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PREDICTING THE FINANCIAL CRISIS VOLATILITY

Abstract. *A volatility model must be able to forecast volatility even in extreme situations. Thus, the main objective of this paper,* ***and due to the most recent increase in international stock markets' volatility****, is to check which one of the most popular autoregressive conditional heteroskedasticity models (GARCH, GJR, EGARCH or APARCH) is more able to predict the extreme volatility in 2008 considering the daily returns of eight major international stock market indexes: CAC 40 (France), DAX 30 (Germany), FTSE 100 (UK), NIKKEI 225 (Japan), HANG SENG (Hong Kong), NASDAQ 100, DJIA and S&P 500 (United States). Goodness-of-fit measures demonstrate that EGARCH and APARCH models are able to correctly fit the conditional heteroskedasticity dynamics of the return's series under study. In terms of volatility forecast comparisons, using the Harvey-Newbold test for multiple forecasts encompassing and the ranking of forecasts based on the coefficient of determination* (R^2) *resulting from the Mincer-Zarnowitz regression, we conclude that EGARCH dominates competing standard asymmetric models.*

Key words: *Forecasting volatility, EGARCH, APARCH, GJR..*

JEL Classification: C52, C53

1. Introduction

In the last years the stock markets prices have been characterized by unusual variations leading to several periods of extreme high volatility, especially in the second half of 2008. As one can see in figure 1, the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), the most prominent indicator of investors' expectations on future market volatility, achieved its 80.86 greatest value in November 20, 2008. Recall that VIX values greater than 30 are generally associated with high volatilities a result of investors fear or uncertainty, while values below 20 generally correspond to less stressful, even complacent, times in the financial markets.

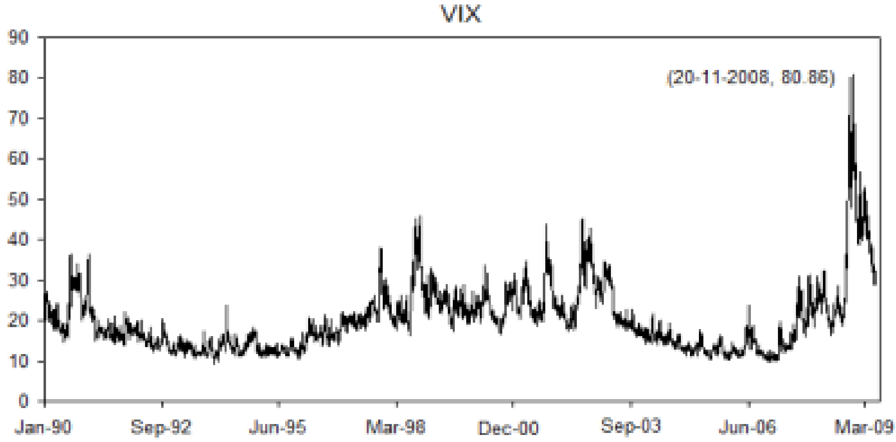

Figure 1. Historical values of Volatility Index (VIX)

For the lack of investors' confidence we can refer the bankruptcy of some important American financial institutions (Bear Sterns and Lehman Brothers, among others), the rescue of many others by the governments' support (AIG, Fannie Mae, Freddie Mac, among others) and the economic recession where most of the western economies dived in. The "speculative bubble" in the real estate sector with the properties overpricing and the high risk derivatives applications are pointed as the main causes for the huge amounts of "toxic" assets write-off that "threw down" the financial institutions.

The sharply volatility increase in financial markets during 2008 is also documented by the yearly boxplots of S&P 500 daily returns (see figure 2). This graphical representation also confirms the dramatically changes in volatility throughout the analyzed period.

