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# A GENETIC FUZZY EXPERT SYSTEM TO OPTIMIZE PROFIT FUNCTION

***Abstract*** *In this paper we develop a fuzzy system to obtain the demand based on input variables of price, warranty length and lead time of order delivery. We use linguistic terms to illustrate the situation of input and output variables and construct a fuzzy rule base. The fuzzy rule base is integrated with a genetic algorithm to construct a hybrid solution method. The best value of price, warranty length and lead time are obtained by maximizing the manufacturer profit. Finally a numerical example is presented to show the performance of proposed approach.*

***Keyword*** *fuzzy expert system; genetic algorithm; warranty; pricing; lead time; profit optimization.*

**JEL Classification: C630**

## 1. Introduction

In the competitive markets that customers have authority and analysis power to select products. Therefore manufacturers have to persuade customers to purchase their products. Hence identifying and analyzing the important parameters that effect on demand is vital for companies. The more effective and important factors on customer's demand are: price, lead time, quality level, advertising, discount of price, and after sales service or warranty. One of the other challenges of a manufacturer is demand forecasting based on effective parameters. In the real world, finding a mathematical function for demand that considers these parameters is very difficult and complex. Several studies assume a deterministic demand function that depends to one or two of parameters (Guowei et.al, 2010; M.esmaeili et.al, 2009). The other studies consider a stochastic demand function for demand and maximize the total profit (Yingxue Zhao et.al, 2010; G.Huang, L.Liu, 2006). Considering a mathematical function for demand is not exact due to uncertainty in market. In the uncertain environment, expert's judgments are vital in order to decision making and optimization and usually their judgments are in linguistic terms form. Fuzzy expert system can be used to formulate the human knowledge and analyze the imprecise data.

Gulcin Buyukozkan (2002), considered the fuzzy logic in decision making of new product development to reduce risks for decision makers. Hassan Shavandi and Parisa alizadeh (2010) developed a hybrid forecasting model which combines artificial intelligence and fuzzy inference system to predict short-term stock price index. David Peidro et.al (2009), considered a supply chain which supply, demand and process are uncertain and formulated the model as a fuzzy mixed-integer linear programming. Chaug–Ing Hsu and Yuh–Horng Wen (2005), determined the frequency of air line flight to maximize the profit with consider the competitive interaction using fuzzy logic. Alireza Haji and Morteza Asadi (2007) estimated the price of new product with using expert knowledge and fuzzy logic method. Tomasz Korol and Adrian Korodi (2010) use the fuzzy logic to analyze the usage in predicting bankruptcy for one year, two and three years before the failure of companies. Jing-Shing Yao, Teng-San Shing (2002), investigated revenue maximizing problem and optimum price in fuzzy environment with non linear function of revenue. Yu-Jie Wang and Chao-Shun Kao (2008), investigate overbooking which is a revenue management policy for air industries. Bin Li (2011), studied inventory control problem under uncertain demand with considering a belief-rule-base method and using manager's knowledge and operational data. Resad Nuhodzic et.al (2010), presented a model for choosing adequate marketing campaigns of a rail company by using the Fuzzy logic Hierarchy Process and multicriteria decision making. R.A. Aliev et.al (2007), considered aggregate production-distribution planning in supply chain with uncertain market and uncertain environment for manufacturer. They developed a fuzzy model using fuzzy function for demand and capacity.

In this paper, we consider the price, warranty length and lead time as input parameters and demand as an output variable in a fuzzy expert system. We estimate the demand by developed fuzzy expert system and integrating it with a genetic algorithm to optimize the variables of price, warranty length, lead time for product.

The structure of this paper is as follows: Section 2 presents the designed fuzzy expert system to estimate the amount of demand based on decision variables. In section 3, we formulate the profit function and describe its various cost function. Section 4, presents the genetic algorithm and hybrid solution method with fuzzy expert system to maximize the profit. The numerical experiments and results analysis are presented in section 5. The final section is discussion and conclusion.

## 2. Fuzzy expert system for demand

In many cases, decision making and optimization are in uncertain environment and a mathematical deterministic equation does not exist. In this situation, human knowledge and experiment have been utilized to decision making and optimization.

Generally, human knowledge can be categorized in two groups: one is unconscious knowledge and the other one, conscious knowledge. In unconscious knowledge, experts cannot describe their knowledge as linguistic terms but in conscious knowledge, experts can explain their knowledge as linguistic terms. Zadeh (1965, 1996, 1999), first introduced the concept of fuzzy logic "computing with words". Fuzzy logic can be used to formulate the human knowledge. A fuzzy expert system contains both expert knowledge and fuzzy inference that we describe it as follow:

### 2.1. Definition

Consider the some concept of fuzzy sets (zadeh 1965) as following

**Definition 2.1**: Let U be a universe set. A fuzzy set X of U is defined by a membership function $\mu_X(x) \in [0\ 1]$, where $\mu_X(x), \forall x \in U$ indicates the degree of x in X.

**Definition 2.2**: Let X be a fuzzy set of U, where U is a real line. X is normal, if and only if $sup_{x \in U}\, \mu_X(x) = 1$.

**Definition 2.3**: Let X be a fuzzy set of U, where U is a real line. X is convex, if and only $\mu_X(\lambda x + (1-\lambda)y \geq (\mu_x(X)\, ^\wedge \mu_y(X)), \forall x, y \in U, \forall \lambda \in [0\ 1]$ where the symbol ^ denotes the minimum operator.

