4 presents the out-of-sample forecast evaluations based on RMSE, MAE and TIC. For the purpose of comparison, the forecast evaluations also conducted using EGARCH and FIEGARCH for logarithmic volatility. As indicates in Table 4, both the HAR and ARFIMA models perform substantially better than the daily GARCH models. Comparing the three evaluation criteria across 14 models show that the ARFIMA-type models perform marginally better than the HAR-type models with the same volatility proxies (, and). Among the models, both the ARFIMA-GARCH (GED) and HAR-GARCH (GED) provides the best forecasts using 5-minutes , follows by and lastly . As a summary, the overall ranking based on the forecast evaluation criteria is presented in Table 4.

[3] Due to space scarcity, only the ARFIMA-GARCH (GED) is presented in Table 3.

Table 2. The maximum likelihood estimation for Heterogeneous Autoregressive GARCH

Estimation	HAR-normal		HAR-GARCH GED					
	HAR	HAR-GARCH	$\sigma^2_{RV\,05}$	$\sigma^2_{RBP\,05}$	$\sigma^2_{RPV\,05}$	$\sigma^2_{RV\,15}$	$\sigma^2_{RBP\,15}$	$\sigma^2_{RPV\,15}$
θ_0	-0.078266*	-0.086778*	-0.092830*	-0.099270*	-0.048983*	-0.124075*	-0.136926*	-0.068613*
	(0.023035)	(0.021228)	(0.020454)	(0.021493)	(0.014399)	(0.024155)	(0.025674)	(0.017686)
θ_{day}	0.433007*	0.402121*	0.399388*	0.393625*	0.410664*	0.293922*	0.292144*	0.309937*
	(0.039915)	(0.039518)	(0.037728)	(0.038286)	(0.037484)	(0.038302)	(0.038438)	(0.038387)
θ_{week}	0.372515*	0.412192*	0.402507*	0.427460*	0.398759*	0.460525*	0.450318*	0.452273*
	(0.059916)	(0.062804)	(0.059919)	(0.058951)	(0.059355)	(0.064044)	(0.064792)	(0.063787)
θ_{month}	0.149767*	0.135304*	0.148317*	0.130506*	0.143928*	0.188862*	0.203745*	0.183606*
	(0.044715)	(0.047381)	(0.046472)	(0.045663)	(0.046312)	(0.053033)	(0.053593)	(0.052958)
		0.007272*	0.008040*	0.008752*	0.006211*	0.012888*	0.014715*	0.010403*
		(0.002827)	(0.004114)	(0.004532)	(0.003157)	(0.008337)	(0.010511)	(0.006541)
		0.035745*	0.040096*	0.040717*	0.039548*	0.031570*	0.027666*	0.031461*
		(0.010318)	(0.014466)	(0.015749)	(0.014485)	(0.015226)	(0.015613)	(0.015319)
		0.939057*	0.931965*	0.929019*	0.933198*	0.935596*	0.935795*	0.934873*
		(0.015399)	(0.022796)	(0.025290)	(0.022469)	(0.030576)	(0.036059)	(0.030585)
v			1.577542*	1.609570*	1.564075*	1.779607*	1.744644*	1.770366*
			(0.082792)	(0.076477)	(0.082576)	(0.108496)	(0.116041)	(0.109053)
Model selection								
L	-789.9075	-775.6803	-767.8609	-765.1635	-647.3451	-942.5871	-957.2887	-821.3927
AIC	1.576778	1.554479	1.540935	1.535578	1.301579	1.887959	1.917157	1.647255
SIC	1.596300	1.588643	1.579980	1.574623	1.340624	1.927003	1.956202	1.686299
HIC	1.584195	1.567460	1.555770	1.550413	1.316414	1.902793	1.931992	1.662089
Diagnostic								
$\tilde{a}_t$, LB (12)	15.777(0.202)	16.815(0.157)	16.768(0.159)	17.415(0.135)	15.938 (0.194)	15.289 (0.226)	21.087 (0.049)	14.803 (0.252)
$\tilde{a}_t^2$, LB (12)	40.504* (0.000)	16.708(0.161)	15.440(0.218)	7.5167(0.822)	14.981(0.242)	8.8966(0.712)	9.8786(0.627)	8.8302(0.717)
LM-ARCH(12)	2.936* (0.0005)	1.338(0.1909)	1.226(0.2593)	0.620(0.8261)	1.188(0.2865)	0.778(0.6729)	0.894(0.5519)	0.774(0.6771)

*Notes 1. $\tilde{a}_t$ represents the standardized residual. Ljung Box Serial Correlation Test (Q-statistics) on $\tilde{a}_t$ and $\tilde{a}_t^2$: Null hypothesis – No serial correlation; LM ARCH test: Null hypothesis - No ARCH effect 2. For estimation, the parentheses values represent standard error;3. For diagnostic, the parentheses values represent p-value 4. * denotes 5% level of significance.*

