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THE IMPACT OF STRUCTURAL CHANGE ON THE CALIBRATION OF INTEREST RATES MODELS IN TAIWAN

Abstract. *Structural change makes the underlying time series process violate the mean-reversion property. This paper proposes the Narayan and Popp (2010) model to determine the structural break date for interest rate data in Taiwan. The break date coincides with the well-known financial crisis at the end of 2008. The empirical results in this paper show that the calibration model with the dataset after the structural break date captures the downward trend and mean reversion pattern better than with the whole historical dataset. Overall, if the interest rate is indeed nonlinear but is modeled on a simple mean reversion model, the resulting interest rate scenarios can be biased and inaccurate. Because of such a structural break nature, risk analysts must be cautious when specifying a time series process for consistent market interest rate scenarios.*

Keywords*: Calibration, financial crisis, mean reversion, unit root tests, structural change.*

JEL classification: C32; C33

1. Introduction

To reveal the economic value of insurance policies, modern risk management Solvency II requires insurers to employ risk-based capital management. For this purpose, insurers will have to choose between using the Solvency II standard formula or building their own internal model to create interest rate models that assess the market risk. The mean reversion property of the term structure of the interest rate plays an important role in the modeling of the market interest rates, especially in the life insurance industry, which sells long-term contracts. For example, the Vasicek (1977) and CIR (1985) models are two important and widely used models of short rates. In practice, the American Academy of Actuaries (AAA) adopts the mean reversion stochastic interest rate models to implement a liability adequacy test with C3-Phase III interest rate scenarios[1]. The standard formula for solvency capital requirements (SCR) proposed by the German GDV[2] suggests the Black-Karasinski (BK) model assuming mean reversion for the logarithm of 10-year government bonds. Similarly, the Actuarial Institute in Taiwan adopts the term structure scenarios based on CIR (1985) stochastic short rate models which are the typical and representative models with a mean reversion property[3].

The mean reversion property is fundamental in the theoretical models and scenario generators of interest rates. However, many practical issues and empirical studies challenge this. For example, the central bank may employ loosened monetary policies to stimulate economic growth. Conversely, a strict monetary policy may be used to slow down economic growth. Unexpected monetary policies will change the interest rate level abruptly rather than gradually. Figure 1 indicates that there were significant drops in the interest rates at the end of year 2008 due to the Taiwan government's monetary and financial policies during the financial crisis. Academic literature suggests that such a structural change caused by exogenous

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[1] Refer to the *Report from the American Academy of Actuaries' Economic Scenario Work Group To The National Association of Insurance Commissioners' Life Risk Based Capital Working Group and Life and Health Actuarial Task Force*. Available at http://www.actuary.org/pdf/life/lbrc_dec08.pdf

[2] GDV: Gesamtverband der Deutschen Versicherungswirtschaft e. V; The German Insurance Association.

[3] The technical report is available at http://www.airc.org.tw/newsfiles/ICED.pdf

factors destroys the mean reversion property in the market price process. The broken-trend stationary model was proposed originally by Perron (1989) to examine the effects of the Great Crash in 1929 and the oil-price shock in 1972 on macroeconomic data series. After this, many empirical studies proved that the broken-trend or structural change (break) makes the underlying time series process violate the mean-reversion property. This paper studies this crucial component of the model from a different perspective by focusing on the structural break or change of the interest rate process.

As for methodologies, recent econometric studies have found that conventional unit root tests, such as the Augmented Dickey and Fuller (1981, ADF), the Phillips and Perron (1988, PP) and the Kwiatkowski et al. (1992, KPSS) tests, have lower power against stationary alternatives. Perron (1989) argues that if there is a structural break, the power to reject a unit root decreases when the stationary alternative is true and the structural break is ignored. Accordingly, Zivot and Andrews (1992, ZA), Perron (1997) and Lumsdaine and Papell (1997, LP) account for endogenous structural breaks. However, Lee and Strazicich (2001, 2003, LS) argue that the ZA and LP models do not allow for a break under the null and that Perron (1997) does not model the break as an innovational outlier (IO), which may result in an over rejection of the unit root null. To handle this problem, Lee and Strazicich (2001, 2003, LS) use a minimum Lagrange multiplier (LM) unit root test. Popp (2008) points out that the cause of over rejection is that the parameters associated with structural breaks have different interpretations under the null and alternative hypotheses of testing models. Following Schmidt and Phillips (1992), Narayan and Popp (2010) consider two innovational outlier (IO) type specifications, that is, two breaks in the level and two breaks in the level and slope of a trending data series with unknown break times. The Narayan and Popp (2010) test allows the generation of a new ADF-type unit root test and generates critical values by assuming unknown break dates with correct size and stable power and by identifying the structural breaks accurately. Therefore, this paper adopts Narayan and Popp (2010) test.

This paper uses the Narayan and Popp (2010) test examine whether the market interest rates for the term structure (yield curve) in the Taiwan market are best described by mean reversion models. Narayan and Popp (2010) test enables us to determine the structural break date exogenously and examine whether the market interest rates are mean reverting or not. Empirical results show a significant structural break occurring around the financial crisis at the end of 2008. To examine the impact of structural change, the interest rate model is calibrated with two datasets, that is, all the available data and the recent data after the structural break date. Both for in-sample and out-sample validations, the calibrated models with data after the financial crisis generates a range of interest rate scenarios closer to the historical interest rates than that of the whole data set. The calibration model with the dataset after the structural break date captures the downward trend and mean reversion pattern better than that of the whole historical dataset. Overall, if the interest rate is indeed nonlinear but is modeled by a simple mean reversion model, the resulting interest rate projection can be biased and inaccurate.

