Associate Professor Hassan SHAVANDI, PhD
Department of Industrial Engineering
Sharif University of Technology, Tehran, Iran
Assistant Professor Amir Abbas NAJAFI, PhD[1]
Department of Industrial Engineering
K.N. Toosi University of Technology, Tehran, Iran
E-mail: aa_najafi@yahoo.com
Alireza MOSLEHIRAD, M.Sc.
Department of Mathematics
Qazvin Islamic Azad University, Qazvin, Iran

FUZZY PROJECT SCHEDULING WITH DISCOUNTED CASH FLOWS

Abstract*. Project scheduling to maximize the net present value of the cash flows has been a topic of recent research. These researches assume that the duration of activities is known either with certitude or at least with some probability. However, in many applications the structure of the duration of activities is imprecise. In this paper, the project scheduling problem with discounted cash flows under fuzzy environment is considered. The duration of activities are assumed triangular fuzzy numbers. The derived mathematical model is a multi-objective non-linear programming model and for solving it the Taylor expansion is used to transform the objective function to a linear form. To interpret the fuzzy solution of the model, a procedure which can be used for other similar problems is proposed. Finally, an example is solved and the proposed procedure is applied to interpret the fuzzy solutions.*

Keywords*:* *Net Present Value; Scheduling; Fuzzy Sets; Fuzzy Duration.*

JEL Classification: **C44, E40**

1. Introduction

Project scheduling is a main objective of most models to aid planning and management of projects. Initially, the study of project scheduling has been done assuming deterministic activity durations, for example critical path method (CPM). The critical path method was proposed at the beginning of the 1960s (Kelley 1961). The CPM has become one of the tools that are most useful in practice and are applied in the planning

[1] Corresponding Author

of the projects. What is essential in the CPM is that the activities duration times are deterministic and known. In practice, this assumption not always can be fulfilled with satisfactory accuracy. In order to cope with such uncertainties, the duration of an activity is assumed to be a stochastic variable or a fuzzy number. The originators PERT (Project Evaluation and Review Technique) proposed a stochastic approach to cope with probabilistic activity durations. Malcolm et al. (1959) presented to use three estimates for activity duration. They modeled activity duration as a stochastic variable with an appropriate beta distribution and they proposed a simple approximate method to calculate the expectation and the variance of the project. Since the pioneering work of Zadeh (1965), authors have started to reject the stochastic approach and recommend the use of fuzzy models for the activity durations. Due to the uniqueness of some types of projects, historical data about activity durations are often not available. As a result, probability distributions for the activity durations are unknown. As activity duration is estimated by human experts, sometimes under unique circumstances, project management is often confronted with judgment statements that are vague and imprecise. In those situations, which involve imprecision rather than uncertainty, the advocates of fuzzy set approach reject the use of probability estimates and recommend the use of fuzzy numbers for modeling activity durations (Demeulemeester and Herroelen, 2002). Historically, first fuzzy scheduling procedures were concerned with the PERT model. The Fuzzy PERT was originated from Chanas and Kamburowski (1981). They presented the project completion time in the form of fuzzy sets in the time space. Based on the given possibility distributions of activity durations, the possibility distributions of the project completion time can be derived. Gazdik (1983) assumes that in a fuzzy network the duration of activities and some other input variables are imprecise and biased, and the imprecision is summarized to four classes. This work proposed a technique called FNET based on a combination of fuzzy sets and the theory of graphs. The membership function expressing the activity duration time for FNET depends on such diverse factors as expert opinions; the availability of means of production, materials, or staff; and personal experience. Nasution (1994) presented Fuzzy CPM, and showed that fuzzy numbers can be exploited further in the network. This was done first by considering simple interactive fuzzy subtraction in the backward calculation, and then by observing that if time were represented by fuzzy numbers. The study of a fuzzy model of resource-constrained project scheduling has been initiated in Hapke et al. (1994) and Hapke and Slowinski (1996). They have extended the priority rule based serial and parallel scheduling schemes to deal with fuzzy parameters. For a comprehensive survey of fuzzy project scheduling problems refer to Herroelen and Leus (2005).