**Definition 2.4**: A fuzzy number X is a fuzzy set that is both normal and convex in the universe U.

**Definition 2.5**: Let U be a universe set. Fuzzy sets of $A^1, A^2, \ldots, A^N$ in the U are completeness of fuzzy sets if for each $x \in U$, there is at least one $A^j$ as $\mu_{A^j}(x) > 0$.

**Definition 2.6**: A triangular fuzzy number X can be defined by (a, b, c) as shown in Fig1. The membership function $\mu_X(x)$ is presented as below :( Fig 1)

$$\mu_X(x) = \begin{cases} (x-a)/(b-a) & ; \quad a \leq x < b \\ 1 & ; \quad x = b2 \\ (c-x)/(c-b) & ; \quad b < x \leq c \end{cases} \qquad (1)$$

**Definition 2.7**: A trapezoidal fuzzy number Y can be defined by (a, b, c, d) as shown in Fig. 2. The membership function $\mu_X(x)$ is presented as following: (Fig 2)

$$\mu_X(x) = \begin{cases} (x-a)/(b-a) & ; \quad a \leq x < b \\ 1 & ; \quad b \leq x < c \\ (d-x)/(d-c) & ; \quad c \leq x \leq d \end{cases} \qquad (2)$$

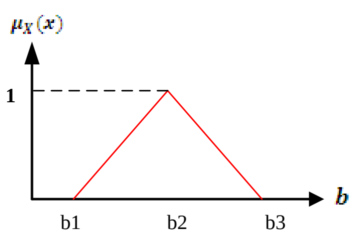


**Fig. 1. Triangular fuzzy number**

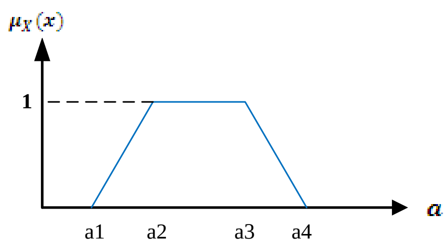


**Fig. 2. Trapezoidal fuzzy number**

## 2.2. Identify input and output variables

In this article, we design a fuzzy system to estimate the amount of demand based on price, warranty length and lead time. Thus, input variables for fuzzy system are price, warranty length and lead time and also, output variable is the amount of demand. Demand of product is decreasing in price and lead time but increasing in warranty length (See figures 3 and 4).

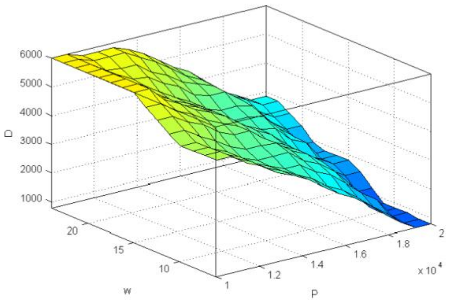


**Fig. 3. Relation plot of demand with price and lead**

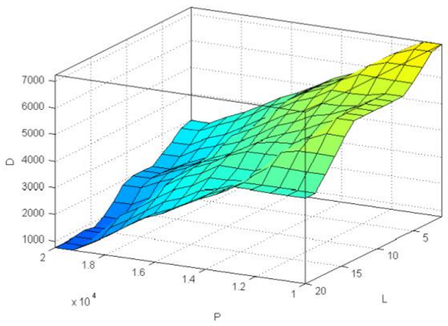


**Fig. 4. Relation plot of demand with price and warranty length**

## 2.3. Fuzzification interface

The fuzzifier has been utilized for convert a point of input variable into a fuzzy set in universe set. In this paper, we use the singleton fuzzification as a conversion of crisp value $x^* \in U$ into a fuzzy set A with the Membership function in relation (3) and presented by figure 5 ( Kwang H. Lee, 2005).

$$\mu_A(X) = \begin{cases} 1 & : \quad X = X^* \\ 0 & : \quad \text{other wise} \end{cases} \tag{3}$$

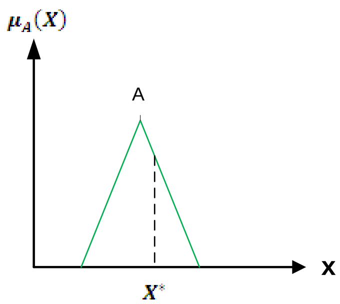


**Fig 5. Fuzzification function for singleton fuzzifier**

### 2.4. Knowledge base

#### 2.4.1. Fuzzy partition of input and output space

The linguistic variable that we use in this article is described in table 1.

**Table 1: Definition of linguistic variable and its fuzzy set**

| Linguistic term | very low | Fairly low | Low | Medium | High | Fairly High | Very high |
|---|---|---|---|---|---|---|---|
| abbreviation | VL | FL | L | M | H | FH | VH |
| Fuzzy set | Trapezoidal | Triangular | | | | | Trapezoidal |

**Difinition2.8** Assume the universe set P for price input variable and its fuzzy sets ($X^j$) are normal. The fuzzy sets of price contain very low, low, medium, high and very high linguistic terms. (Fig 6).

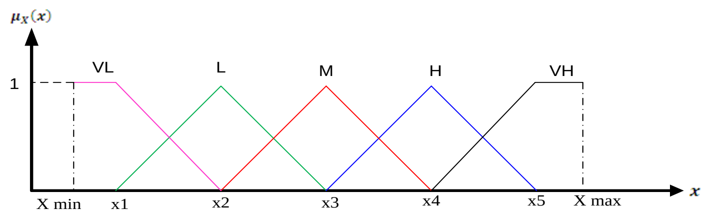


**Fig. 6. Price Membership Function**

where $x_{min}$ and $x_{max}$ in figure 6 are minimum and maximum values of price. Membership degrees for each fuzzy set are determined by statement 1 and 2.

