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# SUPPLY CHAIN INTEGRATION UNDER VENDOR MANAGED INVENTORY MODE OF OPERATION CONSIDERING STOCKOUT

***Abstract.*** *This paper formulates a two-echelon single-vendor multi-buyer supply chain model assuming unsatisfied demand at the vendor to be backordered. The vendor and buyers apply Vendor Managed Inventory (VMI) mode of operation. The vendor gives the product to the buyers. The operational parameters are sales quantity, sales price and maximum number of stockouts for each buyer at the vendor's location. Channel profit of the supply chain and contract price between the vendor and buyers are determined based on the operational parameters. In order to find out the optimal values of the parameters, a mathematical model is formulated. Two heuristics are proposed to solve the addressed problem. Some numerical problems are provided and each problem is solved utilizing the heuristics. We have also solved the sample problems utilizing LINGO optimization solver in order to evaluate the performance of the proposed algorithms.*
***Keywords:*** *Supply Chain; Vendor Managed Inventory; Stockout; Genetic Algorithm; Simulated Annealing*

**JEL Classification: C60; C61, C62, C63, C65, C67, C68, C69**

## 1. Introduction

Vendor Managed Inventory (VMI) is a modern type of vendor-buyer integration in which the vendor makes decisions about the buyer(s) inventory system in such a way as to maximize the total profit of the system. In such a system, the vendor usually determines the order quantity and the reorder point.

The integration in VMI may be of different types. In some cases, the vendor is only responsible for replenishing the inventory and the ownership of the inventory is transferred to the buyer as its arrival at the buyer's site. In a more integrated case, the ownership of the inventory remains with the vendor until the withdrawal from the buyer's site occurs. The first case can be referred to as Vendor Managed Replenishment (VMR) and the latter is the true VMI (Vigtil (2007)).

Wal-Mart and Procter & Gamble (P&G) integration was one of the initial successful experiences of VMI in 1985. The integration caused sizable improvements in on time deliveries from P&G to Wal-Mart. The mentioned experience along with the other ones such as Dillard Department stores and JCPenney coordination show up to 20-25 % increase in sales and up to 30 % increase in the inventory turnover (Buzzel and ortmeyer (1995)).

Some of the advantages from utilizing VMI are as following (Danese (2006)):

- Reduction in customer demand uncertainty;
- Reduction of inventory levels;
- Reduction of stockout number and frequency;
- More flexibility in production and distribution planning
- Improvement in customer services

This paper deals with the operational issues of a two-echelon single-vendor multi-buyer supply chain model under VMI mode of operation. The vendor gives the product to the buyers. The operational parameters are sales quantity, sales price and maximum number of stockouts for each buyer at the vendor's location. Channel profit of the supply chain and contract price between the vendor and buyers are determined based on the operational parameters. In order to find out the optimal values of the parameters, a mathematical model is formulated.

Although VMI was initially introduced in the 1970's, it did not become popular until the 1990's while IT-based systems such as Electronic Data Interchange (EDI) were sufficiently advanced to support the supply chain processes (Yao et al. (2007a)).

Goyal (1976) was one of the initial researches which proposed an integrated inventory model for a single vendor-single buyer structure assuming a uniform demand for a single item. Lead times for both the vendor and buyer were assumed to be zero with no demand to be lost. The results indicated considerable savings for both the vendor and the buyer.

Banerjee (1986) studied a single vendor-single buyer inventory model. An exact mathematical model was developed to find the optimal lot sizes which minimized the total joint costs. The vendor undertakes a production setup cost every time the buyer places an order based on a lot-for-lot inventory policy. The results indicated that the implementation of a jointly optimal ordering policy could be of economic benefit to both parties.

Goyal (1988) extended Banerjee's model generalizing the lot-for-lot policy of the vendor so that the vendor's production quantity to be an integer number of that of the buyer. One of the major assumptions was that the vendor could not split an order. The results indicated that further reduction in total cost could be achieved.

Chatterjee and Ravi (1991) proposed a model in which the vendor dispatched the products manufactured from a single batch in sub-batches of different sizes to the buyer. The time required to transport a sub-batch was assumed zero. In addition, the inventory holding cost was assumed identical for both the vendor and buyer..

Maloni and Benton (1997) stated that the main focus of the revenue sharing should be on sharing the integration generated revenues/profits according to the responsibilities which each member adopts. Viswanathan (1998) analyzed the relative performance of the two strategies namely 'Identical Delivery Quantity' (IDQ) and 'Deliver What is Produced' (DWP) for delivering goods with the objective of minimizing the joint average annual cost in an integrated vendor–buyer inventory model.

Dong and Xu (2002) formulated a single-vendor single-buyer model operating under VMI. They investigated the short-term and long-term effects of implementing VMI on the whole supply chain and each member. Disney et al. (2003) investigated the impact of VMI policy upon transportation operations in a supply chain. Plambeck and Zenios (2003) considered VMI in the context of principal-agent setting. In their contingency model, the retailer motivates the manufacturer to control its production rate in a manner that minimizes the retailer's own costs. Lee and Chu (2005) analyzed the expected payoff by transferring demand uncertainty risk in a two-member supply chain. All of these studies discuss the integration of two-echelon supply chain inventory models with the objective of minimizing the total costs of supply chain concentrating on independent mode of operation except Dong and Xu (2002) which formulated a single-vendor single-buyer model operating under VMI mode of operation.

Danese (2006) found how VMI could be extended for both upstream and downstream echelons in a supply network to coordinate the material and information flows among a number of different suppliers, manufacturers and distributors. Nachiappan and Jawahar (2006) provided a two-echelon single vendor-multiple buyers supply chain under VMI mode of operation. They formulated a mathematical model to find out the optimal sales quantity for each buyer, the optimal sales price and the acceptable contract price. The research in this paper is an extension of Nachiappan and Jawahar (2006) so that stockout is permitted.