Many of the recent researches in project scheduling focus on maximizing the Net Present Value (NPV) of the project using the sum of positive and negative discounted cash flows throughout the life cycle of the project. Russell (1970) introduced the problem of maximizing NPV in project scheduling. He proposed a successive approximation approach to solve the problem. Grinold (1972) added a project deadline

to the model, formulated the problem as a linear programming problem, and proposed a method to solve it. Doersch and Patterson (1977) presented a zero-one integer-programming model for the NPV problem. Their model included a constraint on capital expenditure of the activities in the project, while the available capital increased as progress payments were made. Russell (1986) considered the resource-constrained NPV maximization problem. He introduced priority rules for selecting activities for resource assignment based upon information derived from the optimal solution to the unconstrained problem. Smith-Daniels and Smith-Daniels (1987) extended the Doersch and Patterson Zero-one formulation to accommodate material management costs. Icmeli and Erengus (1996) introduced a branch and bound procedure to solve the resource constrained project scheduling problem with discounted cash flows. Najafi et al. (2009) and Shahsavar et al. (2010) extended the NPV maximization upon the resource investment. Najafi et al. (2010) developed the project scheduling problem with discounted cash flow under inflation environment and proposed two different situations due to type of contract between contractor and client.
To summarize, one can categorize the characteristics of the fuzzy project scheduling models in the reviewed researches as follows:

- The objective function is minimization of the project duration
- No payments made for the project during its life cycle
- They do not involve the concept of the time value of money throughout the life cycle of the project

Considering the fact that in real-world projects, the time-value-of-money of not only the project costs, but also the payments made for the project is very important for a project manager, In this research, a fuzzy project scheduling problem is considered in which the goal is to maximize the net present value of the project cash flows, the cash flows being the project costs and the payments made for the project during the life cycle of the project. This problem is called the Fuzzy Project Scheduling Problem with Discounted Cash Flows (FPSDCF).

2. Problem Formulation

An exact definition of the FPSDCF problem investigated in this paper is as follows: A project is given with a set of *n* activities indexed from *1* to *n*. Activities *1* and *n* are dummies that represent the start and completion of the project, respectively. Precedence relations of activities are shown by an activity on node network with no loops. Each activity *i* has a set of predecessor activities *p(i)*. Also, assume CF_i is the cash flow associated with activity *i* and it occurs at the finish of activity *i*. The activities are to be scheduled such that the make span of the project does not exceed a given due date (*DD*). Also, α is the discount rate.

In the classical models, duration of an activity is a crisp number, where the duration of activity *i* is denoted by d_i , but in the fuzzy model, $\tilde{d}_i$ is defined as the fuzzy duration of activity *i* with membership function $\mu_{\tilde{d}_i}(x)$ .The membership function is used to represent the degree of possibility that the activity duration is x, for all x belonging to a time scale. The values belonging to the core of $\tilde{d}_i$ are considered as the most possible values for $\tilde{d}_i$ and the values outside the support of $\tilde{d}_i$ are the least possible ones. The values outside the core and inside the support of $\tilde{d}_i$ are in between.

To formulate the problem, let us define $\tilde{S}_i$ as the fuzzy start of activity *i*. Now, the FPSDCF problem can be formulated as follows:

$$Max \ \ \tilde{Z} = \sum_{i=1}^{n} CF_i \ e^{-\alpha\,(\tilde{S}_i + \tilde{d}_i)} \tag{1}$$

Subject to

$$\tilde{S}_j + \tilde{d}_j \le \tilde{S}_i \qquad ;\forall \ j \in p(i) \qquad ;i = 1,2,\ldots,n \tag{2}$$

$$\tilde{S}_n \le DD \tag{3}$$

$$\tilde{S}_i \ge 0 \quad ;\, i = 1,2,\ldots,n \tag{4}$$

The objective function (1) maximizes the net present value of the project. Equation (2) enforces the precedent relations between activities. Constraint (3) ensures that the project ends by the latest allowable completion time. Finally, equation (4) denotes the domain of the variables.

In the literature of fuzzy project scheduling, the triangular and trapezoidal fuzzy numbers were used in the most researches to model the activity duration as a fuzzy number. In this article, according to the practical way of estimating the fuzzy activity duration as well as computational efficiency, the triangular fuzzy number is used.

3. Basic Definitions

In this section, some basic definitions are quoted which are used to formulate the problem (Zimmermann, 2001).