**Difinition2.9:** Assume that universe set W is for warranty length input variable and it's fuzzy sets ($Y^j$) are normal. The fuzzy sets of warranty length contain very low, low, medium, high and very high linguistic terms. (Fig. 7)

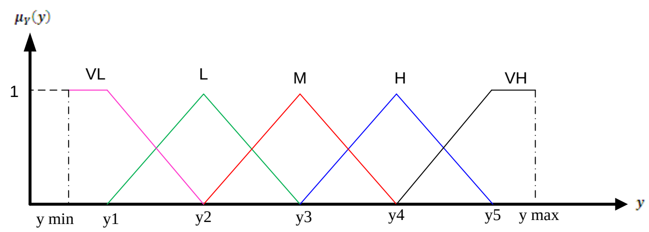


**Fig. 7. Warranty length Membership Function**

where $y_{min}$ and $y_{max}$ in figure7 are minimum and maximum lengths of product warranty. Membership Degrees are determined by statement 1 and 2.

**Difinition2.10:** Assume the universe set L for lead time input variable and it's fuzzy sets ($Z^j$) are normal. The fuzzy sets of lead time contain very low, low, medium, high and very high linguistic terms. (Fig. 8).

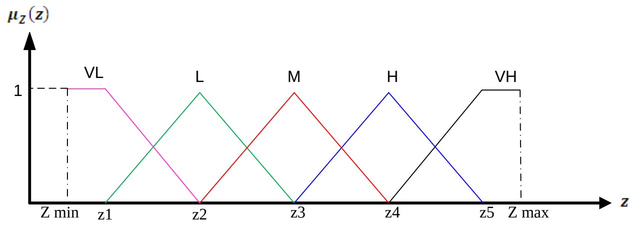


**Fig. 8. Lead time Membership Function**

where $Z_{min}$ and $Z_{max}$ in figure8 are minimum and maximum values of lead time for product. Membership degrees are determined by statement 1 and 2.

**Difinition2.11:** Assume the universe set D for demand output variable and its fuzzy sets ($U^j$) are normal. The fuzzy sets of demand contain very low, fairly low, low, medium, high, fairly high and very high linguistic terms. (Fig. 9).

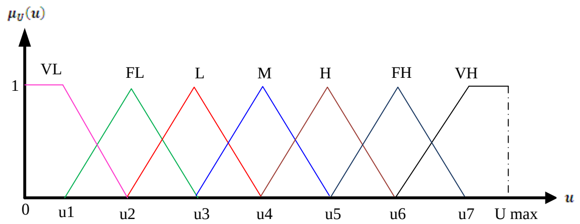


**Fig. 9. Demand Membership Function**

Where $u_{min}$ and $u_{max}$ in figure 9 are minimum and maximum values for demand. Membership degrees are determined by statement 1 and 2.

### 2.4.2. Fuzzy rule base

The rule is interpreted as an "implication" and consists of the "antecedent" (if part) and "consequent" (then part). The general form of fuzzy rule is given in the following:

If $x$ is A, Then $y$ is B (4)

where A and B are linguistic term defined by fuzzy sets on universe of discourse $X$ and$Y$, respectively.

In our paper price, warranty length and lead time are in "if part" of rules and demand is in "then part". Thus, we have "multiple input single output" rule that is named MISO.

A fuzzy rule base consists of a set of fuzzy "if-then" rules that are collected in forms of linguistic terms by expert's opinions. A sample of fuzzy rule is indicated in statement (5):

If $x \in p$ is $M$ and $y \in w$ is $L$ and $z \in L$ is $H$ Then $D$ is $L$ (5)

Statement (5) represents that:

If price is very low and warranty length is medium and lead time is very high then demand is Low.

Since the price, warranty length and lead time parameters consist of five linguistic variables, then we have $5 \times 5 \times 5 = 125$ "If – Then" rules for designing fuzzy rule base. In Table 2, the 125 fuzzy rules are constructed to determine the demand. These fuzzy rules show the expert's opinions for estimating the demand under different conditions.