The remainder of this paper is organized as follows: Section 2 introduces the structural change models and calibration methodology. Section 3 presents the empirical findings. Finally, Section 4 presents some concluding remarks and economic implications.

2. **Models and Methodologies**

2.1 Narayan and Popp (NP) Models

Perron (1989) argues that if there is a structural break, the power to reject a unit root decreases when the stationary alternative (mean reversion) is true and the structural break is ignored. To handle this problem, following Narayan and Popp (2010), the DGP of a time series y_t is described as:

$$y_t = d_t + u_t, \tag{1}$$

$$u_t = \rho u_{t-1} + \varepsilon_t, \tag{2}$$

$$\varepsilon_t = A(L)^{-1} B(L) e_t, \tag{3}$$

with d_t being the deterministic component, u_t being the stochastic component and $e_t \sim i.i.d.(0, \sigma_e^2)$. It is assumed that the roots of the lag polynomials $A(L)$ and $B(L)$, which are of order p and q, respectively, lie outside the unit circle. The NP unit root test considers two specifications, both for trending data: one allows for two breaks in level (denoted M1 hereafter), and the other allows for two breaks in level as well as slope (denoted M2 hereafter). The IO-type models for M1 and M2 are given as follows, respectively:

$$d_t^{M1} = \alpha + \beta t + \; A \; L \;^{-1} B \; L \;\; e_t \;\; L \;\; \theta_1 DU'_{1,t} + \theta_2 DU'_{2,t} \;\; , \tag{4}$$

and

$$d_t^{M1} = \alpha + \beta t + \; A \; L \;^{-1} B \; L \; e_t \;\; L \;\; \theta_1 DU'_{1,t} + \theta_2 DU'_{2,t} + \gamma_1 DT'_{1,t} + \gamma_2 DT'_{2,t} \tag{5}$$

with $T_{B,i}^{'}$, i = 1,2 denotes the structural break dates and $DU'_{i,t} = I_{\;t > T_{B,i}^{'}}$ and $DT_{i,t}^{'} = I_{\;t > T_{B,i}^{'}} \;\; t - T_{B,i}^{'}$, i = 1,2. The parameters θ_i and γ_i indicate the magnitude of the level and slope breaks, respectively. The inclusion of $A \; L \;^{-1} B \; L \; e_t$ in equations (4) and (5) enables the breaks to occur slowly over time. The unit root test models for M1 and M2 are presented, respectively, as follows:

$$\begin{aligned} y_t = \rho y_{t-1} + \alpha_1^* + \beta_1^* t + \delta_1 DU'_{1,t-1} + \delta_2 DU'_{2,t-1} + \theta_1 D \;\; T'_B \;\;_{1,t} + \theta_2 D \;\; T'_B \;\;_{2.t} \\ + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t \end{aligned} \tag{6}$$

and

$$\begin{aligned} y_t = \rho y_{t-1} + \alpha_1^* + \beta_1^* t + \delta_1 DU'_{1,t-1} + \delta_2 DU'_{2,t-1} \\ + \theta_1 D \;\; T'_B \;\;_{1,t} + \theta_2 D \;\; T'_B \;\;_{2,t} + \gamma_1 DT'_{1,t-1} + \gamma_2 DT'_{2,t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t . \end{aligned} \tag{7}$$

NP tests the unit root null hypothesis of $\rho = 1$ against the alternative hypothesis of $\rho < 1$. Specifically, NP makes use of a sequential grid search procedure comparable to Kapetanios (2005) according to the maximum absolute *t*-value of the break dummy coefficient θ_1 under the restrictions $\theta_2 = \delta_2 = 0$ for M1 and $\theta_2 = \delta_2 = \gamma_2 = 0$ for M2. That is:

$$\hat{T}_{B,1} = \arg\max_{T_{B,1}} \left| t_{\hat{\theta}_1}\left(T_{B,1}\right) \right|. \tag{8}$$

Under the restriction of the first break, $\hat{T}_{B,1}$, NP estimates the second $\hat{T}_{B,2}$ analogously to the first break by:

$$\hat{T}_{B,2} = \arg\max_{T_{B,1}} \left| t_{\hat{\theta}_1}\left(\hat{T}_{B,1}, T_{B,2}\right) \right|. \tag{9}$$

The new ADF-type test is invariant approximately to level and slope breaks in finite samples by means of Monte Carlo simulations.

2.2 Interest Rates Model for Calibration

Following German GDV, the description of the interest rate curve evolution at the one-year horizon, a subset of *n* tenors (or maturities) is used to approximate the full term structure. For simplicity, this paper adopts the Vasicek-type stochastic process for each tenor. To avoid the probability of negative interest rates, the logarithm of the zero coupon bond rate, $r_t^{(i)}$, at each tenor is assumed to follow an Ornstein-Uhlenbeck process:

$$d\ln r_t^{(i)} = \kappa^{(i)}\left(\ln\theta^{(i)} - \ln r_t^{(i)}\right)dt + \sigma^{(i)} dW_t^{(i)},\ i = 1, \ldots n. \tag{10}$$

Meanwhile, with the natural logarithm transformation, each zero curve tenor is mean-reverting and governed by the mean-reversion speed $\kappa^{(i)}$, the mean-reversion level $\ln\theta^{(i)}$, and the volatility $\sigma^{(i)}$. Each tenor is driven by the Wiener processes $W_t^{(i)}$ under the real measure whose codependence structure is captured by the correlation of their Brownian increments: $\left(dW_t^{(i)} . dW_t^{(j)}\right) = \rho_{ij} dt$.