***Definition 1**:* Approximate numbers can be defined as a triangular fuzzy number, such as "approximate 5" that would normally be defined by a triangular fuzzy number as {3, 5, 7} where the membership degree of 5, is 1, while for 3 and 7 it is zero. The membership degrees for the other real numbers between 3 and 5, and between 5 and 7 are between zero and 1. In general, suppose $\widetilde{A}=(a^p, a^m, a^o)$ is a triangular fuzzy number that is defined as Fig. 1.

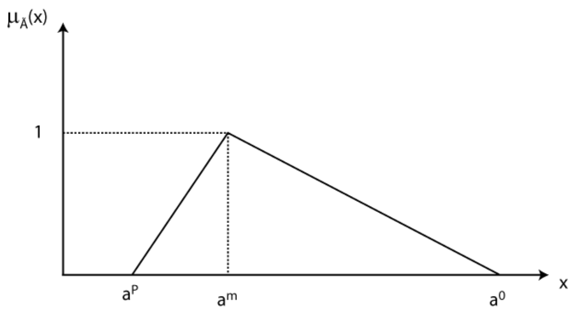


Figure 1: A triangular fuzzy number

Note that, the membership function of $\widetilde{A}$ has the following form:

$$\mu_{\tilde{A}}(x)=\begin{cases}1-\dfrac{a^m-x}{a^m-a^p} & , \quad a^p\le x\le a^m\\ 1-\dfrac{x-a^m}{a^o-a^m} & , \quad a^m\le x\le a^o\\ 0 & , \quad otherwise\end{cases} \tag{5}$$

Definition 2: Suppose $\widetilde{A}=(a^p, a^m, a^o)$ and $\widetilde{B}=(b^p, b^m, b^o)$ are triangular fuzzy numbers so the arithmetic operations on them can be shown as:

$$\widetilde{A}+\widetilde{B}=(a^p+b^p, a^m+b^m, a^o+b^o) \tag{6}$$

$$\tilde{A} - \tilde{B} = (a^p - b^o \ , a^m - b^m \ , a^o - b^p) \tag{7}$$

$$\tilde{A} * \tilde{B} = (a^p * b^p \ , a^m * b^m \ , a^o * b^o) \tag{8}$$

$$\tilde{A} / \tilde{B} = (a^p / b^o \ , a^m / b^m \ , a^o / b^p) \tag{9}$$

$$\exp(\tilde{A}) \approx \left(\exp(a^p), \exp(a^m), \exp(a^o)\right) \tag{10}$$

Definition 3: The maximum of two fuzzy numbers is defined as (11) and for two triangular fuzzy numbers it can be estimated as (12).

$$Max(\tilde{A}, \tilde{B})(z) = SUP_{z=Max(x,y)} Min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\} \tag{11}$$

$$Max(\tilde{A}, \tilde{B}) \approx \left(Max(a^p, b^p), Max(a^m, b^m), Max(a^o, b^o)\right) \tag{12}$$

4. **A Solution Procedure**

In order to solve the derived model in previous sections, it is converted to the crisp and linear model. To do this, $\tilde{s}_i = (s_i^p \ , s_i^m \ , s_i^o)$ and $\tilde{d}_i = (d_i^p \ , d_i^m \ , d_i^o)$ are considered as triangular fuzzy numbers. The objective function (1) can be transformed to the crisp form by using definitions 2 and 3. If all of the activity cash flows are equal or greater than zero ($CF_i \geq 0, \ \forall i$), then the objective function is transformed to the following form.

$$\text{Max } \tilde{Z} = \left(\sum_{i=1}^{n} (CF_i \ e^{-\alpha(s_i^p + d_i^p)}), \sum_{i=1}^{n} CF_i \ e^{-\alpha(s_i^m + d_i^m)}, \sum_{i=1}^{n} (CF_i \ e^{-\alpha(s_i^o + d_i^o)}) \right)$$

(13)

Since in the real projects, the activity cash flow may be positive (incomes) or negative (costs), therefore CF_i^+ and CF_i^-, are defined by the following assumption:

- If $CF_i \geq 0$ then $CF_i^+ = CF_i$ and $CF_i^- = 0$
- If $CF_i < 0$ then $CF_i^- = CF_i$ and $CF_i^+ = 0$