Zhang et al. (2007) considered a single-vendor multiple-buyer model and proposed a joint relevant cost for such a system in which the vendor purchases and processes raw materials and then delivers finished items to multiple buyers.

Yao et al. (2007a) developed a single-vendor single buyer discrete model to evaluate the effects of the supply chain parameters on cost savings applying VMI mode of operation. Van der Vlist et al. (2007) completed Yao et al. (2007a) considering some additional parameters such as the cost of shipment from supplier to buyer. Vigtil (2007) identified what types of advanced demand data would be valuable to the vendor for successful replenishment planning and also the frequency and means of information exchange. Yao et.al (2007b) developed a mechanism under VMI by which a vendor provides an incentive contract to a buyer to convert lost sales stockouts into backorders.

Sari (2007) studied the performance of a supply chain utilizing VMI by different levels assuming demand uncertainty. Bichescu and Fry (2007) analyzed decentralized supply chains which follow general continuous review inventory policy *(R, Q)* subject to VMI agreements where the supplier selects the order quantity $Q$ and the retailer selects the reorder point $R$. Yao and Dresner (2008) showed that information sharing, Continuous Replenishment Programs (CRP) and VMI provided varying inventory costs savings to the firms. They indicated that the savings were not consistently distributed among the firms. The research also showed that how managers might decide the product mix and the replenishment frequency under CRP and VMI. Southard et al. (2008) used data from two agricultural corporations to run discrete event simulation model of fuel delivery systems. They compared the results from this model with operating costs of technology-enabled systems such as inventory, delivery and stockout costs under a variety of VMI implementation alternatives.

The remainder of the paper is structured as follows: In Section 2, assumptions and notation is given. Problem formulation is explained in Section 3. Section 4 gives the solution methodology and in Section 5, some numerical problems are designed and solved. Section 6, represents the conclusion and some further research.

## *2. Assumptions and notation*

The major assumptions of the under study model are as following:

- The system includes a single vendor- multiple buyers in which the vendor replenishes a specific item for the buyers.
- Lost demand at the buyers is backordered.
- The vendor makes decisions on inventory for all buyers (VMI mode of operation).
- Delivery of orders to buyers is instantaneous which means that the lead time is zero.

The following notations are presented:

| | |
|---|---|
| $j$ | Buyer identifier (*j=1 to n*) |
| $N$ | Number of the buyers |
| $H_s$ | Holding cost of the vendor in independent mode |
| $H_{b_j}$ | Holding cost of *j*th buyer in independent mode |
| $S_s$ | Setup cost of the vendor per order in independent mode |
| $S_{b_j}$ | Setup cost of the *j*th buyer per order in independent mode |

| | |
|---|---|
| $a_j$ | Intercept value for the demand pattern of the *j*th buyer |
| $c_j$ | Cost slope of the demand pattern of the *j*th buyer |
| $y_j$ | Sales quantity of the *j*th buyer |
| $y_{j_{\min}}$ | Minimum expected sales quantity of the *j*th buyer |
| $y_{j_{\max}}$ | Maximum expected sales quantity of the *j*th buyer |
| $y_{j_{opt}}$ | Optimal sales quantity of the *j*th buyer |
| $\pi_j$ | Cost of one unit stockout |
| $\pi'_j$ | Cost of one unit stockout per time unit |
| $b_j$ | Maximum number of the stockouts of the *j*th buyer in each period of time |
| $PR_j$ | Revenue share ratio between the vendor and the *j*th buyer |
| $P(y_j)$ | Sales price of the *j*th buyer corresponding to sales quantity $y_j$ |
| $W_j$ | Contract price between vendor and buyer *j* |
| $Q_j$ | Replenishment quantity to buyer *j* |

### *3. Problem formulation*

As pointed out earlier, the under study model in this paper is an extension to that of Nachiappan and Jawahar (2006) so that unsatisfied demands at the buyers are backordered. The major parameters of the model are: sales quantity *'y'*, the sales price at buyer's market *'P(y)'*, the contract price between the vendor and the buyer *'W'* and the maximum number of the stockouts for each buyer. The sales quantity of any product at a particular location is greatly influenced by its sales price and it depends on the factors like the necessity of the commodity, the purchasing power of the customers, the nature of the product (perishable or storable), and so on. The general observation is that higher the sales price lower the sales quantity and vice versa. The relationship between *'P(y)'* and *'y'* may be assumed to behave linearly and is given as (Nachiappan and Jawahar (2006)):

$$P(y) = a - cy, \tag{1}$$

Where *a* and *c* are the intercept of *'P(y)'* axis and the slope of sales curve, respectively, in the sales price vs. sales quantity graph shown in Fig. 1.

Besides, the sales quantity lies inside a specific range between $y_{j_{\min}}$ and $y_{j_{\max}}$ and the validity of the linear demand assumption function holds very well within this range. Since the buyers are not necessarily identical, the demand function for *j*th buyer may be stated as:

$$P(y_j) = a_j - c_j y_j \tag{2}$$

$$s.t: y_{j_{\min}} \leq y_j \leq y_{j_{\max}} \tag{3}$$

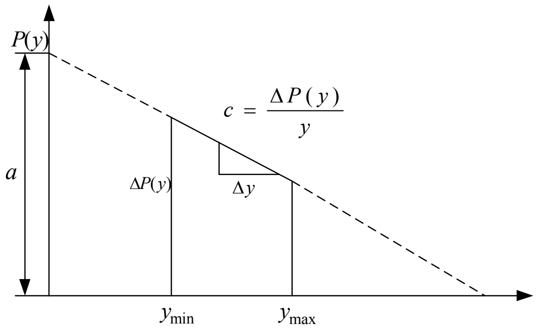


**Figure 1. Relationship between the sales price and the sales quantity**

The other parameter which plays an important role in the profits of the both vendor and buyer(s) is the contract price. It is a price which is mutually agreed between the vendor and the buyer(s). It is logical to assume it between the sales price and the cost of manufacturing. The nature of the product, the demand and the logistic costs play a critical role in determination contract price. The commodities which have good reputation and higher demand are usually fast moving and involve low risk. In these circumstances, the buyer accepts the contract price closer to sales price. However in other cases where the product is new and the demand is not yet stabilized, the contract price is expected being settled at low level, closer to the cost of production. As Nachiappan and Jawahar (2006) state, the contract price is a variable dependent on location, the competitiveness of the products, the production and the operational costs between vendor and buyer(s). The contract price between vendor and *j*th buyer is addressed as $W_j$.