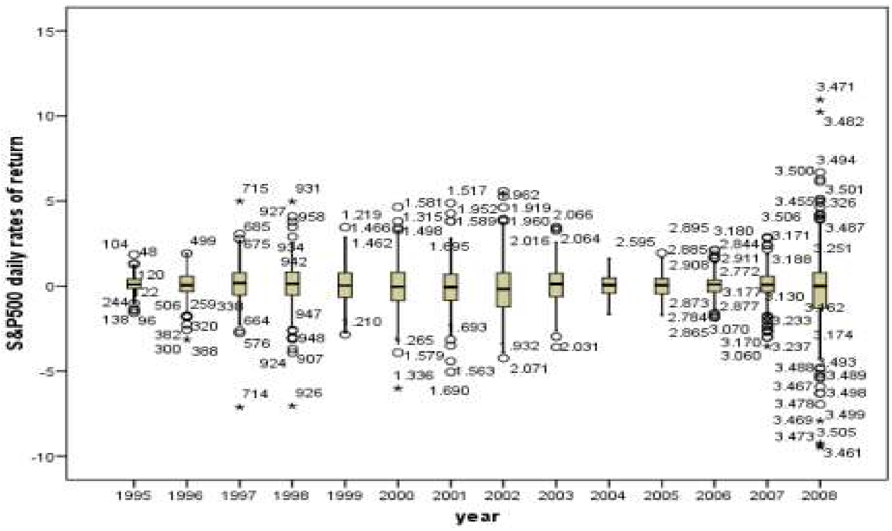

Figure 2. S&P 500 yearly boxplots

This feature of stock markets' volatility suggests that returns appear to be drawn from a time-dependent heteroskedastic distribution. As early noted in the pioneering studies of [20] and [11], financial time series vary systematically with time and tend to display periods of unusually large volatility, followed by periods of low volatility.

Despite these early studies, and the importance of modeling and forecasting volatility in financial markets (see for example [21]), the efforts to model volatility dynamics have only been developed in the last decades. In fact, the variance of the disturbance term was assumed to be constant in conventional econometric models, i.e., financial time series modelling was centered on the conditional first moment, with any temporal dependencies in the higher order moments treated as a nuisance.

However, the increased importance played by risk and uncertainty considerations has recently spurred a vast literature on modeling and forecasting return's volatility (see for example [6] and [19]). The trade-off between risk and return, where risk is associated with the variability of the random (unforeseen) component of a time series (volatility), constitutes one of the cornerstones of modern finance. In effect, finance and economics are nowadays fields where the explicit modeling of uncertainty takes on a particularly significant role, since valuation models for the majority of assets are essentially based on the first two moments of the return series: mean, variance and covariances. Moreover, due to the compelling theoretical and empirical results supporting the efficient market theory, academicians and practitioners have to some extent ignored the question of return's forecastibility in recent decades; concentrating, instead, on exploring the question of risk. Understanding the statistical properties of volatility is currently considered an important area of interest, given the impact of volatility changes, namely, in risk analysis, portfolio selection, market timing and derivative pricing.

Recent studies on stock return's volatility have been dominated by ARCH models (Engle, 1982; Bollerslev, 1986), which stand for autoregressive conditional heteroskedasticity. These models have been extremely successful in accounting for the main characteristics of financial data time series. As the conditional variance is specified as a weighted average of past squared errors, ARCH and GARCH models are able to capture the volatility clusters stylized fact of returns and they also partially describe the fat tails exhibited by these series. However, their simple structure has two major drawbacks. First, as the GARCH model assumes that only the magnitude — and not the sign — of the innovations determines the conditional variance, it fails to incorporate the leverage effect (see for example [28]). As observed by [3], volatility responds asymmetrically to the sign of the change in the price of the financial asset, i.e., volatility increases more after negative changes than after positive changes of the same magnitude, naming this phenomenon as the leverage effect (also referred as the Fisher-Black effect). Another drawback is the parameters non-negativity restrictions.

These drawbacks led [29] to propose the Threshold GARCH (TGARCH) model, which is similar to the GARCH model except in an additional term to capture the leverage effect. This model is very similar to the GJR of [12], modeling the standard-deviation instead of the conditional variance. A different approach to capture the leverage effect is presented by [24], with his Exponential GARCH (EGARCH) model. [8] propose a model which allows the power of the heteroskedasticity equation to be estimated from the data, and name it as Asymmetric Power ARCH (APARCH). This model encompasses seven other models (see [8] for a proof).