Notably, the model is not calibrated to the market but on the basis of a time series or time calibration, which is also suggested by the GDV (in Annex 8) for the consideration of stability, transparency, simplicity and compatibility. This paper adopts a mean reversion process for the log transformation of the interest rates of six tenors (maturities), namely, 1y, 3y, 5y, 10y, 15y and 20y. All the maturities are not used because a high dimensional multivariate normal process would decrease the accuracy of the simulation. The interest rates of the other maturities can be derived by some interpolation or smoothing techniques. Furthermore, for the interest rates with maturities of over 20 years, the analysts may employ some extrapolation techniques proposed by the GDV.

3. **Numerical Results**

The data used for empirical analysis was the annualized yield to the maturity of the government bond price at a weekly frequency taken from GreTai Securities Market, which is the over-the-counter market in Taiwan, for the period of January 2006 to December 2011. Firstly, three traditional unit root tests were employed, ADF, PP and KPSS tests. Table 1 shows that ADF and PP tests fail at the significant level of 5% to reject the null of non-stationary interest rates for all maturity, whether or not the time trend is present. The KPSS test yields similar results; only the twenty-year maturity can reject the null of non-stationary.

Table 2 demonstrates the estimated and significant tests for the Narayan and Popp (2010) models (M0, M1 and M2). A common break date ranges from September to November in 2008 in response to the global financial crisis at that time. For the tenors 1, 3 and 5 years, allowing for breaks in the trend function, the null hypothesis of a unit root is rejected, implying a regime-specific mean reverting time series process divided by the 2008 financial crisis. Besides, the breaking coefficients of these short tenors are generally significant, which may be caused by some monetary policies on the short term capital market. Furthermore, the coefficients for the regressor y_{t-1} are all less than one and statistically significant, indicating a strong mean reversion pattern after being detrended with the structural break function. Although the estimated time trends are all positive, those are all non-significant.

To exam the effect of the after-break models with the Narayan and Popp

(2010) model, we firstly calibrate the interest rate models of equation (10) separately, based on two datasets: all available data (from Jan. 2006 to Dec. 2010) and the after-break dataset (from Nov. 2008 to Dec. 2010) for the in-sample estimation. Then the out-sample period of 2011 is used for model validation. For simplicity, we choose the last break date, 2008/11/7, according to Table 2.

Tables 3 and 4 summarize the in-sample calibration results for the whole dataset and the post-2008 dataset. In each table, the upper half gives the process parameters for each tenor, while the correlation data is shown in the lower half of each table. The mean reversion speed and volatility are estimated by the maximum likelihood estimation (Brigo, 2009). Correlation estimates are then obtained from the time series of these stochastic increments. Comparing Tables 3 and 4, mean reversion speeds are higher in the post-2008 data than that of the whole data. Conversely, the mean reversion levels are lower in the post-2008 data than that of the whole data. This phenomenon reflects a relatively low and stable interest rate environment in the Taiwan market. It is remarkable that the one-year horizon projection for insurance liabilities based on all available data may incur a too optimistic capital requirement due to a higher interest rate level. Moreover, the low mean reversion speed may produce a large range of yield scenarios, which may not be specific to the current low interest rate environment in Taiwan. The volatility parameters are much the same with the exception of the short-term 1 year tenor. The correlation matrices from the two datasets are similar to each other, but with a significant exception for the 1 and 20 year tenors, 0.029 and -0.023 respectively. As shown in Figure 1, the negative correlation with the after-break data may reflect the reverse correlation between 1 and 20 year tenors in the Taiwan market after the 2008 global financial crisis. Overall, it could be expected that the calibration model based on the after-break data may generate more specific results for the current low interest rate environment in Taiwan.

In addition to the comparison of the estimated parameters, this paper further implements scenario validation. For model validation, 1,000 yield curve scenarios from 2006 to 2010 at a weekly frequency are generated based on the estimated results in Tables 3 and 4, respectively. This paper performs a one-year-shock validation for in-sample validation. For validating the model calibration against these historical shifts, we start by calculating one-year returns, $x_t = r_{t+1y} - r_t$, from the zero rate time series. As shown in Figure 2, every historical one-year shift is plotted, x_t (symbolized as a black circle), against their initial level r_t. For the simplicity of a visual representation, the 5% quantile (red triangle) and 95% (green cross) quantile of the 1,000 scenarios are plotted to correspond to each in-sample time *t* and in-sample r_t. The upper part of the horizontal line at $x_t = 0$ indicates a positive shock or increase in yields, and vice versa. For short-term tenors, 1, 3, and 5 years, the horizontal line of the right panel is higher than that of the left panel. Hence, a higher proportion of the right panel is downward shock, implying that the after-break data is more conservative in terms of the downward shocks than that of the whole data. Furthermore, the downward trend in the one-year shock indicates the mean reversion property, that is, the high interest rate may tend to decrease (negative shock) while the low interest rate may tend to increase (positive shock). Especially for short-term tenors, 1 year, 3 years and 5 years, the generated scenarios from the after-break data fit the downward trend of the one-year shock well, compared to that of the whole data. For the remaining tenors, 10, 15 and 20 years, the patterns of the scenarios from the two datasets are similar but with a narrower range from the after-break data.