An other parameter in this model is the maximum number of stockouts for jth buyer which is notated as $b_j$. It is determined minimizing the total cost function in the mode of VMI operation.

***3.1. Vendor operations and costs***

Disney and Towill (2002) state that in VMI mode, the vendor has more responsibility than the buyers and acts as a leader. The vendor monitors, manages and replenishes the inventory of all locations (Achabal et al. (2000)). The costs associated are production cost, distribution cost, order cost and stock maintenance cost. Production cost is derived from the amount spent for producing a single unit '$\delta$' and the aggregate demand '$y$' (i.e., $y = \sum_{j=1}^{n} y_j$). Therefore; the total production cost is $\delta y$. The distribution cost is the multiplication of flow and transportation resource cost. The flow cost is the direct mileage and the carrier contract cost per unit of the *j*th buyer '$\theta_j$' and the transportation resource cost is the indirect cost such as mode of transport, human router cost and administrative costs and termed as '$\upsilon_j$' per unit demand for the *j*th buyer (Dong and Xu (2002)). Therefore; the distribution cost would be '$\theta_j y_j \upsilon_j y_j$'. In this paper it is assumed that the products to all locations are delivered by road and the value of '$\upsilon_j$' is taken as 0.5 per unit (Dong and Xu (2002)).

The production and distribution costs '$PD_j$' to the vendor for meeting sales '$y_j$' of buyer *'j'* is given as in (4)

$$PD_j = \delta y_j + 0.5\theta_j y_j^2 \tag{4}$$

While VMI is implemented, the vendor monitors the *buyers'* inventory position and freights the inventory batched as needed at the buyers. Thus, the order cost per replenishment "$S_{j_{VMI}}$" associated to continuously monitor the stock status is assumed as sum of order cost of vendor "$S_s$" and order cost of buyer 'j' "$S_{b_j}$" (Dong and Xu (2002))

$$S_{j_{VMI}} = S_s + S_{b_j} \tag{5}$$

Thus, the total cost of replenishing the batches "$Q_j$" for buyer *'j'* is equal to $(S_s + S_{b_j})y_j/Q_j$ .

While no stockout is permitted at the vendor, the inventory position is as Fig. 2. Therefore, the average inventory which is hold by the vendor for *j*th buyer is $Q_j/2$ .

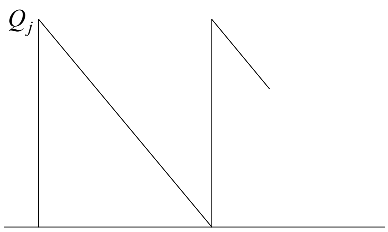


**Figure 2. Vendor inventory position**

Since unsatisfied demand at the buyers is backordered, the inventory position is as Fig. 3. Therefore, the average inventory of buyer *'j'* is equal to $(Q_j - b_j)^2/2Q_j$ (Hadley and whitin (1963)).

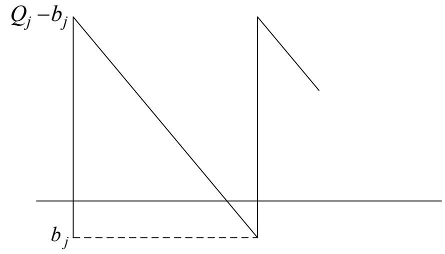


**Figure 3. Buyer inventory position**

Thus, sum of the inventory holding cost is given as in (6).

$$Sum\ of\ the\ inventory\ holding\ \cos t = H_s Q_j/2 + H_{b_j}(Q_j - b_j)^2/2Q_j \quad (6)$$

The sum of the stockout cost of *j*th buyer is given as in (7) (Hadley and whitin (1963)):

$$Sum\ of\ the\ stockout\ \cos t = \pi b_j y_j/Q_j + \pi' b_j^2/2Q_j \quad (7)$$

The sum of order cost, average inventory holding cost and stock out cost of the vendor for buyer '*j*' thus becomes:

$$TRC_j = \left(S_s + S_{b_j}\right) y_j/Q_j + H_s Q_j/2 + H_{b_j}(Q_j - b_j)^2/2Q_j + \pi b_j y_j/Q_j + \pi' b_j^2/2Q_j \quad (8)$$

Equating the first differential of $TRC_j$ to zero, the optimal values of $Q_j$ and $b_j$ is obtained.

$$\frac{\partial TRC_j}{\partial Q_j} = 0 \rightarrow \frac{-y_j(S_s + S_{b_j})}{Q_j^2} + \frac{H_s}{2} + \frac{H_{b_j}}{2}\left[\frac{Q_j^2 - b_j^2}{Q_j^2}\right] - \frac{\pi b_j y_j}{Q_j^2} - \frac{\pi' b_j^2}{2Q_j^2} = 0 \quad (9)$$

$$\frac{\partial TRC_j}{\partial b_j} = 0 \rightarrow -H_{b_j}\left[\frac{Q_j - b_j}{Q_j}\right] + \frac{\pi y_j}{Q_j} + \frac{\pi' b_j}{Q_j} = 0 \quad (10)$$