**Table 2: Fuzzy rule base from the experts' opinions**

| rule number | IF part | | | Then part | rule number | IF part | | | Then part |
|---|---|---|---|---|---|---|---|---|---|
| | P | w | L | D | | P | w | L | D |
| 1 | VL | VL | VL | H | 66 | M | VL | H | L |
| 2 | VL | L | VL | FH | 67 | M | L | H | L |
| 3 | VL | M | VL | VH | 68 | M | M | H | M |
| 4 | VL | H | VL | VH | 69 | M | H | H | M |
| 5 | VL | VH | VL | VH | 70 | M | VH | H | H |
| 6 | VL | VL | L | H | 71 | M | VL | VH | FL |
| 7 | VL | L | L | H | 72 | M | L | VH | L |
| 8 | VL | M | L | FH | 73 | M | M | VH | L |
| 9 | VL | H | L | FH | 74 | M | H | VH | M |
| 10 | VL | VH | L | VH | 75 | M | VH | VH | M |
| 11 | VL | VL | M | H | 76 | H | VL | VL | L |
| 12 | VL | L | M | H | 77 | H | L | VL | L |
| 13 | VL | M | M | FH | 78 | H | M | VL | M |
| 14 | VL | H | M | FH | 79 | H | H | VL | M |
| 15 | VL | VH | M | FH | 80 | H | VH | VL | H |
| 16 | VL | VL | H | M | 81 | H | VL | L | FL |
| 17 | VL | L | H | H | 82 | H | L | L | L |
| 18 | VL | M | H | FH | 83 | H | M | L | L |
| 19 | VL | H | H | FH | 84 | H | H | L | M |
| 20 | VL | VH | H | FH | 85 | H | VH | L | H |
| 21 | VL | VL | VH | L | 86 | H | VL | M | FL |
| 22 | VL | L | VH | L | 87 | H | L | M | L |
| 23 | VL | M | VH | M | 88 | H | M | M | L |
| 24 | VL | H | VH | H | 89 | H | H | M | M |
| 25 | VL | VH | VH | H | 90 | H | VH | M | M |
| 26 | L | VL | VL | H | 91 | H | VL | H | FL |
| 27 | L | L | VL | H | 92 | H | L | H | FL |
| 28 | L | M | VL | FH | 93 | H | M | H | L |
| 29 | L | H | VL | FH | 94 | H | H | H | L |
| 30 | L | VH | VL | VH | 95 | H | VH | H | M |
| 31 | L | VL | L | M | 96 | H | VL | VH | VL |
| 32 | L | L | L | H | 97 | H | L | VH | FL |

| rule number | IF part | | | Then part | rule number | IF part | | | Then part |
|---|---|---|---|---|---|---|---|---|---|
| | P | w | L | D | | P | w | L | D |
| 33 | L | M | L | FH | 98 | H | M | VH | FL |
| 34 | L | H | L | FH | 99 | H | H | VH | L |
| 35 | L | VH | L | FH | 100 | H | VH | VH | L |
| 36 | L | VL | M | M | 101 | VH | VL | VL | FL |
| 37 | L | L | M | M | 102 | VH | L | VL | FL |
| 38 | L | M | M | H | 103 | VH | M | VL | L |
| 39 | L | H | M | H | 104 | VH | H | VL | L |
| 40 | L | VH | M | FH | 105 | VH | VH | VL | L |
| 41 | L | VL | H | L | 106 | VH | VL | L | VL |
| 42 | L | L | H | M | 107 | VH | L | L | FL |
| 43 | L | M | H | M | 108 | VH | M | L | FL |
| 44 | L | H | H | H | 109 | VH | H | L | L |
| 45 | L | VH | H | H | 110 | VH | VH | L | L |
| 46 | L | VL | VH | L | 111 | VH | VL | M | VL |
| 47 | L | L | VH | L | 112 | VH | L | M | VL |
| 48 | L | M | VH | M | 113 | VH | M | M | FL |
| 49 | L | H | VH | M | 114 | VH | H | M | FL |
| 50 | L | VH | VH | H | 115 | VH | VH | M | L |
| 51 | M | VL | VL | M | 116 | VH | VL | H | VL |
| 52 | M | L | VL | M | 117 | VH | L | H | VL |
| 53 | M | M | VL | H | 118 | VH | M | H | VL |
| 54 | M | H | VL | H | 119 | VH | H | H | FL |
| 55 | M | VH | VL | FH | 120 | VH | VH | H | FL |
| 56 | M | VL | L | M | 121 | VH | VL | VH | VL |
| 57 | M | L | L | M | 122 | VH | L | VH | VL |
| 58 | M | M | L | H | 123 | VH | M | VH | VL |
| 59 | M | H | L | H | 124 | VH | H | VH | VL |
| 60 | M | VH | L | H | 125 | VH | VH | VH | FL |
| 61 | M | VL | M | L | | | | | |
| 62 | M | L | M | M | | | | | |
| 63 | M | M | M | M | | | | | |
| 64 | M | H | M | M | | | | | |
| 65 | M | VH | M | H | | | | | |

### 2.5. Fuzzy inference system

Fuzzy inference uses the fuzzy logic to composite if-then rules of rule base and to convert the input variables into the output variable. The famous fuzzy inference that used in literature is Mamdani and TSK method that each of them has applied in special fields (Klir & Yuan, 1995; Zimmermann, 1996). In this paper we used the Mamdani method as fuzzy inference and explain it in following subsections.

#### 2.5.1. Convert fuzzy singleton into fuzzy sets

Assume that $x_0 \in P$ , $y_0 \in W$ $and$ $z_0 \in L$ are fuzzy singleton and crisp values for price, warranty length and lead time respectively. First, we convert singleton fuzzifier of $x_0$ ,$y_0$ $and$ $z_0$ into the fuzzy sets of price, warranty length and lead time (figure1). Then, we considered fuzzy sets of input variable (price, warranty length and lead time) that include input singleton fuzzifier. Next, we establish possible rules of rule base from the composition of these fuzzy sets. We named this rules as active rules. For example, if two fuzzy sets of P include $x_0$and if two fuzzy sets of W include $y_0$ and if one fuzzy set of L includes $z_0$, Then we will have $2 \times 2 \times 1 = 4$ active rule. Therefore, $\mu_X(x_0), \mu_Y(y_0) and\ \mu_Z(z_0)$ are calculated using equations (1) and (2) for each active rule and named them $h_1, h_2\ and\ h_3$ respectively.