We conclude this brief introduction by emphasizing that the predictability of volatility is required very often by financial activities, such as risk management, derivative pricing and hedging, market making, market timing and portfolio selection [10]. Therefore, due to the importance of modeling and forecasting volatility in financial markets, the main purpose of this paper is to check which one of the most popular autoregressive conditional heteroskedasticity models (GARCH, GJR, EGARCH or APARCH) is more able to predict the extreme volatility in 2008, considering the daily returns of eight major international stock market indexes: CAC 40 (France), DAX 30 (Germany), FTSE 100 (UK), NIKKEI 225 (Japan), HANG SENG (Hong Kong), NASDAQ 100, DJIA and S&P 500 (United States). GARCH-type models have been also used recently by Predescu and Stancu (2011) in the Value-at-Risk estimation.

The paper is organized as follows. Next section presents the econometric approach and section 3 describes the data sets. Section 4 discusses estimation results and compares out-of-sample evaluation results for the GARCH, GJR, EGARCH and APARCH models. Finally, section 5 presents some concluding remarks.

2. Econometric Approach

The empirical distribution of a financial asset return can be described as the sum of a predictable part with an unpredictable part:

$$r_t = E[r_t|\Phi_{t-1}] + u_t, \tag{1}$$

where Φ_{t-1} is the relevant information set until, and including, $t-1$. For the conditional mean, $E[r_t|\Phi_{t-1}]$, our first intuition was to assume a white noise process, since the empirical distributions of returns under study represent the most liquid and efficient financial markets in the world — as far equities are concerned — and since this work is primarily dedicated to the dynamics of the variance equation. However, anticipating our findings in the data analysis section, we shall also specify the conditional mean equation as a fifth-order autoregressive process, AR(5), in order to remove the observed linear dependency in returns:

$$r_t = c + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \phi_3 r_{t-3} + \phi_4 r_{t-4} + \phi_5 r_{t-5} + u_t, \tag{2}$$

where $u_t = z_t \sigma_t$ and the standardized innovations (z_t) are assumed to be independently and identically distributed (i.i.d.) with Student's t distribution. This statistical distribution has a long tradition in the econometrics literature as a popular choice of a fat-tailed distribution, since it has finite second moment (in contrast to stable non-Gaussian distributions), its mathematical properties are well known, it is undemanding to estimate, and is often found capable of capturing the excess of kurtosis observed in financial time-series.

For the conditional variance of u_t: $E[u_t^2|\Phi_{t-1}] = \sigma_t^2$, we have considered the most popular conditional heteroskedastic specifications: the symmetric GARCH and the asymmetric APARCH, GJR and EGARCH models.
Despite the theoretical interest of (p,q) models, the $(1,1)$ specification is, in general, satisfactory when modeling financial assets returns volatility (see [5] and more recently [15]). Thus, in this paper all conditional heteroskedastic models are of $p = 1, q = 1$ order:

$$\text{GARCH:} \quad \sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{3}$$

$$\text{EGARCH:} \quad \ln\, \sigma_t^2 = \omega + \alpha_1 \frac{|u_{t-1}|}{\sigma_{t-1}} + \gamma_1 \frac{u_{t-1}}{\sigma_{t-1}} + \beta_1 \ln\, \sigma_{t-1}^2, \tag{4}$$

$$\text{GJR:} \quad \sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \gamma_1 I_{t-1} u_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{5}$$

$$\text{APARCH:} \quad \sigma_t^{\delta_1} = \omega + \alpha_1 (|u_{t-1}| - \gamma_1 u_{t-1})^{\delta_1} + \beta_1 \sigma_{t-1}^{\delta_1}, \tag{6}$$