Multiplying Equation (9) by $Q_j$, the result is as given in Equation (11).

$$\frac{-y_j(S_s + S_{b_j})}{Q_j} + \frac{H_s Q_j}{2} + \frac{H_{b_j}}{2}\left[\frac{Q_j^2 - b_j^2}{Q_j}\right] - \frac{\pi b_j y_j}{Q_j} - \frac{\pi' b_j^2}{2Q_j} = 0 \quad (11)$$

And accordingly Equation (12) :

$$\frac{y_j(S_s+S_{b_j})}{Q_j}+\frac{\pi b_j y_j}{Q_j}+\frac{\pi' b_j^2}{2Q_j}=\frac{H_s Q_j}{2}+\frac{H_{b_j}}{2}\left[\frac{Q_j^2-b_j^2}{Q_j}\right] \tag{12}$$

Replacing the corresponding sections of Equation (8) with (12), $TRC_j$ can be obtained from (13).

$$\begin{aligned}TRC_j &= Q_j H_s+\frac{H_{b_j}}{2}\left[\frac{Q_j^2-b_j^2}{Q_j}\right]+\frac{H_{b_j}(Q_j-b_j)^2}{2Q_j}\\ &=\frac{1}{2Q_j}\left[2Q_j^2 H_s+H_{b_j}(Q_j^2-b_j^2)+H_{b_j}(Q_j^2+b_j^2-2Q_j b_j)\right]\\ &=\frac{1}{2Q_j}\left[2Q_j^2 H_s+H_{b_j}(Q_j^2-b_j^2+Q_j^2+b_j^2-2Q_j b_j)\right]\\ &=\frac{1}{2Q_j}\left[2Q_j^2 H_s+2H_{b_j}(Q_j^2-Q_j b_j)\right]\\ &=Q_j H_s+H_{b_j}(Q_j-b_j)\end{aligned} \tag{13}$$

Multiplying Equation (10) by $Q_j$ and replacing the corresponding section of Equation (13), $TRC_j$ can be obtained from (14).

$$TRC_j = Q_j H_S+\pi y_j+\pi' b_j \tag{14}$$

Multiplying Equation (9) by $Q_j^2$, $Q_j^2$ can be obtained from (15).

$$Q_j^2=\frac{2y_j(S_s+S_{b_j})+2\pi b_j y_j+b_j^2(H_{b_j}+\pi')}{H_s+H_{b_j}} \tag{15}$$

$b_j$ can be easily obtained from (10) as is given in (16):

$$b_j=\frac{H_{b_j}Q_j-\pi y_j}{H_{b_j}+\pi'} \tag{16}$$

Replacing $b_j$ in Equation (15) with the corresponding value in (16), $Q_j^2$ can be obtained from (17).

$$Q_j^2 = \frac{1}{H_s + H_{b_j}} \left[ 2y_j(S_s + S_{b_j}) + 2\pi y_j \left(\frac{H_{b_j} Q_j - \pi y_j}{H_{b_j} + \pi'}\right) + (H_{b_j} + \pi')\left(\frac{H_{b_j} Q_j - \pi y_j}{H_{b_j} + \pi'}\right)^2 \right]$$

$$\Rightarrow Q_j^2 = \frac{1}{H_s + H_{b_j}} \left[ 2y_j(S_s + S_{b_j}) + \left(\frac{H_{b_j} Q_j - \pi y_j}{H_{b_j} + \pi'}\right)(\pi y_j + H_{b_j} Q_j) \right]$$

$$\Rightarrow Q_j^2 = \frac{1}{(H_s + H_{b_j})(H_{b_j} + \pi')} \left[ 2y_j(S_s + S_{b_j})(H_{b_j} + \pi') + (H_{b_j}^2 Q_j^2 - \pi^2 y_j^2) \right]$$

$$\Rightarrow Q_j^2 \left[ H_s H_{b_j} + H_s \pi' + H_{b_j}^2 + H_{b_j} \pi' \right] = \left[ 2y_j(S_s + S_{b_j})(H_{b_j} + \pi') + (H_{b_j}^2 Q_j^2 - \pi^2 y_j^2) \right]$$

$$\Rightarrow Q_j^2 \left[ H_s H_{b_j} + H_s \pi' + H_{b_j}^2 + H_{b_j} \pi' \right] = \left[ 2y_j(S_s + S_{b_j})(H_{b_j} + \pi') + (H_{b_j}^2 Q_j^2 - \pi^2 y_j^2) \right]$$

$$\Rightarrow Q_j^2 \left[ H_s H_{b_j} + H_s \pi' + H_{b_j} \pi' \right] = \left[ 2y_j(S_s + S_{b_j})(H_{b_j} + \pi') - \pi^2 y_j^2 \right]$$

$$\Rightarrow Q_j^2 = \frac{2y_j(S_s + S_{b_j})(H_{b_j} + \pi') - \pi^2 y_j^2}{H_s(H_{b_j} + \pi') + H_{b_j} \pi'} \tag{17}$$

Replacing $b_j$ in Equation (14) with the corresponding value in (16), $TRC_j$ can be obtained from (18).

$$TRC_j = Q_j H_s + \pi y_j + \pi'\left(\frac{H_{b_j} Q_j - \pi y_j}{H_{b_j} + \pi'}\right)$$

$$TRC_j = \frac{Q_j H_s (H_{b_j} + \pi') + \pi' H_{b_j} Q_j - \pi' \pi y_j}{H_{b_j} + \pi'} + \pi y_j$$

$$TRC_j = \frac{Q_j \left[ H_s H_{b_j} + H_s \pi' + \pi' H_{b_j} \right] - \pi' \pi y_j}{H_{b_j} + \pi'} + \pi y_j \tag{18}$$