For out-sample validation, the realized quarterly-end yield curves in 2011 are used to compare the generated scenarios, as shown in Figure 3. Note that the scale of the interest rate of the left panel (from the whole data) is much larger than that of the right panel (from the after-break data), implying a narrow range of the scenarios generated based on the after-break data, especially for the short-term tenor, such as 1 and 3 years. For example, at the end of 2011/Q1, the interest rates of the one-year tenor calibrated with the whole data disperse from 0.5% up to more than 12%, while that of the after-break data spreads from 0.5% to 1.75%. Note that the short-term interest rate is an important component in the shape of the yield curve. Although the ranges of long maturities, such as 10, 15 and 20 years, are similar for these two datasets, the large difference in the short-term interest rates will generate discrepant term structure scenarios between these two cases. In Figure 3, the reversed term structure scenarios would probably distort some interest-rate-sensitive cash flow projections, such as prepayment, policy surrender and withdrawal. By observing the simulated scenarios (the yield curves with gray color in Figure 3), although both the two methods skew upward, implying an underestimation for the reserve of the liability, the bias of the post-2008 data is much lower than the whole data. For example, at the end of 2011/Q4, the extreme case of the 10-year tenor is about 2.25% from the after-break data, but nearly 4% from the whole data. The skewness could be cured by more complex interest rate models, such as CIR short rate models. However, this is beyond the scope of this paper.

4. **Conclusions**

It is inevitable to determine a dataset, which is more specific to recent economic conditions for the required one-year capital projection, if the economic condition were expected to endure for the next year, such as the low interest rate environment faced by many countries, including Taiwan. When risk analysts are required to select appropriate datasets for model calibration, intuitively, they can generate these with some subjective or qualitative judgments. With the aid of validation by quantitative methods, analysts would feel more confident in using the selected data. For this purpose, this paper proposes a statistical procedure by Narayan and Popp (2010) model to determine the structural break date for the interest rate data in Taiwan. The break date coincides with the 2008 financial crisis. At that time, the Taiwan Central Bank indeed lowered the interest rates to stimulate economic growth.

It is natural for the model with current data to describe the economic environment more specifically than that with all the historical data. However, this paper analyzes what the impact would be if a risk analyst ignores the information of structural change in the underlying market interest risk. The empirical results in this paper show that the calibration model with the dataset after the structural break date captures the downward trend and mean reversion pattern better than that of the whole historical dataset. Such a distinction is important because the interest rate scenarios generated from these two cases could be very different. Overall, if the interest rate is indeed nonlinear but is modeled by a simple mean reversion model, the resulting interest rate scenarios can be biased and inaccurate. Because of such a structural break nature, risk analysts must be cautious when specifying a time series process for market consistent interest rate scenarios.

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Table 1 **Unit Root Test with ADF, PP and KPSS tests**

	Maturity	1y	3y	5y	10y	15y	20y
ADF	Level	-1.119	-0.845	-0.635	-0.716	-1.298	-2.226
	Trend	-1.917	-1.885	-1.809	-2.011	-2.177	-2.418
PP	Level	-1.006	-0.932	-0.91	-0.972	-1.468	-2.226
	Trend	-1.854	-1.979	-2.038	-2.154	-2.311	-2.418
KPSS	Level	0.741***	0.706**	0.704**	0.662**	0.513**	0.295
	Trend	0.125*	0.149**	0.165**	0.186**	0.160**	0.184***

Table 2. Test statistics of unit root tests for Taiwan Treasury Yields (M0)

$$y_t = \rho y_{t-1} + \alpha_0^* + \delta_1 DU'_{1,t-1} + \delta_2 DU'_{2,t-1} + \theta_1 D\ T'_{B\ 1,t} + \theta_2 D\ T'_{B\ 2.t} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t \qquad \textbf{(M0)}$$

	1 year	3 year	5 year	10 year	15 year	20 year
k	0	0	4	4	0	0
ρ	-0.2805** (-4.662)	-0.1950 (-2.960)	-0.2596 (-3.269)	-0.08617 (-1.175)	-0.1042 (-1.758)	-0.1715 (-2.472)
TB_1	2008/3/3	2008/5/2	2007/9/3	2008/9/1	2008/9/1	2008/9/1
TB_2	2008/11/3	2008/11/3	2008/11/3	2008/11/3	2008/11/3	2008/11/3
δ_1	0.09625 (1.779)	-0.06289 (-1.118)	0.05230 (1.165)	-0.1715 (-1.523)	-0.1470 (-1.306)	-0.1482 (-1.063)
δ_2	-0.4826*** (-5.301)	-0.1909 (-1.747)	-0.3095 (-2.966)	0.08785 (0.7244)	0.08574 (0.7564)	0.08022 (0.5811)
θ_1	-0.5324* (-4.180)	0.3730 (3.338)	0.4009 (3.907)	-0.3271 (-3.050)	-0.3539 (-3.140)	-0.3762 (-2.709)
θ_2	-0.6073** (-4.600)	-0.5708** (-4.693)	-0.5627*** (-5.523)	-0.3598 (-2.366)	-0.2863 (-1.837)	-0.2793 (-1.452)

Notes: * denotes 5% significance levels. The number in parenthesis indicates the t-statistic. Critical values at 5% level of significance are obtained from table 3 of Narayan and Popp (2010), respectively. For simplicity, table 2 ignores all the lagging variables.