where $\omega, \alpha_1, \gamma_1, \delta_1$ and β_1 are unknown parameters, $I_{t-1} = 1$ if $u_{t-1} < 0$ and $I_{t-1} = 0$ if $u_{t-1} \geq 0$. The models are estimated through maximum likelihood (MLE).
The sample is partitioned in two distinct periods: the first 13 years of the sample are retained for the parameters estimation while the remaining year (2008) is considered as the forecast period. Parameters for the conditional variance equation are therefore estimated for the 1995-2007 period (corresponding roughly to 3250 observations). These parameters are used to estimate the daily conditional volatility and together with the diagnostics constitute the in-sample set of results. To estimate the ex-ante out-of-sample predictive power of the models, the estimated parameters are used to forecast the one-day-ahead conditional variance for the year 2008 (corresponding roughly to 250 observations).
The statistically significant returns autocorrelations are removed by fitting the AR(5) model to the series (the conditional mean equation parameters are represented by ϕ_j). In all cases the residuals become white noise.
To compare the conditional in-sample results, three likelihood based goodness-of-fit criteria are used. The first is the maximum log-likelihood value obtained from ML estimation. The second is the AIC: Akaike information criteria [1] and the third is the SBC: Schwarz Bayesian criteria [27].
Out-of-sample volatility forecast evaluation is conducted by applying the modified Diebold-Mariano [7] test proposed by [16] (hereafter HN test) to gauge whether each model encompasses the others three[1]. According to [14], when the comparison involves nested models (GJR nests GARCH when $\gamma_1 = 0$; APARCH nests GARCH when $\delta_1 = 2$ and $\gamma_1 = 0$ and APARCH nests GJR when $\delta_1 = 2$) it is more appropriate to apply a test for equal predictive accuracy (EPA), such as that of [16]. In the null we state that each particular model (GARCH, EGARCH, GJR and APARCH) encompasses its competitors, in the sense that they do not contain useful information not present in the forecasts resulting from the model

[1]The test values are obtained using EVIEWS 6.0-based custom software.

considered in the null.
The volatility forecasts are also ranked based on the coefficient of determination (R^2) from the Mincer-Zarnowitz[2] regression:

$$r_t^2 = \beta_0 + \beta_1 \hat{\sigma}_t^2 + \varepsilon_t, \tag{7}$$

where r_t^2 and $\hat{\sigma}_t^2$ are the proxies and the predictions for the conditional volatility, respectively.
Since volatility itself is not directly observable, establishing the effectiveness of the volatility forecast involves the use of a "volatility proxy" that may constitute an imperfect estimate of the true volatility, as mentioned by [2], [13] and [15]. Following the conventional approach, squared returns (r_t^2) are used as a proxy for the latent volatility process. According to [25] the squared returns on an asset over the period t (assuming a zero mean return) is a conditionally unbiased estimator of the true unobserved conditional variance of the asset over the period t.

3. Statistical properties of returns

The empirical analysis is based on daily closing prices for eight major international stock market indexes from December 31, 1994 through December 31, 2008. The investigated indexes are the CAC 40 (France), DAX 30 (Germany), FTSE 100 (UK), NIKKEI 225 (Japan), HANG SENG (Hong Kong), NASDAQ 100, DJIA and S&P 500 (United States). All data series are drawn from Bloomberg and represent a local currency perspective. Dividends are not included in the calculation of the indexes.
The dataset contains several episodes of regional as well as global "market stress", involving high volatility. Noteworthy examples are the October 1997 Asian mini-crash, the 1998 Russian financial crisis, the March 2000 dot-com bubble crash, the September 2001 post-9/11 crash and the still prevailing subprime crisis.
Following the conventional approach, daily stock returns (r_t) are obtained by taking the logarithmic difference of daily stock index price data:

$$r_t = 100 \times \log\left(\frac{P_t}{P_{t-1}}\right) \tag{8}$$

Table 1 provides a general overview of the data used and presents preliminary descriptive statistics and diagnostics for the return series of each of the eight stock indexes. The mean returns are all positive (except NIKKEI) but close to zero. The unfavorable outcome of Japanese stock returns is attributable to the fact that the Japanese market has been a bear market since 1989. The sample moments for all return series indicate asymmetric empirical distributions with heavy tails relative to the normal. Not surprisingly, the Jarque-Bera test [18] rejects the normality assumption for each of the series.

[2] [22] showed that this ranking is robust to noise in the volatility proxy: r_t^2.