Obtaining $Q_j$ from (17), $TRC_j$ can be computed as

$$TRC_j = \frac{1}{H_{b_j} + \pi'}\left(\sqrt{2y_j(S_s + S_{b_j})(H_{b_j} + \pi') - \pi^2 y_j^2}\sqrt{H_s(H_{b_j} + \pi') + H_{b_j}\pi'} - \pi'\pi y_j\right) + \pi y_j \quad (19)$$

The profit of the vendor '$P_{s_j}$' when supplying the product to *j*th buyer is the difference between revenue to the vendor '$W_j y_j$' and total cost involved '$PD_j + TRC_j$' and is represented as

$$P_{s_j} = W_j y_j - (PD_j + TRC_j) \quad (20)$$

Thus, the total profit to the vendor '$P_s$' by supplying the needed products to all the buyers is as follows:

$$P_s = \sum_{j=1}^{N}\left\{W_j y_j - (\delta y_j + 0.5\theta_j y_j^2) - TRC_j\right\}, \quad (21)$$

Where $TRC_j$ is obtained from (19).

***3.2. Buyer operations and costs***

The buyer acts as an agent for the vendor and provides space to sell the products. The costs associated with the buyers in VMI mode are the sales price and contract price. The sales price for each buyer is determined by using (2). The acceptable contract prices that would satisfy both the vendor and the buyer are derived from the revenue share ratio '$PR_j$'. Thus the profit of *j*th buyer '$P_{b_j}$' in VMI mode is the difference between the sales revenue and the cost of purchase and it is represented as

$$P_{b_j} = y_j(a_j - c_j y_j) - W_j y_j \quad (22)$$

For the known revenue share ratio ($PR_j = P_{s_j} / P_{b_j}$) between the vendor and buyer '*j*', the contract price can be stated as:

$$W_j = \frac{PR_j y_j(a_j - c_j y_j) + (\delta y_j + 0.5\theta_j y_j^2) + TRC_j}{(1 + PR_j)y_j} \quad (23)$$

Where $TRC_j$ is obtained from (19).

### *3.3. Mathematical Model*

The objective criterion is considered as the channel profit of the supply chain ‘ $P_c$ ’ which can be stated as $P_s + \sum_{j=1}^{N} P_{b_j}$ . The mathematical model of the problem can be stated as

$$Max\, P_c = P_s + \sum_{j=1}^{N} P_{b_j} = \sum_{j=1}^{N} \left\{ y_j(a_j - c_j y_j) - (\delta y_j + 0.5\theta_j y_j^{\,2}) - TRC_j \right\} \tag{24}$$

$$S.t: \; y_{j_{\min}} \le y_j \le y_{j_{\max}} \tag{25}$$

$$y_j \ge 0 \tag{26}$$

Where $TRC_j$ is obtained from (19).

The solution to the above problem provides optimal sales quantity for *j*th buyer ‘ $y_{j_{opt}}$ ’. The optimal sales price ‘ $P(y_{j_{opt}})$ ’ and the acceptable contract price ‘ $W_{j_{opt}}$ ’ for *j*th buyer can be obtained from (2) and (23) letting $y_{j_{opt}}$ instead of $y_j$ .

### *4. Solution methodology*

The mathematical model in (24)-(26) belongs to Nonlinear Integer Programming (NIP) problem. Two Genetic Algorithm (GA) and Simulated Annealing (SA) based heuristics are proposed to evolve optimal or near optimal sales quantity for the buyer *‘j’* namely ‘ $y_{j_{opt}}$ ’.

### *4.1. GA based heuristic*

Genetic Algorithm is a class of evolutionary algorithms and is based on a population of solutions. It represents a powerful and robust approach for developing heuristics for large-scale combinatorial optimization problems. It also is a generic optimization method which can be applied to almost every problem. The feasible solutions of the problem are usually represented as strings of binary or real numbers called chromosomes. Each chromosome has a fitness value that corresponds to the objective function value of the associated solution. Initially, there is a population of chromosomes randomly generated. Then, a number of chromosomes are selected as parents for mating in order to produce new chromosomes (solutions) called offspring. Mating of the parents is performed applying a few GA operators, such as crossover and mutation. The selection of parents and producing offspring are repeated until the stopping rule (e.g. elapsing a certain number of iterations) is satisfied (Goldberg (1989)).

Before giving a general outline of the heuristic, some additional notations are defined as follows:

*Pop_size*: Size of the population of solutions that remains constant during the algorithm performance.
*Max_iteration*: Number of generations which should be produced until the algorithm stops.
$p_c$: Crossover rate (which is the probability of selecting a chromosome in each generation for performing crossover)
$p_m$: Mutation rate (which is the probability of selecting a gene or bit inside a chromosome for mutating)
*Fitness_function*: Fitness value of a chromosome (the objective function value of the associated solution)
The general outline of the proposed GA based heuristic is as figure 4.
The initial population is generated randomly. Each chromosome of a population represents the sales quantities of the whole buyers. Sales quantity of each buyer which is a part of the chromosome, is represented in the form of a nine-digit binary number. To decode each chromosome in order to obtain the corresponding fitness value, we need to determine sales quantity $y_j$ for buyer 'j' using Equation (27). In this equation $y_{j_{\min}}$ and $y_{j_{\max}}$ are the minimum and maximum expected sales quantity of the *j*th buyer.

$$y_j = y_{j\min} + \left[\frac{\text{Decimal value corresponding to each gene}}{2^9}\right]\left(y_{j\max} - y_{j\min}\right) \tag{27}$$

Figure 5 depicts each chromosome representation.