Therefore the objective function (13), can be transformed as:

$$\text{Max } \tilde{Z} = (Z^{p}, Z^{m}, Z^{o}) \tag{14}$$

where:

$$\begin{aligned}
Z^{p} &= \sum_{i=1}^{n} \left(CF_i^{+}\, e^{-\alpha(s_i^{o}+d_i^{o})} + CF_i^{-}\, e^{-\alpha(s_i^{p}+d_i^{p})}\right) \\
Z^{m} &= \sum_{i=1}^{n} \left(CF_i^{+} + CF_i^{-}\right) e^{-\alpha(s_i^{m}+d_i^{m})} \\
Z^{o} &= \sum_{i=1}^{n} \left(CF_i^{+}\, e^{-\alpha(s_i^{p}+d_i^{p})} + CF_i^{-}\, e^{-\alpha(s_i^{o}+d_i^{o})}\right)
\end{aligned} \tag{15}$$

By applying the Bellman and Zadeh method (Zimmermann 2001), optimizing the fuzzy triangular form of objective function (14) can be transformed to a multi-objective optimization problem as:

$$\begin{aligned}
&\text{Max } Z_1 = \sum_{i=1}^{n} \left(CF_i^{+} + CF_i^{-}\right) e^{-\alpha(s_i^{m}+d_i^{m})} \\
&\text{Max } Z_2 = \sum_{i=1}^{n} \left(CF_i^{+} e^{-\alpha(s_i^{p}+d_i^{p})} + CF_i^{-} e^{-\alpha(s_i^{o}+d_i^{o})}\right) - \sum_{i=1}^{n} \left(CF_i^{+} + CF_i^{-}\right) e^{-\alpha(s_i^{m}+d_i^{m})} \\
&\text{Min } Z_3 = \sum_{i=1}^{n} \left(CF_i^{+} + CF_i^{-}\right) e^{-\alpha(s_i^{m}+d_i^{m})} - \sum_{i=1}^{n} \left(CF_i^{+}\, e^{-\alpha(s_i^{o}+d_i^{o})} + CF_i^{-}\, e^{-\alpha(s_i^{p}+d_i^{p})}\right)
\end{aligned} \tag{16}$$

The derived model is a multi-objective non-linear programming model, but it can be solved iteratively by successive approximations using Taylor expansion approximation. The Taylor expansion transforms a function f around a point (solution) x_0 as:

$$f(x) = f(x_0) + \frac{f^{(1)}(x_0)}{1!}(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \tag{17}$$

The solving method starts with an initial point x_0 and then the optimal solution derived by approximated function is replaced by x_0 and this process is iterated until the difference between the objective function values reaches to a threshold (e.g. 0.01). By this process, the optimal or a very close to optimal solution is determined. Russell (1970) proved that the process converges in a finite number of iterations. If the first two terms of the Taylor expansion is considered, the objective functions are transformed to linear forms and can be maximized very easily by classic methods.

To apply this manner, a feasible solution is needed that it is assumed as $\tilde{S}^0 = (\tilde{S}'_1, \tilde{S}'_2, ..., \tilde{S}'_n)$, which can be obtained by using Fuzzy CPM method (Nasution 1994). Therefore, the objective functions (14) are transformed as follows:

$$\text{Max } Z_1 = \sum_{i=1}^{n} (F_i^{(1)} + \quad s_i'^m \ \alpha \, F_i^{(1)} - s_i'^m \alpha \, F_i^{(1)})$$

$$\begin{aligned} \text{Max } Z_2 = & \sum_{i=1}^{n} (F_i^{(2)} + F_i^{(3)} - F_i^{(1)}) + \sum_{i=1}^{n} (s_i'^p \ \alpha \, F_i^{(2)} + s_i'^o \ \alpha \, F_i^{(3)} - s_i'^m \ \alpha \, F_i^{(1)}) \\ & + \sum_{i=1}^{n} (s_i^m \ \alpha \, F_i^{(1)} - s_i^o \ \alpha \, F_i^{(3)} - s_i^p \ \alpha \, F_i^{(2)}) \end{aligned} \tag{18}$$

$$\begin{aligned} \text{Min } Z_3 = & \sum_{i=1}^{n} (F_i^{(4)} + F_i^{(5)} - F_i^{(1)}) + \sum_{i=1}^{n} (s_i'^o \ \alpha \, F_i^{(4)} + s_i'^p \ \alpha \, F_i^{(5)} - s_i'^m \ \alpha \, F_i^{(1)}) \\ & + \sum_{i=1}^{n} (s_i^m \ \alpha \, F_i^{(1)} - s_i^p \ \alpha \, F_i^{(5)} - s_i^o \ \alpha \, F_i^{(4)}) \end{aligned}$$