Table 1. Summary statistics of returns

	CAC	DAX	FTSE	NIKKEI	H SENG	NASDAQ	DJIA	SP&500
# Obs	3548	3542	3536	3444	3472	3526	3526	3526
Mean	0.0151	0.0233	0.0104	-0.0232	0.0162	0.0210	0.0235	0.0192
Median	0.0473	0.0872	0.0455	-0.0055	0.0404	0.1251	0.0493	0.0615
Maximum	10.5946	10.7975	9.3842	13.2346	17.2470	13.2546	10.5084	10.9572
Minimum	-9.4715	-7.4335	-9.2646	-12.1110	-14.7346	-10.1684	-8.2005	-9.4695
Std. Dev.	1.4687	1.5505	1.2118	1.5691	1.7862	1.7481	1.1915	1.2452
Skewness	-0.0328	-0.0474	-0.1318	-0.2073	0.1020	-0.0282	-0.1498	-0.2337
Kurtosis	8.1136	7.5015	9.7908	8.8532	13.4440	7.9231	11.6550	12.1810
J-Bera	3866*	2992*	6805*	4941*	15786*	3561*	11019*	12416*
LB Q(10)	56.7*	28.4*	101.4*	26.8*	24.7*	21.7**	59.3*	61.4*
LB $Q^2(10)$	2095.1*	2095.9*	2857.8*	2892.1*	1700.2*	2118.5*	2429.4*	2872.9*
ARCH-LM	707.9*	686.7*	901.3*	964.3*	734.9*	687.7*	838.5*	954.8*

**, ** Denote significant at the 1% and 5% level, respectively*
LB Q(10) is the Ljung-Box test for returns
LB $Q^2(10)$ is the Ljung-Box test for squared returns
ARCH-LM is Engle's Lagrange Multiplier test for conditional heteroskedasticity

According to the Ljung-Box statistics on returns, computed at a tenth-order lag, there is relevant autocorrelation in all of the stock indexes. Thus, we consider a five order autoregressive model to remove the linear dependency in the series.
The Ljung-Box statistic for up to tenth order serial correlation of squared returns is highly significant at any level for the eight stock indexes, suggesting the presence of strong nonlinear dependence in the data. As non-linear dependence and heavy-tailed unconditional distributions are characteristic of conditionally heteroskedastic data, the Lagrange Multiplier test [9] can be used to formally test the presence of conditional heteroskedasticity and the evidence of ARCH effects. The LM test for a tenth-order linear ARCH effect strongly suggest the presence of time-varying volatility, implying that nonlinearities must enter through the variance of the processes [17]. Such behaviour can be captured by incorporating GARCH structures in the model, allowing conditional heteroskedasticity by conditioning the volatility of the process on past information. In the next section we use AR-GARCH models to describe the conditional distribution of returns.

4. Estimation Results

Tables 2 to 5 report in-sample results for the eight stock indexes. Almost all the variance equation estimated coefficients are statistically significant pointing to the conditional heteroskedasticity of returns. The in-sample estimation results confirm that markets become more volatile in response to "bad news" (negative return surprises) as the sign of the parameter estimates proxying for asymmetry (γ_1) are always negative (EGARCH) and positive (GJR and APARCH) in spite of the stock index being considered.

Due to the presence of the leverage effect, the asymmetric models clearly outperform the symmetric GARCH in terms of goodness-of-fit measures (Log-lik, AIC and SBC). Within the asymmetric models set, EGARCH and APARCH dominate GJR, as they have the largest log-likelihood value and the smallest AIC and SBC values. The differences between EGARCH and APARCH are very weak pointing to the same results. Thus, goodness-of-fit measures indicate that EGARCH or APARCH are the models more prone to have generated the data.