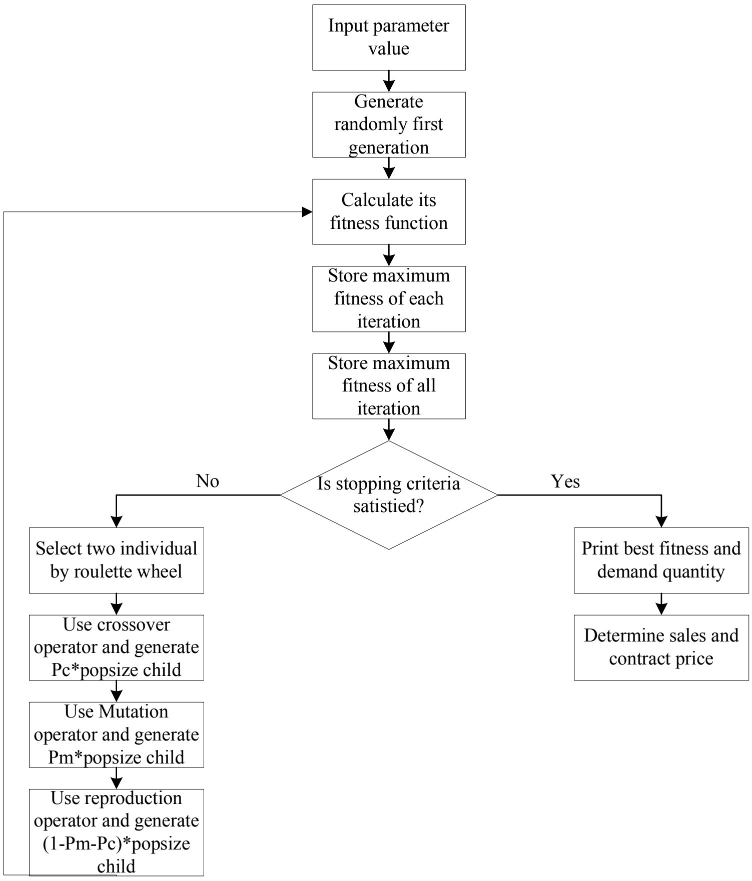


**Figure 4. GA based heuristic**

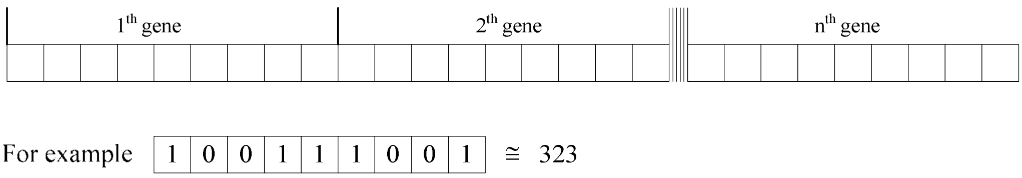


**Figure 5. Chromosome representation**

We have used simple single-point crossover. Mutation is performed by replacing 1 with 0 and inversely 0 with 1. However, the two mentioned operators are performed generating random numbers considering $p_m$ and $p_c$. *Pop_size*, $p_c$ and $p_m$ are determined through full factorial designs as will be explained later on. *Max_iteration* is assumed equal to 200.

***4.1. SA based heuristic***

SA is known to be a random search technique which simulates the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) in optimizing combinatorial optimization problems. SA was first developed in 1983 to deal with highly nonlinear problems. SA approaches the global maximum similar to a bouncing ball which bounces over mountains from valley to valley. It begins at a high temperature which enables the ball to make very high bounces. As the temperature declines, the ball cannot bounce much high and settles to become trapped in relatively small ranges of valleys. A generating distribution generates possible valleys or states which are explored. An acceptance distribution is also defined which depends on the difference between the function value of the presently generated valley to be explored and the last saved lowest one. The acceptance distribution decides probabilistically whether to stay in a new lower valley or to bounce out of it. The generating and acceptance distributions depend on the temperature. It has been proved that by carefully controlling the rate of cooling of the temperature, SA can find the global optimum.

Boltzmann's law was used to determine the probability of accepting a perturbation resulting in a change $\Delta E$ in the energy at the current temperature. Each solution with better fitness value compared with that of the previous solution is accepted with probability of 1 and each solution with worse value is accepted with $e^{-\Delta E/C_B}$ probability Where $C_B$ is the Boltzmann's constant. Iterations in each temperature is continued until balance criteria is satisfied. Balance criteria is usually considered a fixed number of iterations "*bi*". As soon as reaching balance criteria, the temperature should be decreased. The cooling schedule is specified using the way given by Mishmast and Gelareh (2007).

The behavior of the simulated annealing algorithm depends on the temperature *t*. Perhaps the most important thing is how the initial temperature $t_0$ is determinate. In theory SA procedure should be continued until the final temperature "$t_f$" is zero, but in practice other stopping criteria are applied (Mishmast and Gelareh (2007)).

It is necessary to transform the basic solution into a binary form to generating neighborhood solution by changing some binary values (replacing 0 with 1 and vice versa). The number of changed bytes depends on the problem type and size. In this paper, $10\times0.9^{k-1}$ function is used for decreasing the temperature in which *k* is the iterations number and 10 is the initial temperature. The general outline of the proposed SA based heuristic is as figure 6.