Table 2. Test statistics of unit root tests for Taiwan Treasury Yields (M1)

$$y_t = \rho y_{t-1} + \alpha_1^* + \delta_1 DU'_{1,t-1} + \delta_2 DU'_{2,t-1} + \theta_1 D\ T'_{B\ 1,t} + \theta_2 D\ T'_{B\ 2.t} + \beta_1^* t + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t \quad \textbf{(M1)}$$

	1 year	3 year	5 year	10 year	15 year	20 year
k	0	0	4	1	0	0
ρ	-0.3609*** (-6.276)	-0.2189 (-3.366)	-0.2906 (-2.851)	-0.5423* (-4.263)	-0.1037 (-1.738)	-0.1673 (-2.417)
TB_1	2008/3/3	2008/5/2	2007/9/1	2007/6/1	2008/9/1	2008/9/1
TB_2	2008/11/3	2008/11/3	2008/11/3	2008/11/3	2008/11/3	2009/1/5
δ_1	-0.01452 (-0.2600)	-0.1059 (-1.815)	0.07861 (1.120)	0.3177 (3.683)	-0.1349 (-1.168)	-0.1823 (-1.960)
δ_2	-0.7378*** (-7.120)	-0.2957 (-2.522)	-0.3223 (-2.977)	-0.2818 (-2.888)	0.1016 (0.8627)	0.07834 (0.8786)
θ_1	-0.6315*** (-5.412)	0.3359 (3.052)	0.4139 (3.881)	0.3378 (3.132)	-0.3428 (-2.974)	-0.3860 (-2.763)
θ_2	-0.6233*** (-5.272)	-0.5883*** (-4.963)	-0.5598*** (-5.448)	-0.6051*** (-5.237)	-0.2855 (-1.821)	0.4741 (3.052)
trend β_1^*	0.006862 (3.987)	0.003254 (2.113)	-0.0009628 (-0.4899)	-0.006061 (-3.344)	-0.0008035 (-0.5391)	0.0006465 (0.3354)

Notes: * denotes 5% significance levels. The number in parenthesis indicates the t-statistic. Critical values at 5% level of significance are obtained from table 3 of Narayan and Popp (2010), respectively. For simplicity, table 2 ignores all the lagging variables.

Table 2. Test statistics of unit root tests for Taiwan Treasury Yields (M2)

$$y_t = \rho y_{t-1} + \alpha_1^* + \beta_1^* t + \delta_1 DU'_{1,t-1} + \delta_2 DU'_{2,t-1} + \theta_1 D\ T'_{B\ 1,t} + \theta_2 D\ T'_{B\ 2,t} + \gamma_1 DT'_{1,t-1} + \gamma_2 DT'_{2,t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + e_t \quad \textbf{(M2)}$$

	1 year	3 year	5 year	10 year	15 year	20 year
k	0	0	5	0	1	2
ρ	-0.3071*** (-5.193)	-0.2538 (-3.537)	-0.3413 (-3.742)	-0.3145 (-3.340)	-0.3519*** (-4.352)	-0.4538 (-3.883)
TB_1	2008/3/3	2008/5/2	2008/1/2	2008/9/1	2008/9/1	2008/9/1
TB_2	2008/11/3	2008/11/3	2008/11/3	2008/11/3	2008/11/3	2009/3/2
δ_1	0.2478 (2.213)	-0.006701 (-0.05409)	0.02679 (0.3391)	-0.3688 (-2.292)	-0.4401 (-4.404)	-0.6107 (-3.402)
δ_2	-0.4704 (-3.267)	-0.2213 (-1.567)	-0.2350 (-2.071)	0.02800 (1.740)	-0.1673 (-1.675)	-0.3081 (-1.740)
θ_1	-0.5687** (-4.703)	0.3103 (2.662)	-0.3561 (-3.241)	-0.4255 (-3.613)	-0.4394* (-4.056)	-0.4884 (-3.542)
θ_2	-0.4210 (-3.018)	-0.4806 (-3.111)	-0.4449 (-3.919)	-0.3734 (-2.320)	-0.5655 (-5.659)_	-0.6342 (-3.002)
γ_1	-0.04822 (-2.306)	-0.04210 (-1.189)	-0.04715 (-3.841)	0.01697 (1.055)	0.1788 (1.789)	0.1174 (1.865)
γ_2	0.05304 (2.536)	0.03940 (1.128)	0.02904 (2.517)	-0.02691 (-1.672)	-0.1959 (-1.960)	-0.1372 (-2.240)
trend β_1^*	0.002585 (0.6315)	0.005625 (1.547)	0.01656 (3.837)	0.007946 (2.225)	0.01101 (3.340)	0.01403 (2.929)

Notes: * denotes 5% significance levels. The number in parenthesis indicates the t-statistic. Critical values at 5% level of significance are obtained from table 3 of Narayan and Popp (2010), respectively. For simplicity, table 2 ignores all the lagging variables.