Table 3. The maximum likelihood estimation for ARFIMA-GARCH

Estimation	ARFIMA-GARCH (GED)					
	$\sigma^2_{RV\,05}$	$\sigma^2_{RBP\,05}$	$\sigma^2_{RPV\,05}$	$\sigma^2_{RV\,15}$	$\sigma^2_{RBP\,15}$	$\sigma^2_{RPV\,15}$
	-1.602159 *	-1.777830*	-0.858241*	-1.431951*	-1.546094*	-0.710389*
	(0.36267)	(0.41318)	(0.33161)	(0.38103)	(0.40969)	(0.35231)
	0.580430 *	0.586508 *	0.580899 *	0.552808 *	0.559165 *	0.554268*
d	(0.034901)	(0.034671)	(0.034995)	(0.033614)	(0.033719)	(0.033942)
	-0.116631 *	-0.122088 *	-0.103703 *	-0.172796 *	-0.180319 *	-0.158486*
	(0.044955)	(0.044371)	(0.045244)	(0.042589)	(0.043723)	(0.043183)
	0.009707 *	0.012688 *	0.007536 *	0.013410	0.015520	0.010930
	(0.0047683)	(0.0064004)	(0.0036899)	(0.0089981)	(0.012129)	(0.0073070)
	0.048974 *	0.055217 *	0.048867 *	0.035713 *	0.032377	0.036407 *
	(0.016591)	(0.020186)	(0.016691)	(0.016252)	(0.017156)	(0.016655)
	0.917279 *	0.900876*	0.918064 *	0.930001 *	0.928825 *	0.928144 *
	(0.025794)	(0.033787)	(0.025462)	(0.032960)	(0.041786)	(0.034001)
	1.562464 *	1.578438 *	1.551427 *	1.725659 *	1.732796 *	1.725020 *
v	(0.081543)	(0.074281)	(0.081139)	(0.10687)	(0.11508)	(0.10774)
Model Selection						
L	-765.425	-765.688	-646.061	-941.005	-955.828	-820.936
AIC	1.534112	1.534633	1.297042	1.882830	1.912269	1.644362
SIC	1.568276	1.568797	1.331206	1.916994	1.946433	1.678526
HIC	1.547092	1.547614	1.310022	1.895810	1.925250	1.657342
Diagnostic						
$\tilde{a}_t$, LB (12)	12.127(0.354)	12.132(0.353)	11.720(0.385)	12.726(0.311)	11.230(0.424)	12.287(0.342)
$\tilde{a}_t^2$, LB (12)	12.457(0.255)	6.387 (0.781)	12.113(0.277)	9.069(0.525)	9.296 (0.504)	8.944 (0.537)
LM-ARCH(12)	1.014(0.433)	0.524(0.899)	0.983(0.462)	0.793(0.657)	0.839(0.610)	0.787(0.664)

*Notes: 1. $\tilde{a}_t$ represents the standardized residual. Ljung Box Serial Correlation Test (Q-statistics) on $\tilde{a}_t$ and $\tilde{a}_t^2$: Null hypothesis – No serial correlation; LM ARCH test: Null hypothesis - No ARCH effect;2. For estimation, the parentheses values represent standard error;3. For diagnostic, the parentheses values represent p-value 4. * denotes 5% level of significance.*

Table 4: Forecast evaluation

No.	Model	RMSE	rank	MAE	rank	TIC	rank	Overall ranking
	GARCH-type model:							
1	EGARCH	2.108	14	0.8388	14	0.8017	14	14
2	FIEGARCH	1.111	13	0.6385	13	0.6946	13	13
	ARFIMA-GARCH:							
3	with $\sigma^2_{RV\,05}$	0.3182	3	0.2535	4	0.1618	5	3
4	with $\sigma^2_{RV\,15}$	0.4050	9	0.3334	11	0.2137	9	9
5	with $\sigma^2_{RBP\,05}$	0.3316	6	0.2613	6	0.1892	7	8
6	with $\sigma^2_{RBP\,15}$	0.4233	11	0.3421	12	0.2477	11	12
7	with $\sigma^2_{RPV\,05}$	0.2793	1	0.2231	1	0.0989	1	1
8	with $\sigma^2_{RPV\,15}$	0.3569	7	0.2937	8	0.1291	3	6
	HAR-GARCH:							
9	with $\sigma^2_{RV\,05}$	0.3239	4	0.2602	5	0.1731	6	4
10	with $\sigma^2_{RV\,15}$	0.4194	10	0.3322	10	0.2327	10	10
11	with $\sigma^2_{RBP\,05}$	0.3260	5	0.2491	3	0.1957	8	5
12	with $\sigma^2_{RBP\,15}$	0.4246	12	0.3287	9	0.2614	12	11
13	with $\sigma^2_{RPV\,05}$	0.2820	2	0.2270	2	0.1036	2	2
14	with $\sigma^2_{RPV\,15}$	0.3683	8	0.2921	7	0.1383	4	7

In summary, there are two factors that influence the accuracy of forecasting evaluations, namely the type of model and also the frequency used in the volatility estimation. In this specific study, the ARFIMA allowing for time-varying volatility of realized volatility is marginally provides better forecast accuracy than the HAR-type models. This may due to its simplicity (parsimonious principal) as compares to its counterpart. For volatility estimator frequency, the 5-minute interval shows significant improvement in volatility point forecasts over 15-minute interval for S&P500 index. Due to high liquidity of S&P500 index mature market, the 5-minute information has also been used by other researchers (Maheu and McCurdy,2002; Martens et al.,2004).

5.0 Conclusion

This study has shown that the ARFIMA and HAR realized volatility models allowing of time-dependent heteroskedasticity are outperformed in both the in-sample and out-of-sample forecasts than the standard ARFIMA, HAR, EGARCH and FIEGARCH models. Moreover, the heavy-tailed distributed error terms in the model specification has also gained better fitting evaluation based on the in-sample estimation information criteria. Besides the model specification, the data frequency for volatility estimator is also played an important role to ensure superiority in forecast evaluation, for this specific study, the 5-minute frequency data. As a conclusion, this study is relevance to risk management and investment portfolio management where the market risk (in term of value-at-risk) and portfolio hedging for single or multi-asset investments can be determined directly from the forecast results.

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