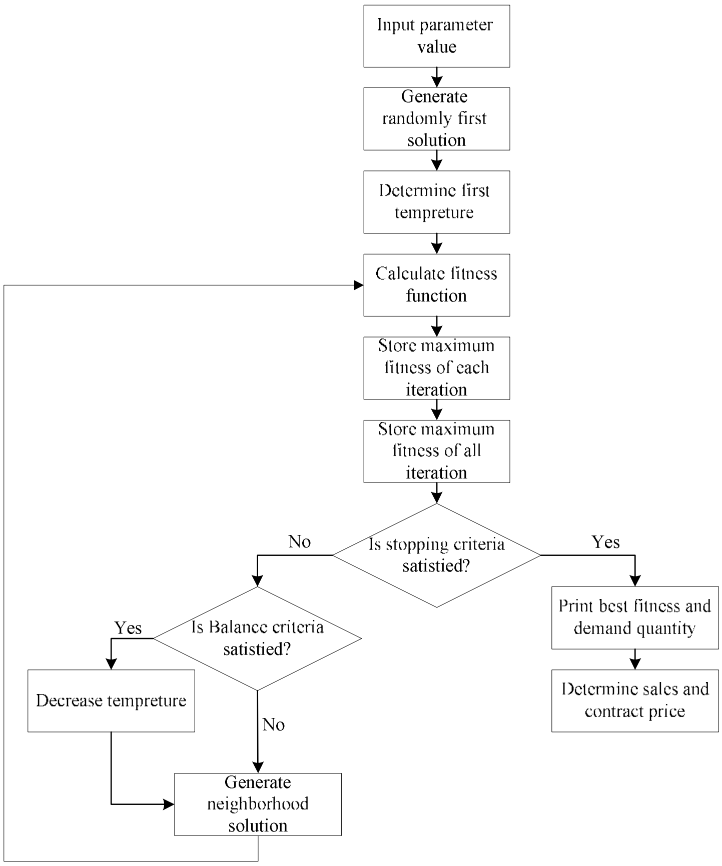


**Figure 6. SA based heuristic**

The acceptance probability of a worse solution in each stage of the algorithm is computed by $e^{\Delta PC/t}$ in which $t$ represents the current temperature and $\Delta PC$ is the difference between the current solution and the last accepted solution. Since the addressed probability becomes very close to zero, we add as a constant value $L$ to the model. The acceptance probability is computed by $e^{\Delta PC/t\times L}$ .The value of $L$ can vary in terms of the problem type so that the acceptance probability of the worse solutions is increased. In this way, SA can search a wider solution space.

### *5. Computational results*

We have designed some numerical problems. The numerical problems are given while there are 3 or 5 buyers in the model. Since the number of parameters is too much, the buyer related parameters are assumed constant. The vendor related parameters vary in a specified range. GA related parameters also vary in a specified range to find the optimal values for each experiment. The values of buyer related parameters in case with three (*j=1,2,3*) and five (*j=1,2,3,4,5*) buyers are given as in Table 1. The value range of the vendor and GA parameters are given as in Table 2.

**Table 1. Related data to the five buyers**

| $j$ | $H_{b_j}$ | $S_{b_j}$ | $a_j$ | $c_j$ | $y_{j_{\min}}$ | $y_{j_{\max}}$ | $\theta_j$ | $\pi_j$ | $\pi'_j$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 24 | 31 | 0.008 | 1600 | 4800 | 0.004 | 0.5 | 62 |
| 2 | 10 | 11 | 35 | 0.004 | 700 | 1400 | 0.008 | 0.4 | 78 |
| 3 | 10 | 29 | 37 | 0.006 | 1200 | 3600 | 0.005 | 0.3 | 59 |
| 4 | 6 | 14 | 32 | 0.003 | 1500 | 3000 | 0.005 | 0.4 | 52 |
| 5 | 7 | 25 | 39 | 0.004 | 900 | 2700 | 0.007 | 0.2 | 63 |

**Table 2. Vendor & GA related data**

| Level | $H_s$ | $S_s$ | $\delta$ | *Pop_size* | $p_c$ | $p_m$ |
|---|---|---|---|---|---|---|
| Low (-1) | 3 | 5 | 3 | 50 | 0.6 | 0.01 |
| Up (+1) | 15 | 40 | 6 | 100 | 0.8 | 0.03 |

We have designed full factorial experiments considering vendor and GA related parameters in Table 2 in order to tune GA related parameters. Since there are three GA related parameters, we have $2^3$ experiments for each numerical problem. We run each experiment for three times and evolve the best solution from among 24 experiments. Table 3-4 gives the results for cases with 3 and 5 buyers in the model.

**Table 3. The best solution of GA based heuristic with 3 buyers**

| No | $H_s$ | $S_s$ | $\delta$ | *Pop_size* | $p_m$ | $p_c$ | Objective function ($P_c$) |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 50 | 0.6 | 0.01 | 79234.29 |
| 2 | 3 | 5 | 6 | 50 | 0.8 | 0.03 | 64560.39 |
| 3 | 3 | 40 | 3 | 100 | 0.8 | 0.01 | 77626.16 |
| 4 | 3 | 40 | 6 | 50 | 0.6 | 0.01 | 62977.54 |
| 5 | 15 | 5 | 3 | 50 | 0.8 | 0.01 | 77978.07 |
| 6 | 15 | 5 | 6 | 100 | 0.8 | 0.03 | 63327.36 |

| No | $H_s$ | $S_s$ | $\delta$ | $Pop\_size$ | $p_m$ | $p_c$ | Objective function ($P_c$) |
|---|---|---|---|---|---|---|---|
| 7 | 15 | 40 | 3 | 50 | 0.8 | 0.01 | 75664.14 |
| 8 | 15 | 40 | 6 | 100 | 0.8 | 0.03 | 61049.72 |

Table 4. The best solution of GA based heuristic with 5 buyers

| No | $H_s$ | $S_s$ | $\delta$ | $Pop\_size$ | $p_m$ | $p_c$ | Objective function ($P_c$) |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 50 | 0.6 | 0.03 | 158539.96 |
| 2 | 3 | 5 | 6 | 100 | 0.6 | 0.03 | 129563.66 |
| 3 | 3 | 40 | 3 | 100 | 0.6 | 0.03 | 155719.03 |
| 4 | 3 | 40 | 6 | 50 | 0.6 | 0.01 | 126832.038 |
| 5 | 15 | 5 | 3 | 100 | 0.8 | 0.03 | 156239.17 |
| 6 | 15 | 5 | 6 | 100 | 0.8 | 0.03 | 127330.14 |
| 7 | 15 | 40 | 3 | 100 | 0.8 | 0.03 | 152063.07 |
| 8 | 15 | 40 | 6 | 100 | 0.6 | 0.01 | 123289.46 |

The value range of the vendor and SA parameters are given as in Table 5. N represents the size of the problem.