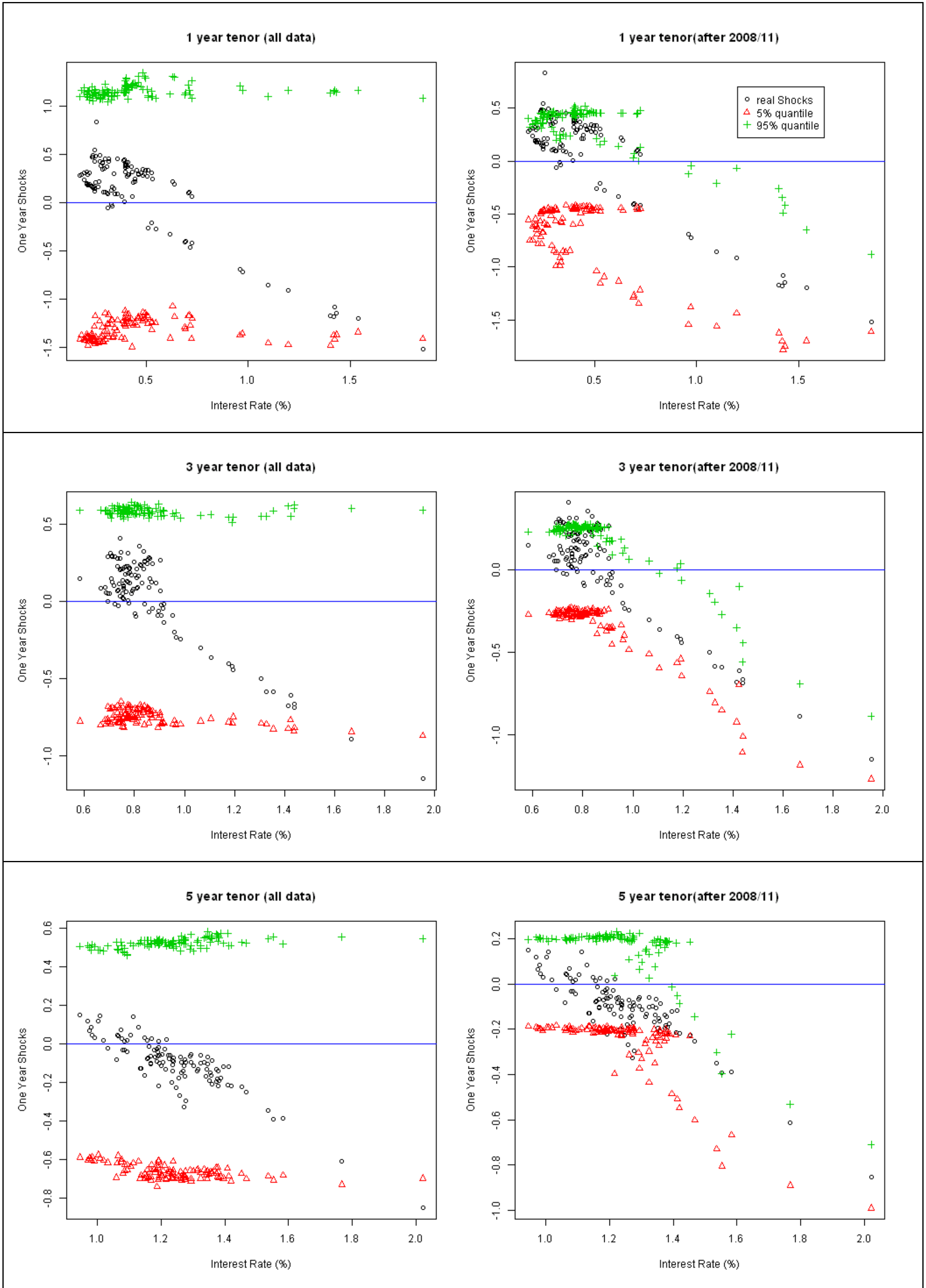
Table 3 Calibration results for Taiwan government curve starting from Jan. 2006 to Dec. 2010

		1y	3y	5y	10y	15y	20y
Process Parameter	Speed	0.479	0.230	0.182	0.460	0.805	1.782
	Level	0.718%	0.862%	1.065%	1.731%	2.038%	2.126%
	Volatility	0.722	0.325	0.258	0.235	0.221	0.269
Correlation Matrix	1y	1.000	0.682	0.168	0.098	0.145	0.029
	3y	0.682	1.000	0.490	0.413	0.387	0.278
	5y	0.168	0.490	1.000	0.754	0.677	0.489
	10y	0.098	0.413	0.754	1.000	0.771	0.580
	15y	0.145	0.387	0.677	0.771	1.000	0.758
	20y	0.029	0.278	0.489	0.580	0.758	1.000

Table 4 Calibration results for Taiwan government curve starting from Dec. 2008 to Dec. 2010

		1y	3y	5y	10y	15y	20y
Process Parameter	Speed	2.559	4.998	6.701	4.073	2.502	4.104
	Level	0.436%	0.853%	1.159%	1.645%	1.877%	1.988%
	Volatility	0.941	0.401	0.268	0.251	0.229	0.296
Correlation Matrix	1y	1.000	0.715	0.192	0.094	0.143	-0.023
	3y	0.715	1.000	0.464	0.349	0.312	0.193
	5y	0.192	0.464	1.000	0.672	0.604	0.493
	10y	0.094	0.349	0.672	1.000	0.712	0.566
	15y	0.143	0.312	0.604	0.712	1.000	0.736
	20y	-0.023	0.193	0.493	0.566	0.736	1.000

Figure 1 Time series plot for the interest rates with maturity 1y, 3y, 5y, 10y, 15y and 20y in Taiwan government bond market.