### 2.5.2. Fuzzy implication and composition

In real system, the inference is determined by two factors: the "implication operator" and "composition operator". For the implication and composition, two operators Mamdani and Larson are proposed in the literature (Kwang H. Lee, 2005).

In this paper we used the Mamdani method as implication and composition. After calculating $h_1$, $h_2$, $and\ h_3$ for each active rule the matching degree ($\alpha_i$) is obtained by equation 5.

$$\alpha_i = min(h_1, h_2, h_3) \qquad i = 1,2, \dots, M \tag{5}$$

Then, utilize max-Min operator between $\alpha_i$ and output fuzzy sets of demand and through this, obtained output fuzzy sets for each active rule and name it, $u_i^{'}$ (Fig. 10).

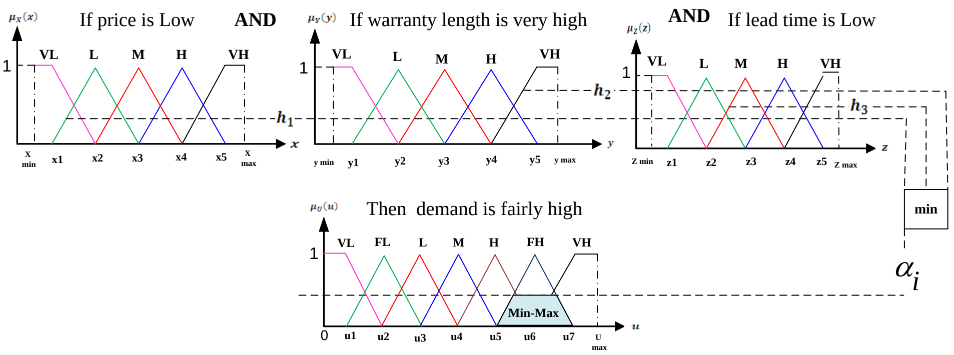


**Fig.10. Mamdani implication for a rule**

### 2.5.3. Aggregation result of active rules

Next step of fuzzy inference is to aggregate all output fuzzy set from active rules into a final output set. The aggregation is done by maximum operator as relation 6.

$$u^{'} = \cup_{i=1}^{M} u_i^{'} \qquad i = 1,2, \dots, M \tag{6}$$

Aggregation of individual rules to total result of inference is show in Fig11.

### 2.6. Defuzzification interface

In the real world, the crisp value of fuzzy system's output is required to make better decision. To do so, the defuzzifier interface has been utilized to convert fuzzy set into the crisp value. Besides, the well known defuzzifiers are as follows: (I) Mean of Maximum Method (MOM), (II) Center of Area Method (COA) (III) Bisector of Area (BOA), (Kwang H. Lee, 2005). We use the COA method in this paper. If $\bar{u}^L$ be the center of fuzzy set of the Lth rule's output, thus COA works as equation 7.

$$u_0 = \frac{\sum_{L=1}^{M} \bar{u}^L \alpha_L}{\sum_{L=1}^{M} \alpha_L} \tag{7}$$

where $u_0$ is the amount of demand and it used for optimizing profit.

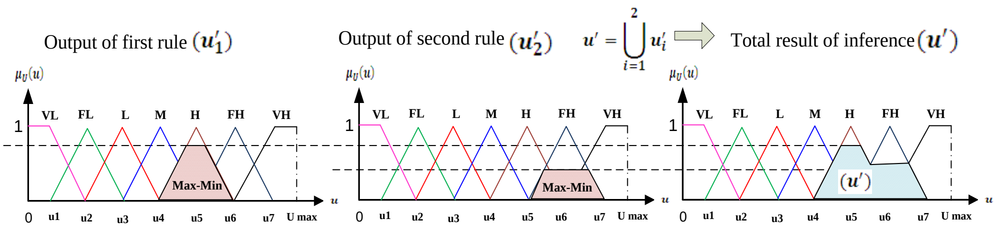


**Fig. 11. A sample aggregation of result with 2 rules**

## 3. Profit function

The input parameters of profit function are production cost ($c_m$), maximum cost of lead time ($c_l$), repair cost per unit failure ($c_w$), minimum cost of quality level ($c_0$) and variable cost of quality level per unit ($c_q$). Decision variables in profit function are price (P), warranty length (W), lead time (L) and quality level (Q) and are formulate as equation 8.

$$\pi(p, w, L, Q) = (p - c_m - f(w) - f(Q) - f(l)) \times D \tag{8}$$

Profit function contains both demand value and cost functions that as mentioned in section 2. Demand value inferences from fuzzy expert system. Cost functions are described in following.

### 3.1. Warranty and quality costs

Manufacturer adopts free repair warranty policy when the product is failed before end of warranty length (w). If w is low, consumers dissuade from purchase and if w is high, the profit per unit of product will be decreased and thus the optimal point of warranty length should be determined.