Table 2. Estimation Results

	CAC 40				DAX 30			
	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH
c	0.083*	0.054	0.057*	0.054*	0.102*	0.074*	0.076*	0.073*
ϕ_1	-0.017	-0.012	-0.012	-0.012	-0.022	-0.02	-0.017	-0.018
ϕ_2	-0.022	-0.016	-0.014	-0.015	0.0005	0.007	0.008	0.008
ϕ_3	-0.051*	-0.046	-0.048*	-0.045*	-0.013	-0.007	-0.008	-0.006
ϕ_4	-0.010	-0.001	-0.005	-0.002	0.004	0.003	0.008	0.004
ϕ_5	-0.048*	-0.044	-0.045**	-0.043**	-0.026	-0.019	-0.022	-0.019
ω	0.013*	-0.097	0.017*	0.017*	0.016*	-0.119*	0.021*	0.022*
α_1	0.070*	0.128	0.019**	0.066*	0.086*	0.159*	0.035*	0.084*
β_1	0.924*	-0.068	0.926*	0.932*	0.908*	0.984*	0.906*	0.914*
δ_1				1.158*				1.183*
γ_1		-0.068	0.085*	0.531*		-0.075*	0.093*	0.475*
tdf	14.452*	17.104	16.322	16.868*	12.762*	13.729*	14.406*	14.283*
LL	-5179.57	**-5151.69**	-5155.2	**-5150.9**	-5150.9	**-5275.714**	-5278.2	**-5272.3**
AIC	3.157	**3.140**	3.143	**3.140**	3.234	**3.222**	3.223	**3.220**
SBC	3.175	**3.161**	3.163	**3.163**	3.241	**3.242**	3.244	**3.243**

Table 3. Estimation Results

	FTSE 100				NASDAQ 100			
	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH
c	0.062*	0.034**	0.038*	0.032**	0.101*	0.070*	0.074*	0.072*
ϕ_1	-0.012	-0.005	-0.009	-0.003	0.034***	0.037**	0.041**	0.039**
ϕ_2	-0.03***	-0.02	-0.025	-0.020	-0.039**	-0.028	-0.03**	-0.030***
ϕ_3	-0.043**	-0.031***	-0.037**	-0.029***	0.007	0.016	0.016	0.016
ϕ_4	-0.014	-0.010	-0.011	-0.010	-0.007	0.002	0.002	0.002
ϕ_5	-0.042**	-0.034**	-0.037**	-0.034**	-0.026	-0.012	-0.012	-0.012
ω	0.010*	-0.091*	0.010*	0.013*	0.010*	-0.097*	0.012*	0.013*
α_1	0.084*	0.111*	0.005	0.060*	0.067*	0.129*	0.020**	0.063*
β_1	0.909*	0.987*	0.929*	0.940*	0.931*	0.990*	0.932*	0.936*

δ_1				0.964*				1.425*
γ_1		-0.090*	0.107*	0.835*		-0.061*	0.084*	0.450*
tdf	16.545*	19.565*	17.355*	19.579*	13.199*	14.643*	14.263*	14.470*
LL	-4324.9	**-4288.7**	-4294.8	**-4287**	-5552.8	**-5530.5**	-5532.2	**-5529.0**
AIC	2.646	**3.624**	2.628	**3.624**	3.403	**3.390**	3.391	**3.390**
SBC	2.664	**3.645**	2.648	**3.646**	3.422	**3.411**	3.412	**3.412**

Table 4. Estimation Results

	DJIA				S&P500			
	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH
c	0.077*	0.056*	0.059*	0.054*	0.086*	0.060*	0.063*	0.060*
ϕ_1	-0.013	-0.007	-0.008	-0.006	-0.023	-0.012	-0.012	-0.013
ϕ_2	-0.039**	-0.025	-0.026	-0.024	-0.049*	-0.031***	-0.03**	-0.032***
ϕ_3	-0.016	-0.009	-0.009	-0.008	-0.028	-0.017	-0.019	-0.018
ϕ_4	-0.010	-0.006	-0.005	-0.005	-0.018	-0.011	-0.006	-0.009
ϕ_5	-0.035*	-0.025	-0.028	-0.024	-0.047*	-0.034**	-0.04**	-0.035**
ω	0.007*	-0.087*	0.010*	0.014*	0.005*	-0.084*	0.009*	0.012*
α_1	0.061*	0.108*	0.004	0.056*	0.060*	0.104*	-0.010	0.048
β_1	0.934*	0.985*	0.934*	0.941*	0.937*	0.984*	0.939*	0.94*
δ_1				1.044*				1.263*
γ_1		-0.084*	0.100*	0.830*		-0.098*	0.120*	0.999*
tdf	7.982*	9.221*	8.850*	9.371*	7.728*	9.174*	9.078*	9.259*
LL	-4319.5	**-4282.8**	-4291.9	**-4281.4**	-4385.3	**-4340.2**	-4343.9	**-4338.3**
AIC	2.649	**2.627**	2.633	**2.627**	2.689	**2.662**	2.664	**2.662**
SBC	2.667	**2.648**	2.653	**2.649**	2.708	**2.683**	2.685	**2.684**