1 year tenor (all data)
1 year tenor(after 2008/11)
real Shocks
5% quantile
95% quantile
3 year tenor (all data)
3 year tenor(after 2008/11)
5 year tenor (all data)
5 year tenor(after 2008/11)
One Year Shocks
Interest Rate (%)

(Continued)

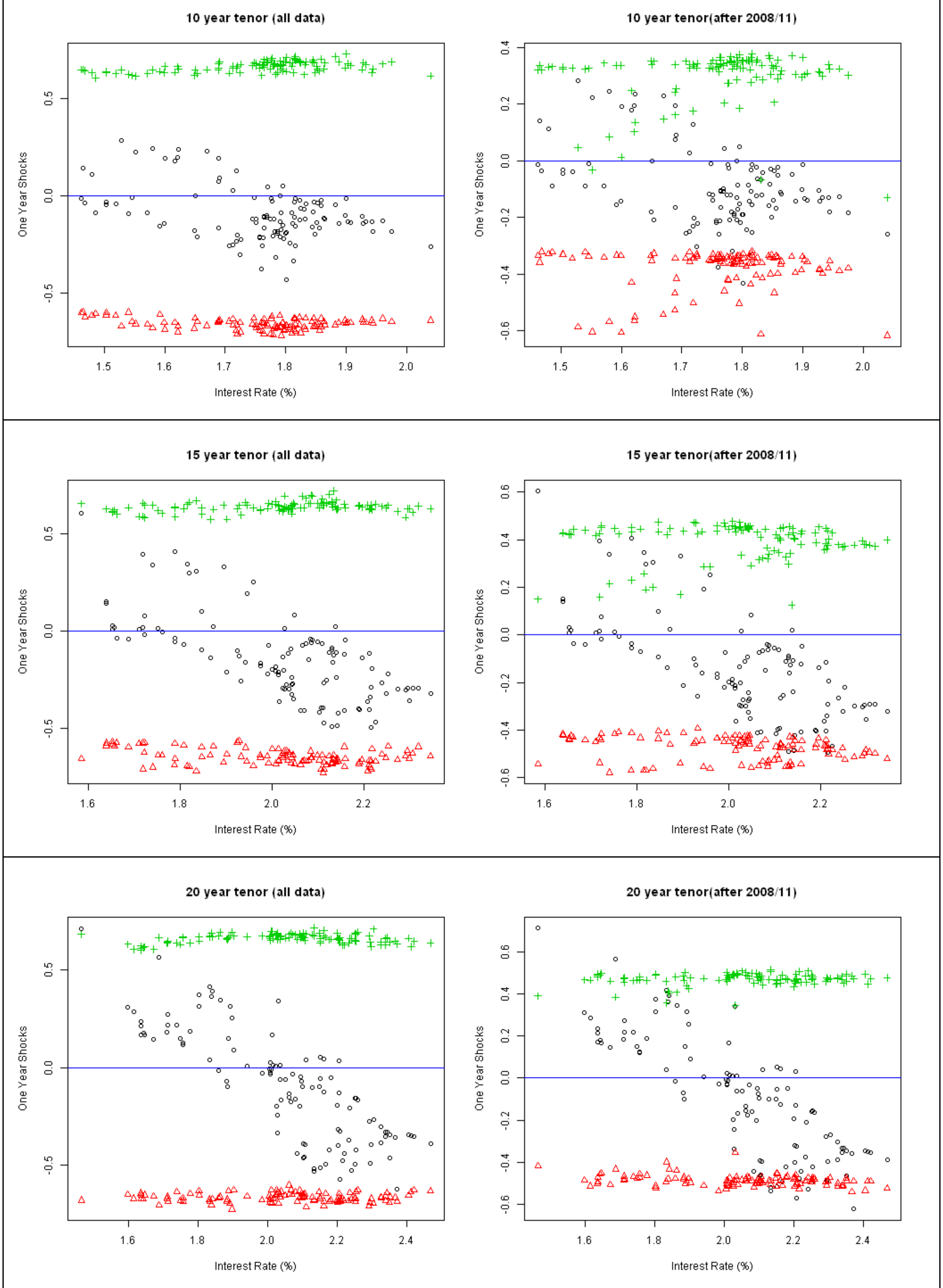
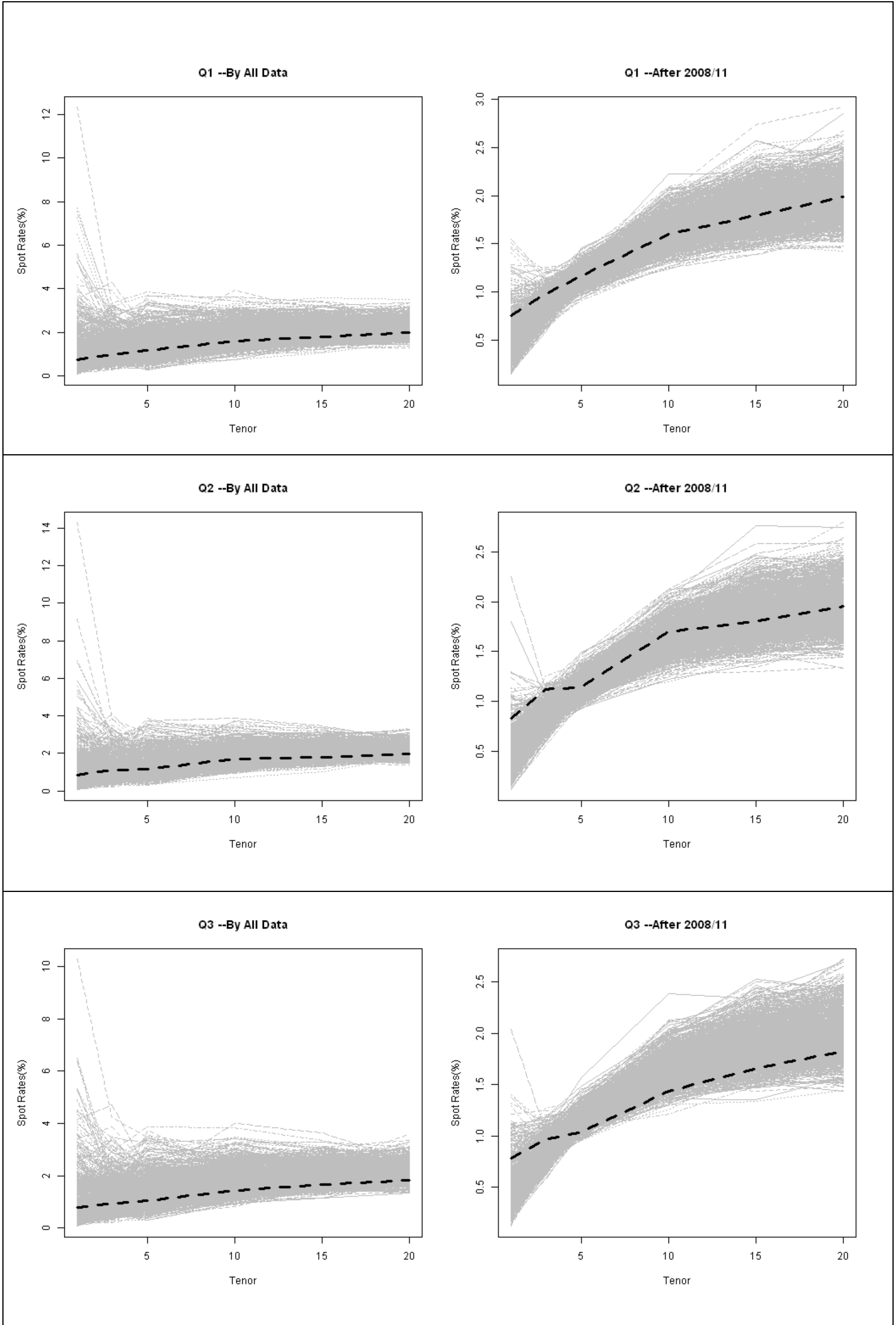