Assume that failure times are independently and cumulative distribution function F (w) as Eq.9 (Chin-Chun Wu et.al, 2009).

$$F(w) = 1 - e^{-(\lambda w)^{\beta}} \tag{9}$$

where, $\lambda$ is failure rate and β>1 is shape parameter. Expected failure number during warranty length, $M(w)$, is calculated by Eq.10.

$$M(w) = (\lambda w)^{\beta} \tag{10}$$

Therefore, the cost of warranty is determined by Eq.11.

$$f(w) = c_w \times M(w) \tag{11}$$

Also, failure rate $(\lambda)$ depends on quality level (Q) as when quality level increases, failure rate decreases and vice versa as Eq.12.

$$\lambda = \frac{K}{Q} \tag{12}$$

Where, K is minimum failure rate in which the quality level is maximum ($Q = 1$), and $0 \leq Q \leq 1$. The cost of quality is formulated as Eq.13.

$$f(Q) = c_0 + c_q Q \tag{13}$$

### 3.2. Lead time cost

Lead time (LT) value is one of the parameters that influence on demand. Demand is decreasing in lead time. The lead time cost is increasing in lead time and is calculated by Eq.14.

$$f(l) = \frac{c_l}{L} \tag{14}$$

Where $c_l$ the maximum LT is cost which in this situation, LT is minimum and if LT grows then LT cost is reduced.

## 4. Genetic algorithm

To optimize the proposed methodology, one of the best-developed metahuiristic algorithms namely genetic algorithms (GA) have been provided. Since the most key factor of proposing metahuristic algorithms are chromosome representation, the chromosome structure is as Fig.12.

| | | | |
|---|---|---|---|
| *P* | *w* | *L* | *Q* |

**Fig. 12. Structure of chromosome**

where p, w, L, and Q are price, warranty length, lead time and quality level respectively. It should be mentioned that all genes are related to the fuzzy system input parameters. In the following subsections, the steps of GA are illustrated.

### 4.1. Initial population

The random policy is considered to generate initial population. In fact, a random number between maximum and minimum of input parameter is produced. In order to evaluate the chromosomes, fitness function value is considered as maximizing profit (Eq.8).

### 4.2. Parents' selection

Rank based proportional selection and Roulette wheel method are used for choose parents in order to produce new generation.

### 4.3. Crossover operator

In this case, uniform crossover is used for mating chromosomes. To do so, after determining parents randomly according to the number of necessary new generation, each time two parents are chosen and thus, the genes which should be exchange are specified by using a random binary number as long as the number of each chromosomes' genes. To do so, the genes are changed that its binary number is one (Fig 13).

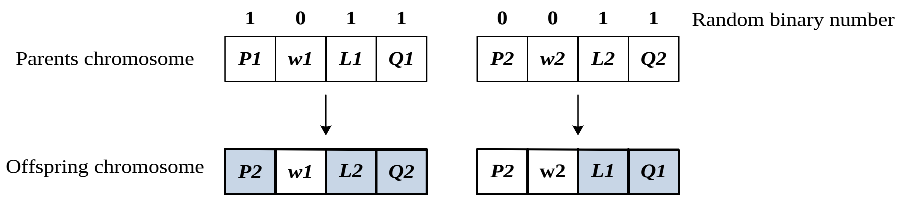


**Fig. 13. A sample of cross over**

### 4.4. Mutation operator

To mute, several members of population are selected and one gene of each member is changed randomly.

### 4.5. Replacement

We use the tournament replacement to place produced children in population. A small group of population is chosen each step and the two of worst members of this group are replaced with two produced children.

### 4.6. Hybrid solution of genetic algorithm and fuzzy expert system

The procedure of optimization is such as flowchart that shown in Fig 14. At first the initial population of solution is generated and these solutions are converted to related fuzzy sets in order to be inputs for fuzzy expert system. The output of fuzzy expert system is the amount of demand respect to each order of P, W, L, and Q. Then the value of demand with the other variables values is entered to the GA to analyze the objective function and produce the new values of P, W, L, and Q. This process is repeated till the stopping criterion is satisfied.