Table 5. Estimation Results

	NIKKEI 225				HANG SENG			
	GARCH	EGARCH	GJR	APARCH	GARCH	EGARCH	GJR	APARCH
c	0.036***	0.006	-0.008	0.006	0.072*	0.053*	0.055*	0.053*
ϕ_1	-0.028	-0.023	-0.018	-0.022	0.038**	0.043**	0.046**	0.044**
ϕ_2	-0.025	-0.018	-0.037**	-0.018	-0.016	-0.011	-0.013	-0.011
ϕ_3	0.015	0.024	0.015	0.024	0.036**	0.043**	0.044**	0.043**
ϕ_4	-0.023	-0.016	-0.031**	-0.015	-0.015	-0.009	-0.013	-0.009
ϕ_5	-0.004	0.003	0.003	0.003	-0.041**	-0.038**	-0.04**	-0.035**
ω	0.021*	-0.088*	0.963*	0.030*	0.011*	-0.091*	0.019*	0.016*
α_1	0.062*	0.130*	0.081*	0.067*	0.054*	0.129*	0.022**	0.064*
β_1	0.929*	0.976*	0.367*	0.925*	0.942*	0.989*	0.933*	0.937*
δ_1				1.119*				1.301*
γ_1		-0.077*	0.062	0.602*		-0.058*	0.073*	0.443*

tdf	8.623*	9.644*	4.763*	9.617*	6.618*	7.207*	7.174*	7.279*
LL	-5408.0	**-5381.5**	-5521.5	**-5380.9**	-5452.4	**-5431.5**	-5435.1	**-5430.6**
AIC	3.392	**3.376**	3.463	**3.376**	3.397	**3.385**	3.387	**3.385**
SBC	3.411	**3.396**	3.484 3	**3.399**	3.416	**3.405**	3.408	**3.407**

*Notes: *, **, *** denote significant at the 1%, 5% and 10% level, respectively. ϕ_j are the conditional mean equation parameters. $\omega, \alpha_1, \beta_1, \delta_1$ and γ_1 are the conditional variance parameters. "tdf" denotes the degrees of freedom for the Student's t distribution. "LL" refers to the maximum log-likelihood value. "AIC" is the Akaike Information Criterion and "SBC" is the Schwarz Bayesian Criterion.*

In the out-of-sample analysis, based on Harvey-Newbold (HN) test (table 6), the results seem to favor the EGARCH model. According to the test results, we fail to reject the null that the EGARCH forecasts encompass, or cannot be improved by combination with, the corresponding GARCH, APARCH and GJR volatility predictions at the 5% significance level in six of the eight stock indexes: CAC, DAX, FTSE, NASDAQ, DJIA and S&P 500. The null is rejected at this significance level in case of the Asian stock indexes: NIKKEI and HANG SENG, implying that combination of the GARCH and/or APARCH and/or GJR predictions with those of EGARCH would lead to an improvement in the NIKKEI and HANG SENG forecast performance. Excluding the case of the FTSE index, the HN test results point to the same conclusions when one tests if APARCH forecasts encompass those of the competing GARCH, EGARCH and GJR. However, in spite of the same conclusion, the significance levels of the test are always lower when compared to the ones of the EGARCH model. In contrast, the hypothesis that the GJR forecasts encompass its rivals is rejected in five of the eight stock indexes.

Thus, even if the failure to reject the null hypothesis of forecast encompassing among multiple forecasts does not necessarily imply that the forecast under the null is superior and dominant with respect to its competitors, this constitutes one legitimate possibility [16]. Based on this, along with the fact that the significance level associated with the HN tests is always higher for the EGARCH model, we can conclude that it is more likely that EGARCH forecasts (when compared to the other models) encompass the corresponding GARCH, APARCH and GJR volatility predictions.