Figure 2 Comparison of One Year Shocks between Whole and After-2008 Datasets


Q1 --By All Data
Q1 --After 2008/11
Q2 --By All Data
Q2 --After 2008/11
Q3 --By All Data
Q3 --After 2008/11
Spot Rates(%)
Tenor

(Continued)

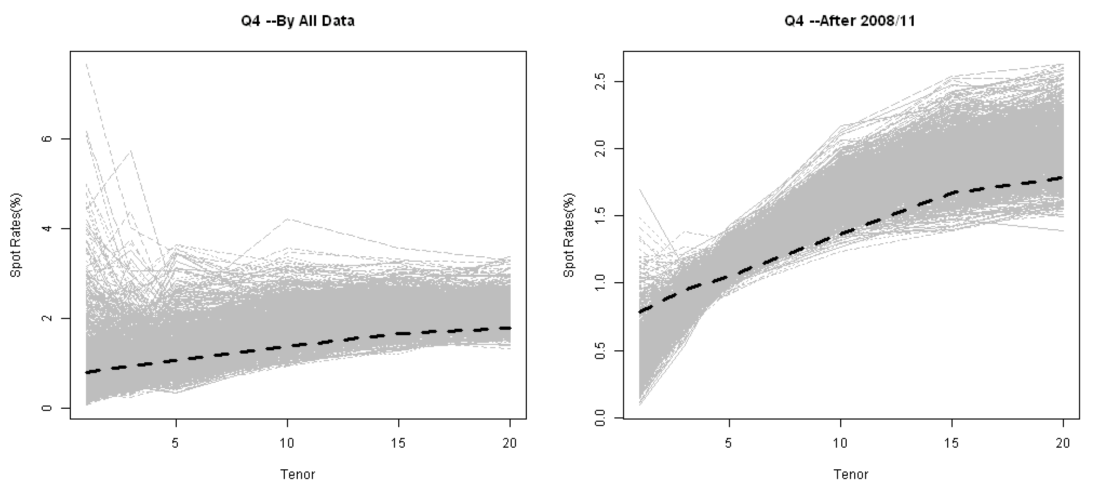


Figure 3 Comparison of Quarterly-end Yield Curves during Year 2011 between Whole and After-2008 Datasets