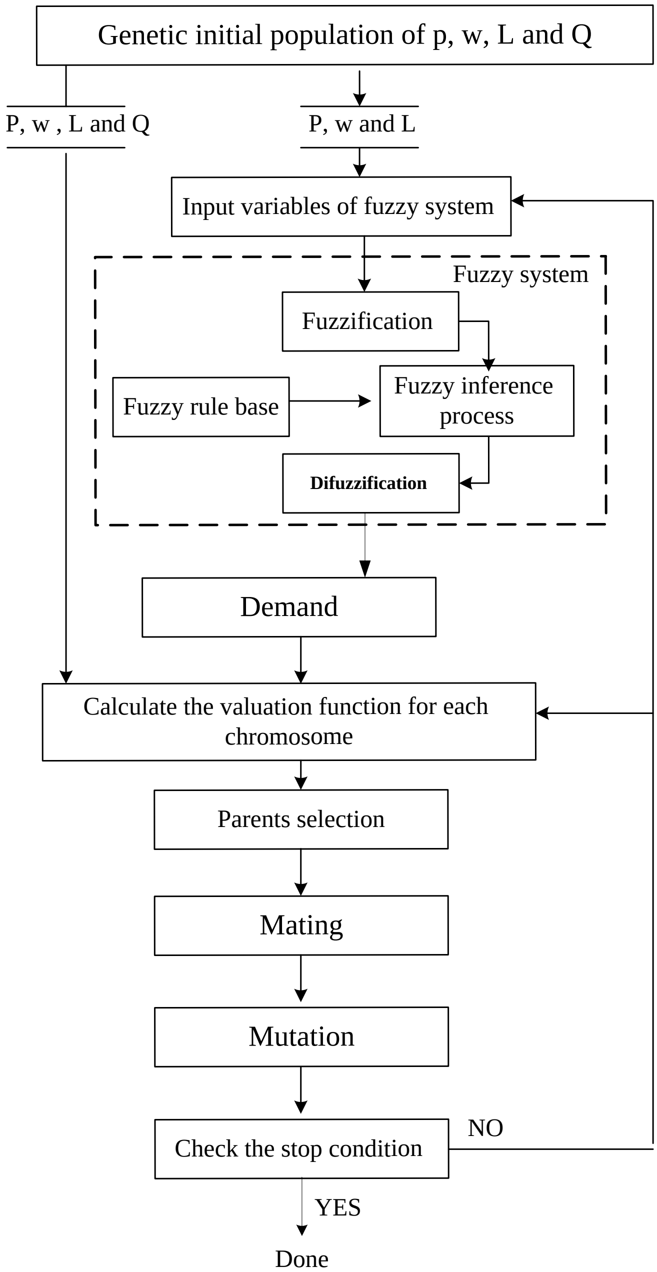


**Fig. 14. The flow chart of hybrid solution of GA and fuzzy system**

## 5. Numerical results

In this section, we study numerical examples to show the performance of proposed solution approach.

### 5.1. Numerical example of fuzzy expert system

The rule base which was designed in table 2 is used for our case study. Assume that partitions of universe sets of price, warranty length, Lead time and demand are such as Fig15 and inputs value for numerical example are as statement 15:

$$[p, w, L] = [1200, 18, 9.5] \quad (15)$$

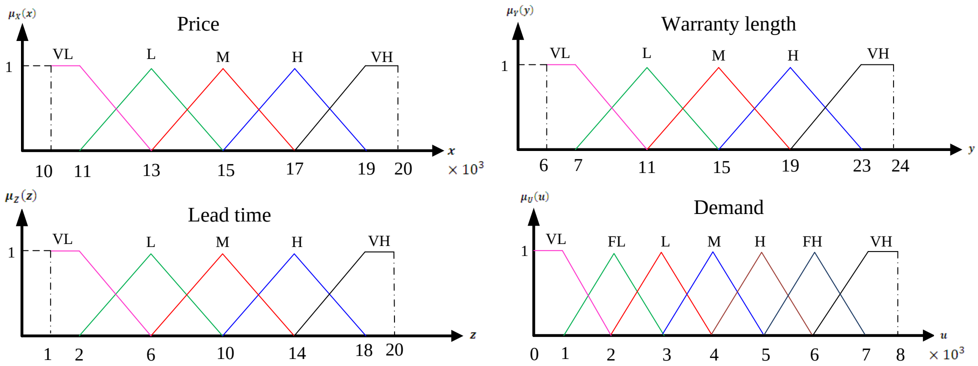


**Fig. 15. Partition of universe sets**

Two fuzzy sets involve with inputs value of each input parameter and then 8 rule of rule base is activated that are shown in table 3. By putting input value of p, w and L in equation 2, their membership degree $(h_j, j = 1,2,3)$ are obtained and then we choose matching degree for each active rule $\alpha_i = min(\{h_{i1}, h_{i2}, h_{i3}\}; \ i = 1, \ldots, 8$.

**Table3: Analyzing the results of numerical example**

| rule number | IF part | | | Then part | $\alpha_i$ | $u_a$ | $u_b$ | $\bar{u}$ | $(\alpha_i \times \bar{u}) \times 10^3$ |
|---|---|---|---|---|---|---|---|---|---|
| | P | w | L | D | | | | | |
| 8 | VL | M | L | FH | 0.125 | 5.125 | 6.875 | 6 | 0.75 |
| 9 | VL | H | L | FH | 0.125 | 5.125 | 6.875 | 6 | 0.75 |
| 13 | VL | M | M | FH | 0.25 | 5.25 | 6.75 | 6 | 1.5 |
| 14 | VL | H | M | FH | 0.5 | 5.5 | 6.5 | 6 | 3 |
| 33 | L | M | L | FH | 0.125 | 5.125 | 6.875 | 6 | 0.75 |
| 34 | L | H | L | FH | 0.125 | 5.125 | 6.875 | 6 | 0.75 |
| 38 | L | M | M | H | 0.25 | 4.25 | 5.75 | 5 | 1.25 |
| 39 | L | H | M | H | 0.5 | 4.5 | 5.5 | 5 | 2.5 |

Next, the result of implication $(u_i'; i = 1, \ldots, 8)$ are utilized with using max-min operator. Figure 16 shows Mamdani implication for first active rule. Finally, we aggregate individual result $(u_i')$ and defuzzify the total result to estimate the demand using equation 16 (See table 3).

$$D = \frac{\sum_1^8 \alpha_i \times \bar{u}}{\sum_1^8 \alpha_i} = \frac{11.25 \times 10^3}{2} = 5625 \quad (16)$$