Table 6. Harvey-Newbold forecast encompassing test (probability values are given in brackets)

Index	GARCH	EGARCH	GJR	APARCH
CAC 40	4.743 [0.0095]	**.898 [0.1520]**	3.709 [0.0258]	2.693 [0.0696]
DAX 30	5.040 [0.0071]	**.722 [0.1807]**	3.595 [0.0289]	2.272 [0.1053]
FTSE 100	6.282 [0.0022]	**.605 [0.0759]**	4.511 [0.0119]	3.354 [0.0365]
NIKKEI 225	7.351 [0.0008]	**.875 [0.0221]**	8.461 [0.0003]	5.026 [0.0073]
HANG SENG	9.458 [0.0001]	**.236 [0.0059]**	8.386 [0.0003]	7.999 [0.0004]
NASDAQ 100	1.990 [0.1389]	**.091 [0.3374]**	1.417 [0.2443]	1.476 [0.2306]
DJIA	2.989 [0.052]	**.437 [0.2396]**	2.173 [0.1160]	1.548 [0.2146]
SP&500	2.860 [0.0591]	**.568 [0.2106]**	2.511 [0.0832]	1.654 [0.1934]

As the decision to reject or not reject the null of the HN test for a 5% significance level points to the same conclusions in seven of the eight stock indexes when volatility forecasts resulting from EGARCH and APARCH models are compared, we proceed next by ranking the forecasts on the basis of the coefficient of determination (R^2) from the Mincer-Zarnowitz regression.
As one can see in table 7 the R^2 value is always higher for the EGARCH model in spite of the stock index being considered. Thus, based on the significance levels of the HN test and the ranking from the Mincer-Zarnowitz regression R^2, we can conclude that EGARCH is the most appropriate model to forecast the financial markets volatility during the extreme volatile year of 2008.

Table 6. The Mincer-Zarnowitz regression

Index	GARCH	EGARCH	GJR	APARCH
CAC 40	0.12867	**.17233**	0.16524	0.16296
DAX 30	0.10976	**.16945**	0.15568	0.16394
FTSE 100	0.15142	**.20380**	0.19164	0.19092
NIKKEI 225	0.18711	**.25820**	0.20042	0.22473
HANG SENG	0.09973	**.19325**	0.16493	0.17294
NASDAQ 100	0.16724	**.19612**	0.19119	0.18558
DJIA	0.17673	**.24599**	0.22321	0.22936
SP&500	0.18131	**.24660**	0.22280	0.22589

5. Conclusions

Volatility forecasts constitute a very important support for many financial activities such as risk management, derivative pricing and hedging, market making, market timing and portfolio selection. Therefore, financial decision-making requires a volatility forecasting model that can be used even in extreme situations.
As conditional heteroscedasticity is a stylized fact of financial returns, the main purpose of this paper is to check which one of the most popular autoregressive conditional heteroskedasticity models (GARCH, GJR, EGARCH or APARCH) is more able to predict the financial markets extreme volatility in 2008.
The empirical analysis is based on daily closing prices for eight major international stock market indexes from December 31, 1994 through December 31, 2008. The investigated indexes are the CAC 40 (France), DAX 30 (Germany), FTSE 100 (UK), NIKKEI 225 (Japan), HANG SENG (Hong Kong), NASDAQ 100, DJIA and S&P 500 (United States).
In order to identify the most appropriate model to forecast volatility in these major stock markets, the sample is partitioned in two distinct periods: the first 13 years of the sample (from 1995 to 2007) are retained for the parameters estimation while the remaining year (2008) is considered as the forecast period.
In terms of finding in-sample results, based on maximum likelihood and information criteria goodness-of-fit measures, we conclude that EGARCH and

APARCH models are able to correctly fit the conditional heteroskedasticity dynamics of the return's series under study.
Out-of-sample results show that, based on Harvey-Newbold encompassing test decision, EGARCH and APARCH models are the best predictors of volatility forecasts for most of the stock indexes under study. However, as the significance levels of the Harvey-Newbold test and the R^2 values from the Mincer-Zarnowitz regression are always higher for the EGARCH model, we can conclude that Exponential GARCH is the most appropriate model to forecast the financial markets volatility during the extreme volatile year of 2008. Thus, this empirical result can be very useful for those who need to forecast the volatility in financial markets during periods of high volatility.

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