where:

$$\begin{aligned} F_i^{(1)} &= (CF_i^+ + CF_i^-)\, e^{-\alpha(s_i'^m + d_i^m)} \\ F_i^{(2)} &= CF_i^+ \, e^{-\alpha(s_i'^p + d_i^p)} \\ F_i^{(3)} &= CF_i^- \, e^{-\alpha(s_i'^o + d_i^o)} \\ F_i^{(4)} &= CF_i^+ \, e^{-\alpha(s_i'^o + d_i^o)} \\ F_i^{(5)} &= CF_i^- \, e^{-\alpha(s_i'^p + d_i^p)} \end{aligned} \tag{19}$$

In addition, the constraints (2) and (3) are transformed to crisp equations as follows respectively (Zimmermann 2001):

$$\begin{cases} s_j^p + d_j^p \le s_i^p \\ s_j^m + d_j^m \le s_i^m \\ s_j^o + d_j^o \le s_i^o \end{cases} \quad ;\forall\ j \in p(i) \quad ;i = 1,2,\ldots,n \tag{20}$$

$$\begin{cases} s_n^p \le DD \\ s_n^m \le DD \\ s_n^o \le DD \end{cases} \tag{21}$$

To ensure the $\widetilde{S}_i$ be a triangular fuzzy number it is needed to add the following constraint to the model.

$$s_i^p \le s_i^m \le s_i^o \quad ;i = 1,2,\ldots,n \tag{22}$$

And finally for the non-negative constraint (4), it is converted to:

$$\begin{cases} s_i^p \ge 0 \\ s_i^m \ge 0 \\ s_i^o \ge 0 \end{cases} \quad ;i = 1,2,\ldots,n \tag{23}$$

Now, the problem formulation can be simplified in the following form:

Optimize Equations (18)

Subject to Equations (20), (21), (22) and (23).

The final model is a linear multi-objective programming model and can be solved by using related methods (Klir and Yoan 1995). After solving the model, the optimal solution is obtained in a fuzzy manner. The earliest start times of activities are triangular fuzzy numbers and it can give opportunities to project manager to plan and control the schedule based on possibility interval for activities. For each possible schedule based on optimal fuzzy schedule, there is a fuzzy NPV as utility function. In real world the meaning of the fuzzy net present value is interesting? In the real world, there is usually a threshold value for the net present value and the project stockholders would be satisfied if the NPV be equal or more than the threshold value. Therefore in fuzzy manner, the possibility of stakeholder's satisfaction (PSS) can be estimated as reach a schedule with NPV which is equal or more than threshold value.

Here a procedure is proposed to analyze the possibility of suitable NPV based on fuzzy scheduling and fuzzy NPV.

Let the fuzzy NPV as $FNPV = (NPV^p, NPV^m, NPV^o)$ be the triangular fuzzy number obtained from solving the model. Considering the threshold NPV (TNPV), three cases may happen:

(a) The TNPV is less than NPV^p

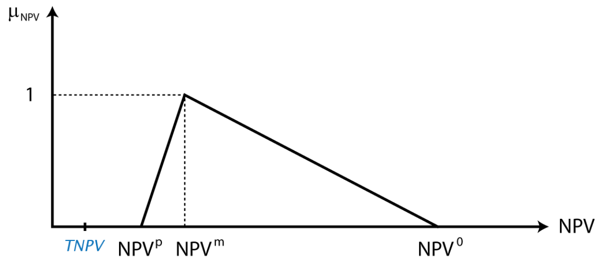
(b) The TNPV is between NPV^p and NPV^o

(c) The TNPV is more than NPV^o

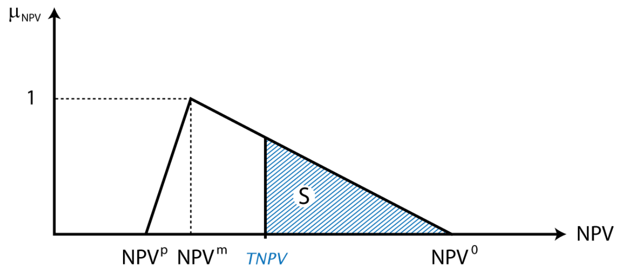
In case (a), the project stakeholders will reach to their threshold target with highest possibility by applying the fuzzy schedule determined by model.