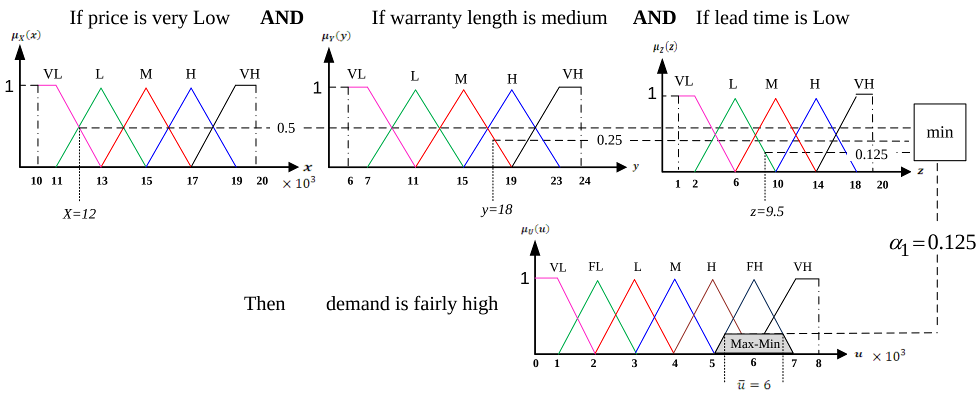


**Fig. 16. Mamdani implication for 1th of active rule of numerical example**

## 5.2. Numerical example of profit optimization

We continue previous example to find best value of price, warranty length, and lead time so that optimizing the profit (Eq.8). Assume that $c_m = 100$ ,$\beta = 1.2$, $c_w = 30$, k=1,$c_0 = 500$, $c_q = 9000$ and $c_l = 8000$. The genetic algorithm and fuzzy expert system performed conjugate such as flowchart of figure 14 and results are shown in table 4.

**Table 4: Result of hybrid solution of GA and fuzzy system performance**

| Run number | profit | P | w | L | Q | D |
|---|---|---|---|---|---|---|
| 1 | 36101425 | 14376 | 12.33 | 8.24 | 0.33 | 4797 |
| 2 | 35795599 | 16370 | 13.26 | 8.55 | 0.36 | 3839 |
| 3 | 35759292 | 16823 | 20.6 | 8.3 | 0.34 | 4428 |
| 4 | 36642655 | 16558 | 22.96 | 5.3 | 0.46 | 5206 |
| 5 | 35760838 | 18229 | 22.97 | 8.54 | 0.41 | 3868 |
| 6 | 35305447 | 16447 | 18.88 | 8.68 | 0.45 | 4299 |
| 7 | 36696871 | 16789 | 22.76 | 6.29 | 0.5 | 4898 |
| 8 | 38008403 | 17056 | 22 | 6.6 | 0.47 | 4792 |
| 9 | 37790340 | 17185 | 23 | 6.33 | 0.48 | 4794 |
| 10 | 37431503 | 17004 | 21.02 | 7.95 | 0.46 | 4502 |
| 11 | 36085272 | 14456 | 12 | 8.09 | 0.31 | 4788 |
| 12 | 37265946 | 14662 | 6 | 4.99 | 0.19 | 4208 |
| 13 | 37932390 | 14989 | 6 | 6.01 | 0.22 | 3995 |
| 14 | 36159318 | 14669 | 11.67 | 7.91 | 0.21 | 4564 |
| 15 | 37915045 | 17131 | 22.92 | 5.73 | 0.49 | 4924 |

As it is indicated in table 4, run number of 8 has obtained maximum profit and the best answer for price, warranty length, and lead time parameters. Genetic algorithm performance isotropy diagram is shown in figure 17.

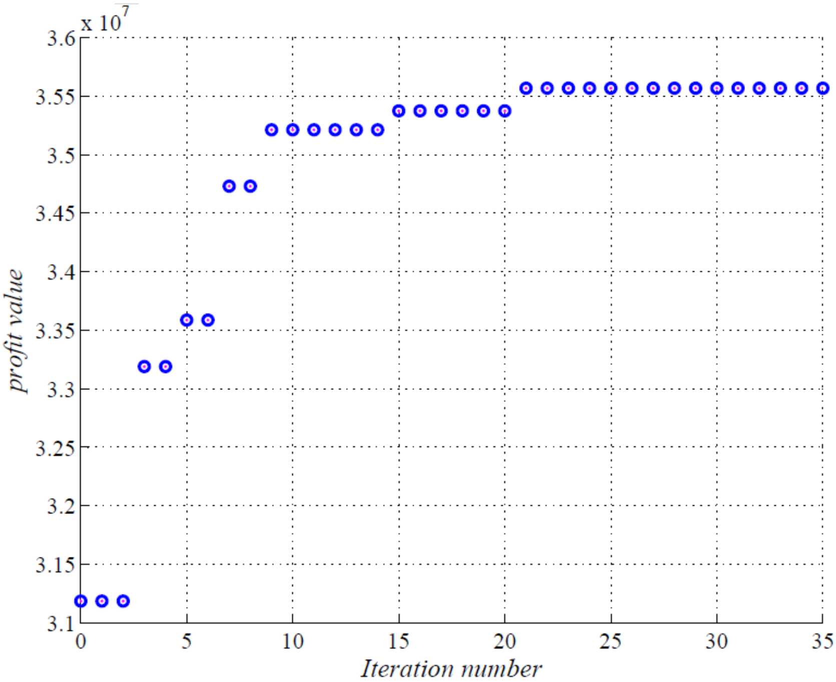


**Fig. 17. Convergence plot of the GA algorithm**

## 6. Discussion and conclusion

One of the most important issues in optimizing the performance of companies is estimating the amount of demand according to effective parameters. The more effective parameters are price, warranty duration, quality level and lead time to deliver the customers orders. Considering a function to estimate the demand based on these parameters is a great challenge for companies. Developing a mechanism to estimate the demand in a more effective and appropriate approach is very valuable for companies. In this article we presented a fuzzy expert system to estimate the demand considering price, lead time, warranty, and quality level. To optimize the decision variables price, lead time, warranty, and quality level we proposed a hybrid solution method integrating the fuzzy expert system and a genetic algorithm.

This research can be extended by considering the other characteristics of companies in supply chain and customer related parameters.

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