**Table 5.Vendor & SA related data**

| Level | $H_s$ | $S_s$ | $\delta$ | $bi$ | The number of changed bytes for neighborhood creation | L |
|---|---|---|---|---|---|---|
| Low (-1) | 3 | 5 | 3 | 100 | N | 500 |
| Up (+1) | 15 | 40 | 6 | 300 | 3N | 1000 |

We have designed full factorial experiments considering vendor and SA related parameters in Table 5 in order to tune SA related parameters. Since there are three SA related parameters, we have $2^3$ experiments for each numerical problem. We run each experiment for three times and evolve the best solution from among 24 experiments. Table 6-7 gives the results for cases with 3 and 5 buyers in the model.

**Table 6. The best solution of SA based heuristic with 3 buyers**

| No | $H_s$ | $S_s$ | $\delta$ | $bi$ | The number of changed bytes for neighborhood creation | L | Objective function ($P_c$) |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 300 | N | 1000 | 79166.91 |
| 2 | 3 | 5 | 6 | 300 | N | 500 | 64500.58 |
| 3 | 3 | 40 | 3 | 300 | N | 500 | 77597.7 |
| 4 | 3 | 40 | 6 | 300 | N | 500 | 62935.34 |
| 5 | 15 | 5 | 3 | 100 | N | 500 | 77959.76 |
| 6 | 15 | 5 | 6 | 300 | 3N | 500 | 63304.41 |
| 7 | 15 | 40 | 3 | 300 | 3N | 500 | 75544.34 |
| 8 | 15 | 40 | 6 | 300 | N | 500 | 61002.44 |

**Table 7. The best solution of SA based heuristic with 5 buyers**

| No | $H_s$ | $S_s$ | $\delta$ | $bi$ | The number of changed bytes for neighborhood creation | L | Objective function ($P_c$) |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 300 | 3N | 1000 | 158256.37 |
| 2 | 3 | 5 | 6 | 100 | 3N | 1000 | 129149.99 |
| 3 | 3 | 40 | 3 | 100 | 3N | 1000 | 155446.73 |
| 4 | 3 | 40 | 6 | 300 | 3N | 1000 | 126609.91 |
| 5 | 15 | 5 | 3 | 100 | 3N | 500 | 155780.36 |
| 6 | 15 | 5 | 6 | 300 | N | 500 | 127065.41 |
| 7 | 15 | 40 | 3 | 100 | N | 1000 | 151873.13 |
| 8 | 15 | 40 | 6 | 300 | N | 500 | 122981.05 |

To evaluate the performance of the proposed algorithms, we have solved them utilizing LINGO optimization solver.
Tables 8-9 gives the objective function values obtained from GA, SA and LINGO for the cases with 3 and 5 buyers respectively.

**Table 8. The best solution of GA and SA based heuristics as wsll as LINGO with 3 buyers**

| No | $H_s$ | $S_s$ | $\delta$ | GA | SA | LINGO |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 79234 | 79166 | 79234 |
| 2 | 3 | 5 | 6 | 64560 | 64500 | 64560 |
| 3 | 3 | 40 | 3 | 77626 | 77597 | 77626 |
| 4 | 3 | 40 | 6 | 62977 | 62935 | 62977 |

| No | $H_s$ | $S_s$ | $\delta$ | GA | SA | LINGO |
|---|---|---|---|---|---|---|
| 5 | 15 | 5 | 3 | 77978 | 77959 | 77978 |
| 6 | 15 | 5 | 6 | 63327 | 63304 | 63327 |
| 7 | 15 | 40 | 3 | 75664 | 75544 | 75664 |
| 8 | 15 | 40 | 6 | 61049 | 61002 | 61049 |

**Table 9. The best solution of GA and SA based heuristics as well as LINGO with 3 buyers**

| No | $H_s$ | $S_s$ | $\delta$ | GA | SA | LINGO |
|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 158539 | 158256 | 158540 |
| 2 | 3 | 5 | 6 | 129563 | 129149 | 129564 |
| 3 | 3 | 40 | 3 | 155719 | 155446 | 155719 |
| 4 | 3 | 40 | 6 | 126832 | 126609 | 126832 |
| 5 | 15 | 5 | 3 | 156239 | 155780 | 156239 |
| 6 | 15 | 5 | 6 | 127330 | 127065 | 127330 |
| 7 | 15 | 40 | 3 | 152063 | 151873 | 152063 |
| 8 | 15 | 40 | 6 | 123289 | 122981 | 123289 |

The results indicate that both GA and SA give results very close to LINGO optimization solver. However, GA gives better solutions compared with SA.

***6. Conclusions and further research***

This paper presents a model under VMI mode of operation. It is an extension to that of Nachiappan & Jawahar (2006) assuming unsatisfied demand is backordered at buyers. The model was stated as a mathematical programming problem with the objective function of channel profit and decision variable of sales quantity. Having the optimal sales quantity, the corresponding optimal sales price and contract price can be determined. Two GA and SA based heuristics were developed to solve the problem. Tuning some selected parameters of each algorithm with respect to a few numerical problems, the near optimal solutions were found. The results showed that both GA and SA gave results very close to LINGO optimization solver. However, GA gave better solutions compared with SA.

Further research can be considering demand to be lost during stockout. Some parameters such as demand and lead time may be assumed stochastic.

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