In case (b), according to Fig. 2, the possibility of satisfying the stakeholders threshold target by applying the fuzzy schedule is a number between zero and one and it is the ratio of area *"S"* to total area of triangular fuzzy number.

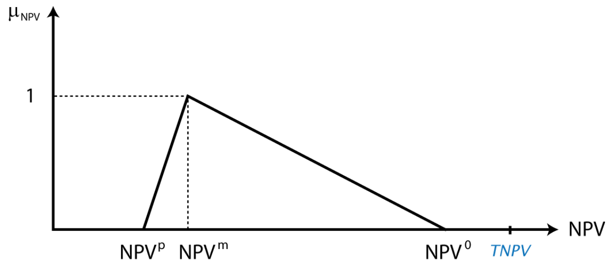
In case (c), the threshold target of stakeholders would not be satisfied by applying the fuzzy schedule determined by model.



(a)



(b)



(c)

Figure 2: Analysis of Fuzzy NPV compared to threshold NPV

5. A Numerical Example

In order to illustrate the proposed method, consider a project network with eight activities. Figure 3 shows the activity-on-node representation of the network with the node numbers denoting the activity numbers. Activities 1 and 8 are defined as dummies.

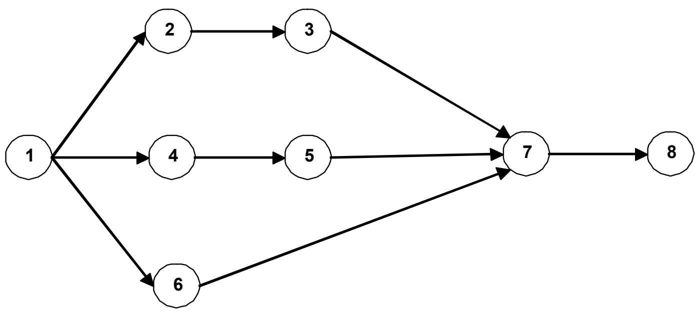


Figure 3: The Project Network of the Example Problem

Table 1 presents the fuzzy durations and the cash flow of the activities. The project deadline is 35 and the discount rate is taken to be 0.01 per period.

Table 1: Activity Data of the Example Problem

Activity(i)	Duration $(\tilde{d}_i)$	Cash Flow (CF_i)
1	(0,0,0)	0

2	(5,8,10)	+20
3	(3,5,8)	+100
4	(8,10,12)	-10
5	(3,4,5)	+50
6	(6,8,10)	-10
7	(1,2,3)	-80
8	(0,0,0)	0

According to the solution procedure, the problem shall be formulated and solved. The final solution which is in fuzzy format is presented in Table 2.

Table 2: Solution of the example problem

Activity(i)	Start Time $(\widetilde{S}_i)$
1	(0,0,0)
2	(0,0,10)
3	(5,15,15)
4	(0,15,15)
5	(8,25,27)
6	(21,21,22)
7	(29,29,32)
8	(30,31,35)

For this solution the value of the fuzzy net present value ,FNPV, is (42.8,67,4,96.3).

If the threshold NPV for project stakeholders defined as 70, the possibility of stakeholder's satisfaction (PSS), can be calculated by applying the fuzzy schedule. According to case (b) in Fig. 2, we will have:

Total triangle area = 13.44

S = 5.29

PSS = 5.29 / 13.44 = 0.39

So by applying the derived fuzzy schedule the stakeholders will be satisfied with possibility of 0.39. It means the derived NPV due to applying the fuzzy schedule will be equal or more than threshold NPV with the possibility of 0.39.

6. Conclusions

In this paper, the project scheduling problem was developed with discounted cash flow under fuzzy environment. The problem was formulated mathematically and shown to be a multi-objective non-linear programming model. To solve the model, the Taylor expansion was used to transform the objective function to a linear form. Then, the problem was converted to a crisp linear programming model. In order to interpret the solution of the final model, a new approach was proposed. Some extensions of this research might be of interest. Since in this paper, the "zero-lag finish-to-start precedent constraints" is considered, some other precedence relation such as generalized precedence may be considered in the project. The other extension of this research would be to investigate a resource constrained project scheduling problem under fuzzy environment in which the goal is to maximize the NPV of the project.

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